Thèse de Doctorat de l'Université Pierre et Marie Curie
Specialité: **Physique Théorique**

Présentée par

Stefano Massai

pour obtenir le grade de
Docteur de l'Université Pierre et Marie Curie

Sujet:

NON-SUPERSYMMETRIC SOLUTIONS OF STRING THEORY

Thèse soutenue le 25 Juin 2013 devant le jury composé de:

Michela PETRINI	Présidente du Jury
Joseph CONLON	Rapporteur
Emilian DUDAS	Rapporteur
Mariana GRAÑA	Directrice de Thèse
Nicholas WARNER	Examinateur

Acknowledgments

I would like to thank my advisor Mariana Graña, to whom I am deeply indebted for her constant guidance and for giving me the freedom in choosing the course of this thesis work.

I am grateful to Iosif Bena, Gregory Giecold, Nick Halmagyi and Stanislav Kuperstein for being such nice collaborators and for all the stimulating discussions I had with them.

I thank Joseph Conlon, Emilian Dudas, Michela Petrini and Nick Warner for agreeing to be part of the jury of this thesis.

Many thanks to all my colleagues in Saclay, with whom I discussed about physics and more: Ibrahima Bah, Roberto Bondesan, Oscar Dias, Maxime Gabella, Stefanos Katmadas, Diego Marquès, Ruben Minasian, Andrea Puhm, Sheer El Showk, Piotr Tourkine, Hagen Triendl, Pierre Vanhove, Thomas Van Riet, Bert Vercnocke, and many others.

A special thanks to Enrico Goi, Francesco Orsi and Andrea Puhm, with whom I shared the office and many useful discussions during these years.

I am grateful to the many colleagues who showed interest in my work. Among them I would like to thank the Cornell University theory group for their hospitality during my short visit there. I am grateful to Matteo Bertolini, Nikolay Bobev, Liam McAllister, Paul McGuirk, Luca Martucci, Eran Palti, Gary Shiu, Alessandro Tomasiello and Timm Wrase for the useful discussions about my research. In particular I would like to thank Alberto Lerda for his support and Anatoly Dymarsky for the many challenging discussions about anti-branes.

I acknowledge the generous financial support from the IPhT and I wish to thank Catherine Cataldi, Laure Sauboy and Sylvie Zaffanella for the help with administrative matters.

Finally, a very big thanks to my parents for their support during these years.

And a special thought for Alice, for her constant encouragement and love.

Résumé

Le sujet de cette thèse est l'étude de solutions non-supersymétriques de la théorie des cordes. Leur utilisation est d'une importance fondamentale dans une variété d'applications : dans la correspondance jauge/gravité, pour construire des duals de vides non-supersymétriques et des modèles de "holographic gauge mediation" ; en cosmologie, pour construire vides de de Sitter et étudier le problème de la constante cosmologique ; pour les trous noirs, pour construire leurs microétats. De façon plus générale, il est important d'étudier l'espace des solutions de la théorie des cordes et de comprendre ses structures mathématiques au-delà des simplifications qui découlent de la supersymétrie.

Nous étudions principalement des solutions dans la limite de supergravité. Nous construisons un vaste espace de perturbations non-supersymétriques autour de solutions de supergravité duals à des théories de jauge supersymétriques confinantes, dans quatre et trois dimensions.

Nous procédons ensuite à une étude rigoureuse et détaillée d'une façon particulière de briser la supersymétrie dans les compactifications avec flux, obtenu ajoutant des branes avec une charge de signe opposé par rapport aux flux. Nous découvrons que la solution de supergravité correspondante est singulière, et nous discutons en détail les possibles résolutions en théorie des cordes de cette singularité.

Nous considérons ensuite les conséquences de ces résultats pour l'existence des états non-supersymétriques métastables dans les théories des champs duals et pour l'existence d'un grand "landscape" de vides de de Sitter en théorie des cordes.

Abstract

The subject of this thesis is the study of non-supersymmetric solutions of string theory. Their use is of fundamental importance in a variety of applications: in gauge/gravity correspondance, to construct gravity duals to non-supersymmetric vacua and models of mediated supersymmetry breaking; in cosmology, to construct de Sitter vacua and to study the cosmological constant problem; for black holes, to construct their microstates. More broadly, it is important to study the solution space of string theory and to understand its deep mathematical structures beyond the simplifications which stem from supersymmetry.

We mainly consider solutions in the supergravity limit. We construct a vast space of non-supersymmetric perturbations around supergravity solutions dual to confining supersymmetric gauge theories, in four and three dimensions.

We then proceed to a rigorous and detailed study of a particular way to break supersymmetry in flux compactifications, namely by adding some branes with charge of opposite sign with respect to the fluxes. We discover that the supergravity solution corresponding to these objects is singular, and we discuss in details possible string theory resolutions of this singularity.

We then consider the consequences of these results both for the existence of metastable non-supersymmetric states in the dual field theories and for the existence of a large landscape of de Sitter vacua in string theory.

Table of Contents

Chapter 1

Introduction

String theory is a quantum theory of gravity, which includes gauge interactions in a consistent and elegant way. This great success calls for an intense study of the foundations of the theory as well as its phenomenological applications. Despite a prodigious amount of effort during the last decades, the mathematical structures at the core of string theory are still being developed, and only a very small corner of the space of solutions has been explored.

String theory is well defined in ten space-time dimensions, a striking feature which is responsible for its richness: some of the space dimensions can be realized as small compact manifolds and, after suitable decoupling limits, one is left with a theory of matter and interactions in the non-compact space-time. Depending on the dimension and the shape of the compact "internal" manifold, we can obtain a large space of different theories. In general, the compact space can support various fluxes and to describe the allowed compact manifolds one needs to use advanced techniques in differential and algebraic geometry. The study of these "flux compactifications" is crucial for phenomenology, for example in the construction of inflationary models and de Sitter vacua.

The techniques developed to study flux compactifications can actually be applied even if the internal space is non-compact. While this situation is not of a direct phenomenological relevance, it is important in the context of gauge/gravity correspondence: string theory on the non-compact transverse space is "dual", in a precise sense that we will review in the following, to a field theory without gravity. The interest in this duality comes from the fact that usually a perturbative regime of the theory with gravity corresponds to a strong coupling regime of the field theory, where traditional field theory techniques are often inadequates to obtain accurate results. Viceversa, one can also use field theory techniques to study quantum properties of gravity.

One of the most important tools in the study of solutions of the theory is supersymmetry. In ten dimensions string theory is necessarily supersymmetric, but a general compactification can break supersymmetry completely. Nevertheless, requiring that supersymmetry is preserved dramatically simplifies the task of constructing explicit solutions, and it also guarantees stability. However, many interesting problems require non-supersymmetric solutions: inflation, the physics of de Sitter vacua and the cosmological constant problem, black hole mysteries, holography and confinement in QCD. While many models have been proposed, the concrete construction of non-supersymmetric solutions has been very limited up to now.

The goal of this thesis is to construct explicit solutions of the supergravity equations of motion which break supersymmetry. It will deal with supersymmetry breaking deformations of non-compact manifolds, known as conifolds, which have a vast range of applications both in phenomenology and in gauge/gravity correspondance. In particular, one of the main focus of the thesis is the rigorous study of a popular way to break supersymmetry, namely by introducing a brane in a given flux compactification with opposite charge dissolved in fluxes.

These solutions find many applications in different contexts:

Gauge/gravity duality. The interest in constructing non-supersymmetric gravity duals is to eventually unveil a gravity description of QCD, the theory of strong forces. We will construct a family of analytic solutions dual to non-supersymmetric theories in four and three dimensions by perturbing gravity backgrounds dual to certain $\mathcal{N} = 1$ supersymmetric gauge theories with interesting properties such as confinement and chiral symmetry breaking. In particular, we will study in great detail the solution corresponding to anti-branes in warped cones, which are widely believed to be dual to metastable non-supersymmetric states in the supersymmetric field theories. In four dimensions, this kind of solutions are also very popular as models of holographic gauge mediation.

String phenomenology. Non-compact conifolds are used as a model of warped throats in string compactifications. Anti-branes in these throats are an essential ingredient to construct a de Sitter vacuum with small cosmological constant, and they are the main evidence on the existence of a large space of de Sitter vacua in string theory, known as the landscape. Anti-branes in warped throats are also widely used as models of brane/anti-brane inflation.

Geometry of string compactifcations. The study of supersymmetric flux compactifications in string theory leads to the unveil of beautiful and elegant mathematical structures, such as those in generalized complex geometry. The first-order equations derived from supersymmetry can be casted in a concise form by using natural variables, which are spinors of an $O(6,6)$-bundle over the internal manifold. We will show how conifold solutions fit into this framework and we will study how the supersymmetric first-order equations are modified for a large class of non-supersymmetric solutions. This is a first step toward a systematic understanding of the geometry of non-supersymmetric compactifications.

Black holes geometries. We will not consider this point in this thesis, but we mention that anti-branes in flux compactifications can be used to construct non-extremal black hole microstates. These are supergravity solutions which are believed to describe the quantum nature of black holes: the singularity of the semi-classical picture is replaced by a macroscopic quantum object of the order of the horizon scale. In the so-called fuzzball approach [136, 32, 154, 11] the constituents are stringy microstate geometries which can be found by various string and supergravity techniques. While many of such microstates are known for supersymmetric and non-supersymmetric but still extremal black holes, the construction of these geometries for non-extremal black holes is much harder. One way is to put anti-branes in an extremal solution, thus breaking supersymmetry much like with anti-D3 branes in the Klebanov-Strassler solution [29, 30].

Let us briefly comment on the above points, by illustrating in details the motivations and the results that will be derived in this thesis.

1.1 Gauge/gravity duality and supersymmetry breaking

The idea of a relation between gauge theory and strings was pioneered by Polyakov [147]. An incarnation of these ideas involves a particular limit in which gravity decouple, and is thus known as gauge/gravity duality. The best understood example of such a gauge/gravity correspondence is the AdS/CFT duality [129, 92, 162] (for a review see for example [2, 113, 35, 130, 145]), which is the conjecture (by now supported by a huge number of checks) that *four dimensional $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory is equivalent to type IIB string theory on $AdS_5 \times S^5$ with N units of 5-form flux.*

The string theory origin of this striking statement is very natural, and descends from the existence of D-branes in the spectrum and their twofold interpretation: on one hand, a D-brane has a description in terms of perturbative open string theory as an hyperplane on which the strings end; on the brane world-volume exists then a confined gauge theory whose excitations are the massless states of open strings ending on the brane. On the other hand, D-branes are sources of closed string states and can be regarded as non-perturbative solitons of the low-energy supergravity equations of motion. By implementing an open/closed duality, one recovers the correspondance between the gauge theory engineered on the brane world-volume and the supergravity solution sourced by the brane.

It is of clear interest to extend this duality to gauge theories with less supersymmetries and which are not conformal, as it is the case for the $\mathcal{N} = 4$ theory. In order to do this, one needs to put the D-branes in non-trivial backgrounds, such as orbifolds or conifolds. An orbifold is a quotient of flat space by a discrete group $\mathbb{Z}_n$; the endpoints of strings on this backgrounds are then matrices which form a

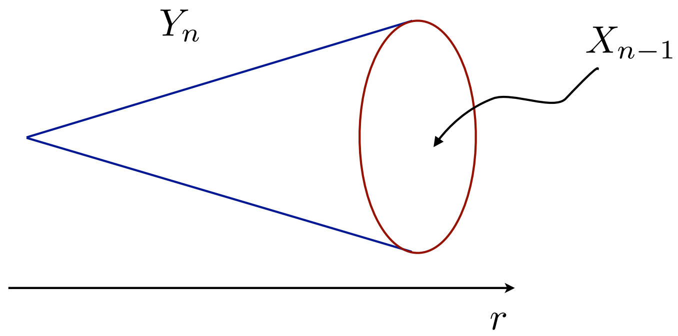


Figure 1.1: A n-dimensional cone Y_n over a compact space X_{n-1}.

representation of this discrete group. As a consequence, there exist branes associated with open strings wich carry irreducible representations of the orbifold group, called fractional branes, which engineer on their world-volume gauge theories with reduced amount of supersymmetry and with non-trivial beta functions (see for instance [62, 112, 34, 36, 35, 105]).

More generally, one can put branes on conifolds, more precisely a Calabi-Yau manifold with conical type singularities [47]. In order to break conformal invariance, one can wrap branes on some cycle of the CY manifold. A fractional D3 brane on an orbifold becomes now a D5 brane wrapped on a 2-cycle inside the CY. The first use of conifolds to extend the Maldacena duality was described by Klebanov and Witten in [117]. They identified the field theory on D3 branes at a conical singularity as the dual theory of type IIB string theory on $AdS_5 \times X_5$, where X_5 is the Einstein manifold base of the cone. For the Klebanov-Witten theory X_5 is a homogeneous space $T^{1,1} = (SU(2) \times SU(2))/U(1)$. Topologically, this space is a product of a three and a two sphere. The corresponding gauge theory is a $\mathcal{N} = 1$ superconformal theory with chiral superfields $A_{1,2}$, $B_{1,2}$ and a superpotential $W = \lambda \epsilon^{ij} \epsilon^{kl} \mathrm{Tr} A_i B_k A_j B_l$. One can generalize this construction to M-theory, where the relevant background is a Calabi-Yau four-fold that is a cone over a compact space known as the Stenzel space [156], which is the quotient $SO(5)/SO(3)$.

In order to break conformal invariance one needs to wrap a stack of M D5 branes on the vanishing cycle of the conifold [114, 118]. In this way we engineer a $\mathcal{N} = 1$ $SU(N) \times SU(N+M)$ gauge theory with a logarithm running of the coupling constant. The renormalization group flow to the infrared was then understood in the work of Klebanov and Strassler [116] (KS in the following). In the supergravity picture we have that the three-form flux sourced by the fractional D3 branes blows up the three cycle of $T^{1,1}$, resolving the singularity of the cone via a geometric transition. The end result is a regular supergravity solution based on the deformed conifold (see Chapter 2, where we will derive these solutions, for more details). The deformation of the cone is the geometrical counterpart of chiral symmetry breaking and confinement in the dual gauge theory.

Having obtained gravity duals to confining supersymmetric gauge theories the next step would be to explore the possibilities to break supersymmetry completely. Chapter 3 is devoted to analyzing this problem. We will introduce a computational tool to handle the second-order equations of motion, provided that the symmetries of the solution are enough to preserve the angular independence as in the Klebanov-Strassler solution. We will then construct a space of analytic non-supersymmetric solutions, obtained as a small (first-order) deformation of the Klebanov-Strassler supersymmetric solution. This set contains solutions dual to theories in which the Lagrangian has been perturbed by operators of various dimensions, corresponding to a particular radial falloff in the ultraviolet region of the supergravity solution. For example, it contains a two parameter family of solutions in which supersymmetry is broken by a small mass for the gauginos.

A particularly important mechanism to break supersymmetry on the deformed conifold, which will be one of the main subjects of this thesis, is to add some sources that do not preserve the supersymmetry of KS. A model proposed by Kachru, Pearson and Verlinde [111] (KPV), uses a stack of anti-D3 branes with world-volume extending along the space-time directions (see also [59]). In a certain limit, in which the interaction between the branes and the background is neglected (known as the probe approximation), and for a particular range of parameters, these anti-branes were found to polarize by the Myers effect [142] into a classically stable expanded NS5 brane source, wrapping a two-cycle on the three-sphere of the deformed conifold (see figure 2.3 and next chapter for more details). This state is quantum mechanically metastable, since it can tunnel to a supersymmetric state with lower energy via non-perturbative bubble nucleation. KPV conjectured that this metastable configuration gives a gravity dual to a metastable non-supersymmetric state in the KS field theory. While adding D3 branes, which are BPS object in KS, gives vacua on the mesonic branch of the theory, the conjecture is that anti-D3 branes uplift the baryonic branch (see [68] for a detailed study of the moduli space of the KS theory).

Chapter 4 and 5 of this thesis contains a detailed study of anti-D3 branes on the deformed conifold. We will first derive and analyze the linearized supergravity solution corresponding to these anti-D3 branes, which should be dual to the metastable state in the field theory. We thoroughly discuss the boundary conditions that correspond to spontaneous supersymmetry breaking, by determining numerically the ratio of the parameters which corresponds to the ratio of confinement scales of the false and true vacuum.

As we will discuss in detail, the study of this system shows however a surprise. The supergravity solution is singular in the infrared region, near the source, and the singularity naively appears to be unphysical. Since in the infrared region the linearized approximation breaks down, we will proceed to solve for the full non-linear backreaction in the near brane region. We find that the singularity, which appear in the three-form fluxes and is not directly sourced by the branes, survive in the full solution.

It is fairly common in string theory to obtain singular supergravity solutions: analogous situations are the enhançon locus in $\mathcal{N} = 2$ gravity duals [108] and the singularity of the GPPZ solution [80] for the $\mathcal{N} = 1^\star$ theory. Studying possible mechanisms which can resolve these singularities usually teaches us deep concepts. In the aforementioned examples, such a mechanism does indeed exist, and it is related to the way strings see the geometry: the appearance of non-commutativity in string coordinates leads to a "fuzzy" geometry which resolves the singularity. In the GPPZ solution, this fuzzy geometry is represented as an expanded brane source which is crucial for the dual interpretation, as described by Polchisnksi and Strassler [146] (PS in the following). We will investigate a similar mechanism, suggested by the KPV probe computation, for the singularity of the anti-brane solutions, extending the PS analysis to a non-supersymmetric setup.

Our result is that the singularity of the anti-D3 solution is not resolved in any obvious way by brane polarization, at least not in the usual Polchinski-Strassler mechanism. We will discuss in detail this puzzle, its possible resolutions and the consequences for the existence of metastable states in the KS field theory.

1.2 String phenomenology

The ability to control supersymmetry breaking in string theory is crucial in the applications of theory to cosmology. Soon after the KPV conjecture, it was realized that the same mechanism can be used to construct a de Sitter compactification. This idea was elaborated in [110] (in the following KKLT). The construction involves two steps: firstly, one needs to find a AdS compactification with all moduli stabilized; then, one put a stack of anti-D3 branes in a highly warped region of the given compactification (see Figure 1.2). This "throat" can be modeled on the deformed conifold solution; the ambient space is thus glued to the ultraviolet region of the Klebanov-Strassler solution and from the field theory perspective corresponds to a UV completion of the KS theory. If the KPV conjecture is true, then the anti-D3 branes will polarize into the NS5 metastable configuration, which is dual to a metastable state in the theory. As we discussed above, the fact that the breaking of supersymmetry is spontaneous translates into the absence of non-normalizable modes in the supergravity solution. This guarantees that when this solution is glued to the ambient compact space, the supersymmetry breaking effects on that space can be made arbitrarily small, essentially by making the throat arbitrarily long. The conclusion seems to be that the anti-branes can contribute by a very tiny amount to the vacuum energy of the given compactification, and thus we can get very generically a de Sitter compactification out of an AdS one. This process is usually referred to as the "uplift" of the AdS compactification.

We stress that, besides the ability to stabilize all the moduli in the initial AdS configuration, the KKLT model essentially relies on the assumption that anti-branes in

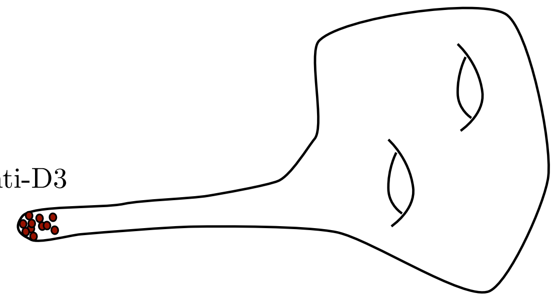


Figure 1.2: Anti-branes in a warped throat are used to break supersymmetry in a controllable way in a flux compactifcation.

warped throat give rise to a non-supersymmetric and arbitrarily long-lived metastable state, and that the breaking of supersymmetry is spontaneous, which translates in the assumption that the supergravity solution corresponding to the metastable state does not have non-normalizable UV modes.

We remark that the uplifting procedure does not solve the hierarchy problem associated to having a dS space with a very small cosmological constant, since one needs to fine tune the length of the warped throat to obtain the experimentally mesured value of $\Lambda \sim 3 \cdot 10^{-120}$ in Planck units. However, even if the KKLT mechanism does not directly help solving the cosmological constant problem, it is at the core of an "anthropic" solution of this problem within string theory. This leads us to the notion of the *string landscape*, introduced by Leonard Susskind in [157]. The idea is that the large number of de Sitter vacua in string theory (obtained by the generic anti-D3 uplift) can be populated by non-perturbative Coleman-De Luccia tunnelling between the different extrema. This has been used to provide a concrete realization of the anthropic solution of the cosmological constant problem [161, 42] (see for example [42] for a review and a list of references). The value of Λ is not regarded as a fundamental physical quantity, but as an environmental quantity which is set statistically, by an appropriate average over the landscape. The basic concept is that the large part of the landscape with big cosmological constant is highly suppressed by the very small probability that life can form in a universe with big Λ. It is important to remember that one usually assumes that string theory has a large landscape of de Sitter vacua, but rigorous constructions are still lacking. While there are different mechanisms to uplift an AdS solution to a dS one (for example F/D-term uplifting [153, 124] and Kahler-uplift [10, 152]), these are generically strongly model-dependent, and thus it is not clear whether they support the existence of a large landscape of vacua. In this sense the KKLT scenario is one of the main evidences for this claim.

In this perspective, our investigation on the existence of metastable vacua in the Klebanov-Strassler theory translates into the question of existence of a large landscape of de Sitter vacua in string theory, and the fact that anti-branes in flux compactifications give singular supergravity solutions is an indication that a rigorous evidence for such large space of vacua is indeed lacking. Moreover, if the singularity is not resolved in string theory, as the results presented in Chapter 5 seem to indicate, it could means that the singularity is a signal of an instability. Various investigations have been performed to elucidate the nature of the singularity. Different setups have been studied, in which one inserts anti-branes of various worldvolume dimensions in flux compactifcations: anti-D6 branes [37, 38, 39, 28, 15], anti-M2 branes [24, 135, 79] (see also Chapter 6), anti-D2 branes [78]. All those examples show the same feature discussed for anti-D3 branes in the KS background, namely an unphysical-looking singularity in the fluxes. For the anti-D6 case, it has also been proven that this singularity is not resolved by brane polarization in string theory [28, 15]. This seems to indicate that there is a universal physical phenomenon underlying the appearance of such singularity, whose nature is still object of debates. It has been suggested that a time-dependent solution will resolve the singularity by showing a perturbative decay toward the supersymmetric solution [40].

It could be interesting to note an analogy with recent investigations about quantum field theory on de Sitter space time. Surprisingly, very little is known about this subject, even if it is of clear direct relevance for cosmology. It has been proposed by Polyakov [148, 149, 121, 150] that interacting particles in de Sitter space create an instability, which should help in solving the cosmological constant problem in a dynamical way by a screening mechanism. For example, the introduction of a coupling like $\lambda\phi^4$ in a scalar field theory on de Sitter space, leads to an explosive particle production, and even in the limit $\lambda \to 0$ one does not recover the free field result. The main reason is a logarithm IR divergency and the breaking of the dS symmetry. These results seem to suggest that pumping energy to obtain a positive cosmological constant will result in an instability of the dS space. It could be interesting to explore whether this general result is related to the appearance of singularities (and instabilities) in the "uplifting" anti-brane backreaction. It is not inconceivable that a solution to the cosmological constant problem will not come from the anthropic landscape argument, but from a more concrete dynamical mechanism which sets an IR/UV mixing. It would be extremely interesting if anti-branes in warped throats could be helpful in this direction.

Another important application of anti-brane supersymmetry breaking is to construct explicit models of inflation in string theory. The idea of considering brane inflation, namely using the attractive potential between branes in a given flux compactification as an inflaton potential, was proposed in [66] and developed in [109] (in the following KKLMMT). This subject has produced a very vast literature, and we refer to some review for a detailed account of references [137, 14, 44, 52]. The basic scenario is to add a D3 brane in a warped throat with a stack of anti-D3 branes. The motion of the D3 branes toward the tip is governed by the inflaton potential of the model. The study of such D3 brane potentials has been intense (see for example [13]) and many different models have been proposed. In general however the presence of an anti-brane as a supersymmetry breaking source is a common ingredient, and thus the fate of anti-branes in a warped flux compactification is of great importance for this class of models. There are principally two problems that can arise in those setups. One is the question of (meta)stability of the non-supersymmetric vacuum obtained by placing anti-branes in the throat. As we discussed before, the inclusion of the backreaction of anti-branes can induce perturbative instabilities and a rigorous computation is needed to prove or disprove the KPV conjecture. Another problem, that we also mentioned, is that even if the anti-branes end up in a long lived metastable state, one should make sure that the supersymmetry breaking contribution is localized in the infrared region of the throat by the warping (in the language of gauge/gravity, this means to have only normalizable UV modes).

This latter property is violated in different supersymmetry breaking setups. For example, in models of axion monodromy inflation [138, 71] which require the presence of a NS5/anti-NS5 brane pair at the end of two long throats, is has been shown in [53] that the backreaction is large in the bulk due to a logarithm grow of a supergravity mode. This means that backreaction effects are not warped down and the scale of the brane/antibrane potential is set by the UV scale (where the throat is glued to the bulk space) and not by the IR scale. This in turn invalidates the model. The mechanism responsible for this behavior is very similar to what happens in a type IIA brane engineering of the Intriligator, Seiberg and Shih (ISS) [106] metastable state of $\mathcal{N} = 1$ SQCD. This model involves D4 and NS5 branes and it reproduces the ISS vacuum in the probe limit (i.e. for $g_s = 0$, the limit in which the branes are rigid). As shown in [16], the backreaction of the branes results in a logarithm bending which destroys the ISS vacuum in the string construction: here again a logarithm mode is responsible for the presence of a non-normalizable mode in the UV. As we will discuss in detail, also in the setup considered in this thesis, namely anti-D3 branes on the KS geometry, there are log modes in the UV. However, whether these modes are dangerous or not is a much more subtle question in cascading theories such as the KS field theory. We will come back to this problem in chapter 4.

1.3 The geometry of non-supersymmetric string vacua

In the previous sections we discussed the importance of non-supersymmetric string compactifications in different contexts: in string phenomenology and string cosmology they provide models of de Sitter vacua and inflation, in the gauge/gravity correspondance they supply solutions which can be used to study metastable vacua in the gauge theory and they are important as a computational tool in models of mediated supersymmetry breaking; they are needed to study microstate geometries of astrophysical non-supersymmetric black-holes.

All this motivates a deeper and formal understanding of string compactifications which break the supersymmetries down to $\mathcal{N} = 0$. While the geometrical structures which arise in the study of supersymmetric compactifications have been intensely studied during the past years, the geometry of non-supersymmetric solutions is a largely unexplored subject.

This is in part motivated by the difficulties in studying supersymmetry breaking solutions. In fact, one of the main simplification which arise from supersymmetry is the saturation of a Bogomol'nyi-Prasad-Sommerfield (BPS) bound, which essentially results in a first-order formulation of the equations of motion: by solving the first-order system found by imposing supersymmetry, one automatically gets a solution of the supergravity equations of motion. The first-order system of supersymmetry equations is also the key to a formal study of the geometry of the solution. As we will review at the beginning of the next chapter, it is possible to reformulate the variations of the fermionic fields in terms of differential forms on the compactification manifold, and the first-order equations can be concisely written as the closure of certain forms under a given differential operator. There is a natural structure which underlies these equations, which is known as generalized complex geometry. This was introduced by Hitchin [104] and refined by Gultieri [91] and it concerns the study of a particular G-bundle over a given manifold $\mathcal{M}$. For a six-dimensional compactifcation manifold we have $G = O(6,6)$, which is the structure group of a bundle which is the direct sum $T \oplus T^\star$ of the tangent and cotangent bundle over $\mathcal{M}$. Generalized geometry unifies two seemingly unrelated geometrical structures: complex and symplectic geometry, which are related in string theory by dualities. Generalized geometry thus provides the natural setup to study string compactifications in full generality. The use of generalized geometry and G-structure techniques in the context of flux compactifcations has been studied by many groups. A partial list of references is [85, 86, 87, 88, 89] and reviews on this subject are [84, 61].

We stress that, besides the inherent beauty and elegance of this formulation, generalized geometry provides very powerful and useful tools to construct explicit solutions in the domains we mentioned above. In fact, most of the techniques of string compactifications are useful if the internal manifold is non-compact as well; this is the case of direct interest for the gauge/gravity correspondance. We will show the powerful of this approach in the next chapter, where we will derive the Klebanov-Strassler solution by using generalized geometry techniques.

It is clear that this formalism is closely related to supersymmetry, and thus the study of general non-supersymmetric compactifications is much more difficult. One can start by asking if there is a subset of non-supersymmetric solutions which share the same integrability properties then supersymmetric flux compactifications; the answer is positive and there exists ways to break supersymmetry where one keeps these integrability properties. In type IIB theories this is for example achieved by turning a particular component of the fluxes [90, 127]; in this case the solution is still described by a first-order system. This configuration is roughly speaking the analogous of what in supergravity are known as "fake" BPS system or extremal non-BPS black holes.

This class of non-supersymmetric compactifications is still a very limited subset of solutions, and for many applications one needs to explore more general solutions. The problem then becomes, if it is possible to find natural variables in order to simplify as much as possible the general second-order equations of motion, and to study the geometrical structures that emerge in this case. The answer to this question is largely unknown.

A first step is to realize that in practical situations, it is often already useful to study small supersymmetry breaking effects around a given and known supersymmetric solution. In this situation we can make use of the natural variables provided by the generalized geometry approach, and use the equations of motion to derive how the first-order description changes in the non-supersymmetric case. In the last chapter of this thesis we will study this problem, and we will use explicit analytic non-supersymmetric solutions based on conifolds to investigate how the first-order equations in the generalized geometry language are modified in the non-supersymmetric case. We will then provide evidence that using an expansion in a small supersymmetry breaking parameter, one can preserve a first-order description for the general background at any order in the series expansion.

Organization of the work

This thesis is organized as follows.

In chapter 2, we give a brief introduction to flux compactification techniques, with the aim of providing a short reference for the following chapters. We also discuss conifold solutions in a somewhat unorthodox way by using pure spinor techniques, we recall their use for the gauge/gravity correspondance and for models of supersymmetry breaking.

In chapter 3 we describe a computational technique to study non-supersymmetric string compactifications perturbatively around a given supersymmetric solution. We then apply these tools to derive a large class of linearized analytic non-supersymmetric perturbations around the Klebanov-Strassler solution.

In chapter 4 we derive the solution for the backreaction of a stack of anti-D3 branes smeared at the tip of the Klebanov-Strassler geometry, in a linear approximation. We discuss in detail anti-brane boundary conditions, and we numerically compute the ratio between the parameters related to the confinement scales of the false and true vacuum. We also discuss a different solution with a new Lagrangian, by adding operators corresponding to small gaugino masses.

Chapter 5 deals with the problem of infrared singularities which are found in the backreacted anti-brane solutions. We first solve for the full non-linear backreaction in the near-brane region, and we then discuss a possible resolution of the singularity via brane polarization.

In Chapter 6 we present a detailed study of the linearized solution corresponding to the backreaction of anti-M2 branes on a warped cone, which is a higher dimensional generalization of the deformed conifold.

We end in Chapter 7 with a collection of results concerning the geometry of the solutions discussed in the previous chapters, in connection to generalized geometry techniques.

Chapter 8 contains a brief conclusion and we add a number of appendices with notations and more technical details.

This thesis is based on the following papers:

1. I. Bena, G. Giecold, M. Grana, N. Halmagyi, S. Massai, *"On Metastable Vacua and the Warped Deformed Conifold: Analytic Results,"* Class. Quantum Grav. 30 (2013) 015003, arXiv:1102.2403 [hep-th].
2. I. Bena, G. Giecold, M. Grana, N. Halmagyi, S. Massai, *"The backreaction of anti-D3 branes on the Klebanov-Strassler geometry,"* submitted to JHEP, arXiv:1106.6165 [hep-th].
3. S. Massai, *"Metastable Vacua and the Backreacted Stenzel Geometry,"* JHEP **1206** (2012) 059, arXiv:1110.2513 [hep-th].
4. S. Massai, *"A comment on anti-brane singularities in warped throats,"* submitted to Phys.Rev.D, arXiv:1202.3789 [hep-th].
5. I. Bena, M. Grana, S. Kuperstein, S. Massai, *"Anti-D3's - Singular to the Bitter End,"* to appear in Phys.Rev.D, arXiv:1206.6369 [hep-th].
6. I. Bena, M. Grana, S. Kuperstein, S. Massai, *"Polchinski-Strassler does not uplift Klebanov-Strassler,"* submitted to JHEP, arXiv:1212.4828 [hep-th].

Chapter 2

Flux compactifications and their use

This chapter gives a brief introduction to the main topics of the thesis. While most of the material covered here is by now standard, we offer a new derivation of the Klebanov-Strassler solution and we present some results that will be used in the following chapters.

2.1 Supersymmetry and generalized geometry

In this thesis we will mainly be interested in the construction of solutions of supergravity theories in different dimensions. These should be viewed as low energy limits of string theory, and the first step toward the understanding of various stringy effects. Constructing a supergravity solution with all possible fields is still an extremely challenging problem, since the source terms in Einstein's equations can be rather cumbersome. The situation gets greatly simplified if one looks for a supersymmetric solution. In this case one should first solve the conditions imposed by supersymmetry, which in general are a set of first-order differential equations. In most of the cases, these BPS conditions imply the equations of motion, thus the task of finding solutions is much simpler for supersymmetry: a first-order system replaces the second-order equations of motion.

This property has been extensively used to construct string compactifications that preserve some supersymmetries, in various contexts: string phenomenology, gauge/gravity duality and black hole solutions. Sometimes, the same structure can be extended to solutions that break supersymmetry: examples are the $(0,3)$-flux supersymmetry breaking [90, 76, 127], the "almost-BPS" black hole solutions [83, 18] and "fake" supergravity flows [70, 56, 50, 107, 159]. However, a vast region of the space of non-supersymmetric solutions remains unexplored, since generically one needs to abandon the hope of a first-order description.

One of the aims of this thesis is to initiate the exploration of general kind of non-supersymmetric solutions in the context of gauge/gravity duality and in string phenomenology. As we will see in the next chapter however, the techniques used to construct supersymmetric solutions are still extremely useful also to solve for a generic non-supersymmetric problem, and we will extensively use these techniques in the following. Thus, in the present section we will very briefly review what is known about supersymmetric flux compactifications.

We will focus on type IIB supergravity, which will plays a major role in this thesis. The field equations for this theory can be derived by the following action in the Einstein frame:

$$\begin{aligned} S_{IIB} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}R - \frac{1}{\kappa^2}\int \Big[d\phi\wedge\star d\phi + e^{2\phi}dC\wedge\star dC + ge^{-\phi}H_3\wedge\star H_3 \\ + ge^{\phi}F_3\wedge\star F_3 + \frac{g^2}{2}F_5\wedge\star F_5 + g^2 C_4\wedge H_3\wedge F_3\Big] + S^{(loc)}\,, \end{aligned} \tag{2.1}$$

supplemented by the on-shell self-duality condition $F_5 = \star F_5$. The general problem is to derive a first-order description for supersymmetric solutions of the type IIB field equations inside a given Ansatz. We are interested in compactifications to four-dimensions, thus we take an Ansatz for the metric which is the most generic one compatible with maximal symmetry in four-dimensions. This Ansatz is a warped product of the form

$$ds_{10}^2 = e^{2\tilde{A}}ds_4^2 + ds_6^2\,, \tag{2.2}$$

where ds_6 is a metric on six-dimensional "internal" manifold $\mathcal{M}_6$, possibly with non–zero fluxes[1]. We will assume for now that ds_4^2 is the Minkowski metric, although the results below can be generalized to an Anti-de-Sitter compactification. The warping $\tilde{A}$ is a function of the internal coordinates. One can perform a reduction of the ten dimensional action (2.1) and derive an effective action for the specific Ansatz (2.2). To find a first-order description for a supersymmetric solution essentially means to perform a "BPS rewriting" of this effective action.

To explain the general idea is better to look at a very simple example. Suppose that we search for a metric ds_6^2 with enough symmetries, in a such a way that the solution will depend on just one internal coordinate. Although this seems a very restrictive assumption, solutions of this kind play an important role in gauge/gravity duality, as we will discuss in the next section. In this situation it is usually possible to perform a reduction of the action to obtain a one dimensional effective action for the dynamics of all the degrees of freedom that participate in the solution. This dynamics can be thought of as a one dimensional motion on a given moduli space, whose coordinates ϕ^a, $a = 1,\ldots,n$ are given by the n degrees of freedom of the problem. Let us assume that the effective action is of the form

$$\mathcal{L} = \frac{1}{2}G_{ab}(\phi)\dot{\phi}^a\dot{\phi}^b + V(\phi)\,. \tag{2.3}$$

[1] We will use the notation $\tilde{A}$ for the warp factor, while the function A is a combination of other metric modes that we will often use in the following chapters.

The equations of motion are simply

$$\nabla_{\dot{\phi}}\dot{\phi} = \mathrm{grad}V\,, \tag{2.4}$$

and if $V = 0$ the solution is a geodesic on the moduli space. It is clear that to find a first-order description of the system, namely a system of first-order ordinary differential equations of the form

$$\dot{\phi}^a = v^a(\phi)\,, \tag{2.5}$$

we need to find a vector field $v = v^a\partial_a$ such that $\nabla_v v = \mathrm{grad}V$. The problem can also be casted in the form of a Hamilton-Jacobi equation. In this case the Hamilton-Jacobi procedure gives a principal function W such that $v = \mathrm{grad}W$. In this case, one can easily show that the effective action can be written in the following BPS form:

$$\mathcal{L} = \frac{1}{2}G_{ab}(\dot{\phi}^a - v^a)(\dot{\phi}^b - v^b) + \frac{dW}{d\tau}\,. \tag{2.6}$$

In general, to find the vector field v or the function W from their defining equations, it is as difficult as to solve directly the second-order system (2.4). This is precisely where supersymmetry plays a crucial role: by looking at the supersymmetry conditions, one gets directly a system of the form (2.5), in an essentially algebraic way. From this point of view, supersymmetry gives one integration of the Hamilton-Jacobi equation.

We can now come back to the general problem. We need to find the natural variables, analogous to the vector field v of the previous example, which permit to use the supersymmetric variation to rewrite the action in a natural first-order, or BPS, formulation. Since a full derivation will require a lengthy explanation, here we limit to state the results of this investigation and we refer the reader to the original references (in particular [84, 87, 88]) for more details.

The starting point are the supersymmetry variations of type IIB supergravity. Since they contain ten dimensional spinors, one need to first decompose the spinors in a 4+6 splitting. The problem is then to get rid of the spinors, and to recast the supersymmetric variations as a first-order system of differential bosonic equations. This means to construct some geometric objects which encode some algebraic structure and which satisfy a first-order equation. Since we are compactifying on a six-dimensional manifold $\mathcal{M}_6$, we expect that such geometrical objects will be sections of a bundle over $\mathcal{M}_6$, on which we can apply a first-order differential operator.

We will now briefly follow the discussion in [87, 88]. The requirements to have an $\mathcal{N} = 1$ supersymmetric compactification involve both algebraic and differential conditions. The supersymmetry variations of type IIB supergravity (we will mainly consider the type IIB string, analogous results can be derived for type IIA) contain two Majorana-Weyl spinors $\epsilon_{1,2}$ in ten dimensions; since we want to preserve $\mathcal{N} = 1$ supersymmetry in the non-compact four dimensional space, we need a conserved spinor ζ in four dimensions, and thus we decompose

$$\epsilon^{1,2} = \zeta_+ \otimes \eta_+^{1,2} + \zeta_- \otimes \eta_-^{1,2}\,, \tag{2.7}$$

where $\eta^{1,2}$ are two six-dimensional Weyl spinors. If these two spinors are just parallel, we get as an algebraic condition the existence of a nowhere vanishing spinor on $\mathcal{M}_6$. This condition is equivalent to a reduction of the structure group $SO(6)$ of the tangent bundle of $\mathcal{M}_6$ to the stabilizer of η, namely $SU(3)$. This case contains the well known example of a Calabi-Yau manifold: in this case the spinor η is also covariantly constant, $\nabla\eta = 0$ and it is the simplest example of a compactification besides the torus. An $SU(3)$ structure can be defined in an equivalent way in terms of differential forms; indeed, there is a correspondance between the pair (g, η), where g is the metric on $\mathcal{M}_6$ and a pair (J, Ω), where J is a real two-form and Ω a volume form:

$$(g, \eta) \leftrightarrow (J, \Omega)\,. \tag{2.8}$$

This is a considerable progress since the explicit form of the metric is generically unknown. We now search for a more general connection between reduction of the structure group and differential forms, also valid in the general case in which the two spinors $\eta^{1,2}$ can have arbitrary relative orientation. It is easy to realize that the tangent bundle T is not the natural framework to study this general case. This is because the bundle of differential forms $\Lambda^\bullet(T^\star\mathcal{M}_6)$ carries a natural representation of the Clifford algebra Cliff$(6,6)$. This algebra is associated to a bundle over $\mathcal{M}_6$ which is the sum of the tangent and cotangent bundle $T \oplus T^\star$, with structure group $O(6,6)$. The study of complex structures on such bundles is the subject of what is known as generalized complex geometry and it was introduced by Hitchin and Gualtieri [104, 91] as a way to unify complex and symplectic structures on T. In this language, one can encode the metric, the B-field and the spinors $\eta^{1,2}$ as a pair of Cilff$(6,6)$ spinors $\Phi_\pm$ which are compatible and pure. Here $\pm$ refers to the chiralities, so that $\Phi_\pm \in \Lambda^{even/odd}(T^\star\mathcal{M}_6)$. One can also show that a pair of compatible pure spinors corresponds to a reduction of the structure group of the generalized bundle from $O(6,6)$ to $SU(3) \times SU(3)$. In terms of the two spinors $\eta^{1,2}$, in general one can write

$$\Phi_\pm = e^B \eta^1_+ \otimes \eta^{2\,\dagger}_\pm\,. \tag{2.9}$$

If $\mathcal{M}_6$ has $SU(3)$ structure, there is a natural set of pure spinors, given by the pair (J, Ω) as follows:

$$\Phi_- = -i\Omega \qquad \Phi_+ = e^{-iJ}\,. \tag{2.10}$$

Since in this thesis we will mainly focus on manifolds with $SU(3)$ structure, it is interesting to classify them. We start by the simplest example of a manifold with $SU(3)$ structure, namely a Calabi-Yau manifold. In this case the Levi-Civita connection has a $SU(3)$ holonomy and the forms J and Ω are closed. In the general case, one can define a connection with $SU(3)$ holonomy which is not torsionless; the torsion is a mesaure of the failure to satisfy the integrability condition of the Calabi-Yau manifold. One usually decompose this torsion in $SU(3)$ representations, as follows:

$$dJ = -\frac{3}{2}\text{Im }(\mathcal{W}_1\bar{\Omega}) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \qquad (2.11)$$
$$d\Omega = \mathcal{W}_1 \wedge J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5 \wedge \Omega\,.$$

The forms $\mathcal{W}_i$ are known as torsion classes and they provide useful information on the geometry of a given manifold. For example, a complex manifold is characterized by the necessary and sufficient condition $\mathcal{W}_1 = \mathcal{W}_2 = 0$. We will discuss more details about the intrinsic torsion in Chapter 7.

This conclude the brief survey on the algebraic conditions imposed by supersymmetry on a given compactification. We now come to the differential part, namely the first-order conditions on the metric and the fluxes which ensure supersymmetry, and give a first-order formulation of the equations of motion. We skip the technical details of the derivation, which can be found in great clarity in Appendix A of [88]. For the type IIB theory, $\mathcal{N} = 1$ supersymmetry on a warped Minkowski 4D compactification requires:

$$e^{-3\tilde{A}+\phi} d_H \left[e^{3\tilde{A}-\phi}\Phi_-\right] = 0 \qquad (2.12)$$
$$e^{-3\tilde{A}+\phi} d_H \left[e^{3\tilde{A}-\phi}\Phi_+\right] + d\tilde{A} \wedge \bar{\Phi}_+ + e^{\phi} \star_6 \lambda(F) = 0\,,$$

where d_H is the H-twisted differential given by $d_H\bullet = d\bullet - H\wedge\bullet$ (H being the NS-NS three form), F is a polyform constructed from the sum of RR fluxes: $F = F_1 + F_3 + F_5$, λ is the following transposition

$$\lambda(X) = \sum_n (-)^{[(n+1)/2]} X_n \qquad (2.13)$$

and ϕ is the dilaton. The main advantage of this pure spinor approach in deriving the conditions imposed by supersymmetry is that they provide directly a set of first–order differential equations, without the need of explicitly computing the supersymmetry variations of the supergravity fields.

The fact that this first-order system implies the supergravity equations of motion can be proved from integrability arguments [119]. One can also show that the pure spinors $\Phi_\pm$ are the natural variable in which a BPS rewriting of the action is possible, and roughly speaking they reduce to the Hamilton-Jacobi result (2.5) in a cone-like compactification. We will come back to these ideas in chapter 7.

2.2 Conifolds and gauge/gravity duality

In this section we would like to show the powerful of the pure spinor equations described above also in the case that the internal manifold is non-compact. Generalized geometry techniques indeed prove very useful to derive solutions of interest for the gauge/gravity duality, where the six-dimensional compactification manifold is a cone Y_6 over a compact five-dimensional base X_{n-1} (see Figure 1.1). The radial direction plays a crucial role, since it is dual to the renormalization group flow in the gauge theory [80, 74, 155]. In this situation, the solution usually preserves enough symmetries so that all the degrees of freedom depend only on the radial coordinate. The supersymmetry equations then simplify considerably and one can hope to find explicit analytic solutions.

We will now apply the machinery described in the previous section to derive the Klebanov-Strassler solution, which is the gravity dual to a confining $\mathcal{N}=1$ gauge theory, first obtained in [116]. We begin by a brief review of the geometry of conifolds; for more details we refer to the reviews [101, 102]. The conifold is described in $\mathbb{C}^4$ by the equation

$$\sum_{n=1}^{4} z_n^2 = 0\,. \tag{2.14}$$

The metric can be written in the following way:

$$ds_6^2 = dr^2 + r^2 ds^2_{T^{1,1}}\,, \tag{2.15}$$

where the space $T^{1,1}$ is the coset space $T^{1,1} = (SU(2)\times SU(2))/U(1)$ and its metric is

$$ds^2_{T^{1,1}} = \frac{1}{9}\Big(d\psi + \sum_{i=1}^{2}\cos\theta_i d\phi_i\Big)^2 + \frac{1}{6}\sum_{i=1}^{2}\Big(d\theta_i^2 + \sin^2\theta_i d\phi_i^2\Big)\,, \tag{2.16}$$

where ψ has range in $[0,4\pi]$ and (θ_i,ϕ_i) parametrize two S^2's. The metric is thus an S^1 fibration over $S^2\times S^2$; the topology of this bundle is $S^2\times S^3$. It is useful to introduce the following basis of one-forms on $T^{1,1}$ [141]:

$$\begin{aligned}
g_1 &= \frac{1}{\sqrt{2}}\Big(-\sin\theta_1 d\phi_1 - \cos\psi\sin\theta_2 d\phi_2 + \sin\psi d\theta_2\Big)\,, \\
g_2 &= \frac{1}{\sqrt{2}}\Big(d\theta_1 - \sin\psi\sin\theta_2 d\phi_2 - \cos\psi d\theta_2\Big)\,, \\
g_3 &= \frac{1}{\sqrt{2}}\Big(-\sin\theta_1 d\phi_1 + \cos\psi\sin\theta_2 d\phi_2 - \sin\psi d\theta_2\Big)\,, \\
g_4 &= \frac{1}{\sqrt{2}}\Big(d\theta_1 + \sin\psi\sin\theta_2 d\phi_2 + \cos\psi d\theta_2\Big)\,, \\
g_5 &= d\psi + \cos\theta_2 d\phi_2 + \cos\theta_1 d\phi_1\,.
\end{aligned} \tag{2.17}$$

The metric then reads

$$ds^2_{T^{1,1}} = \frac{1}{9}g_5^2 + \frac{1}{6}\sum_{i=1}^{4} g_i^2\,. \tag{2.18}$$

One can add a stack of N regular D3-branes and M fractional D3-branes on the conifold, obtaining a gravity description of an $SU(N)\times SU(N+M)$ gauge theory [114, 118]. The fractional D3 branes are just D5 branes wrapped on the S^2 of

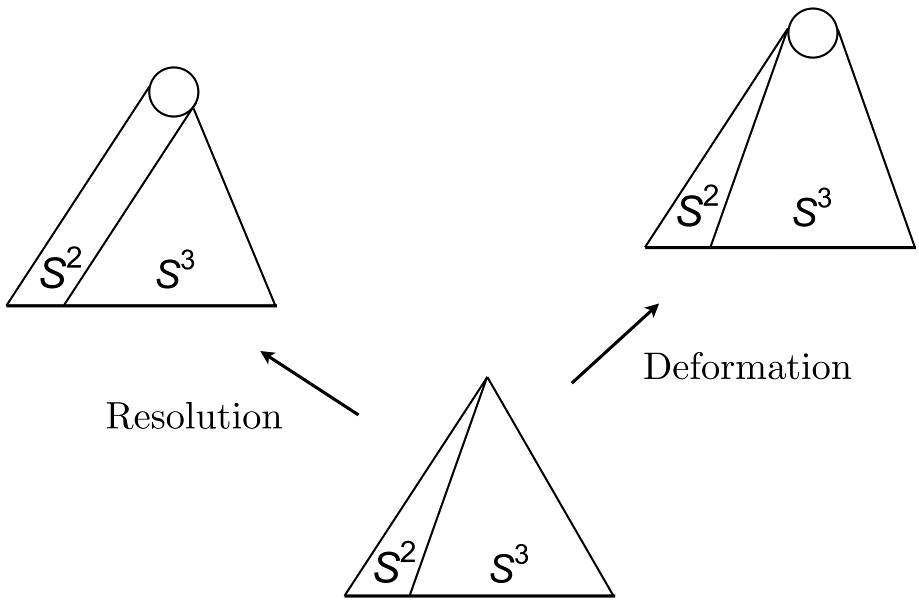


Figure 2.1: The resolution and deformation of the singular conifold.

the conifold, and thus they source a magnetic R-R three-form flux on the S^3, in addition to the N units of five-form flux which come from the regular D3-branes:

$$\frac{1}{(4\pi^2\alpha')^2}\int_{T^{1,1}} F_5 = N\,, \qquad \frac{1}{(4\pi^2\alpha')^2}\int_{S^2} F_3 = M\,. \tag{2.19}$$

The solution of Klebanov and Tseytlin [118] which describes this situation is singular in the infrared. The resolution of this singularity was described by Klebanov and Strassler in [116]. The singularity of the conifold can be repaired in two ways (see Figure 2.1). The first is a deformation in which the apex is replaced by a three-sphere, the second is a small resolution in which the apex is replaced by a two-sphere [47]. The KS solution deals with the deformation of the singularity; an heuristic way to see this is that in the KT solution there are M units of F_3 on the S^3. In the infrared the S^3 shrinks to zero size at the apex of the cone, causing the energy density of the flux to diverge. Clearly, to resolve this singularity one can try to find a solution in which the three-sphere remains of finite size at the apex (deeper reasons actually come from a field theory analysis, which we skip here for simplicity). The deformed conifold is described by the equation

$$\sum_{n=1}^{4} z_n^2 = \epsilon^2\,, \tag{2.20}$$

where we introduced a new parameter ϵ, the radius of the blowed-up three-sphere at the apex of the cone. A Ricci-flat metric on the deformed conifold was derived in [47]; here we will present an heuristic derivation which also includes the fluxes of the Klebanov-Strassler solution, by using the techniques described in the previous section. We start by writing an Ansatz for the fields. As in [116, 144], we need the most general metric compatible with the symmetries of the $T^{1,1}$ space, which are $SU(2) \times SU(2) \times U(1)$. The $U(1)$ symmetry, coming from a shift of the angular ψ coordinate, is broken by the deformation of the conifold down to its $\mathbb{Z}_2$ subgroup acting on the complex coordinates as a reflection $z_k \to -z_k$. The field theory origin of this is the breaking of a $U(1)$ R-symmetry[2]. We thus seek for the most general $SU(2) \times SU(2) \times \mathbb{Z}_2$ symmetric Ansatz for the metric and the fluxes. This was constructed by Papadopolus and Tseytlin (PT) in [144] (they actually wrote a more general Ansatz in which the $\mathbb{Z}_2$ symmetry is completely broken). The solution is parametrized by eight scalars

$$\phi^a = (x, y, p, A, f, k, \phi)\,, \tag{2.21}$$

depending only on the radial direction of the cone τ. The ten dimensional PT metric is:

$$ds^2_{10} = e^{2A+2p-x}\, ds^2_{1,3} + e^{-6p-x}\, d\tau^2 + e^{x+y}\, \left(g_1^2 + g_2^2\right) + e^{x-y}\, \left(g_3^2 + g_4^2\right) + e^{-6p-x}\, g_5^2\,. \tag{2.22}$$

We should require that in the infrared (ie at small τ) this metric approaches a finite size three-sphere, while the S^2 shrinks to zero. The metric on the S^3 is given by

$$d\Omega_3^2 = \frac{1}{2}\epsilon^{4/3}\left(\frac{2}{3}\right)^{1/3}\left[g_3^2 + g_4^2 + \frac{1}{2}g_5^2\right], \tag{2.23}$$

while the S^2 forms are g_1 and g_2. The S^2 should shrinks to zero as τ^2 (this can be shown for instance by solving the equations of motion in the infrared). These infrared boundary conditions fix the leading order behavior of the metric functions in the PT Ansatz (2.22):

$$x \sim c_x + \log\tau\,, \quad y \sim c_y + \log\tau\,, \quad p \sim c_p - \frac{1}{6}\log\tau\,, \quad A \sim c_A + \frac{2}{3}\log\tau\,. \tag{2.24}$$

We now write the Ansatz for the fluxes. In order to find F_3, we note that at the apex the flux should lie within the S^3, so that

$$F_3(\tau = 0) = 2P g_3 \wedge g_4 \wedge g_5\,, \tag{2.25}$$

and in the ultraviolet should approach the KT value

$$F_3(\tau = \infty) = P(g_1 \wedge g_2 + g_3 \wedge g_4) \wedge g_5\,. \tag{2.26}$$

[2]The $U(1)$ is actually broken to a $\mathbb{Z}_{2M}$ subgroup by instanton effects, but this is a $1/M$ effect not visible in the SUGRA approximation, while the breaking to $\mathbb{Z}_2$ is a leading order effect.

Here we are setting $P = M/4$ and $\alpha' = 1$. The simplest interpolating Ansatz is

$$F_3 = F\, g_1 \wedge g_2 \wedge g_5 + (2\,P - F)\; g_3 \wedge g_4 \wedge g_5 + F'\, d\tau \wedge (g_1 \wedge g_3 + g_2 \wedge g_4)\;, \quad (2.27)$$

with $F(0) = 0$ and $F(\infty) = P$. For the NSNS flux, a simple $\mathbb{Z}_2$ symmetric Anstaz for the B-field is

$$B = f\, g_1 \wedge g_2 + k\, g_3 \wedge g_4\,, \quad (2.28)$$

and $H_3 = dB_2$. For the five-form flux, we have

$$F_5 = \mathcal{F}_5 + *\mathcal{F}_5\,, \quad (2.29)$$

where

$$\mathcal{F}_5 = \left[\frac{\pi Q}{4} + (k-f)F + 2Pf\right] g_1 \wedge g_2 \wedge g_3 \wedge g_4 \wedge g_5\,. \quad (2.30)$$

For $Q = 0$, the Ansatz automatically implies $\mathcal{F}_5 = B_2 \wedge F_3$. A nonzero Q measures the number of explicit D3-brane sources present at the tip of the cone.

We are interested in deriving the first-order equations imposed by supersymmetry for the fields of the PT Ansatz discussed in detail above. In order to do that, we will use the pure spinor equations described in the previous section. We will only consider the case of an $SU(3)$ structure, so the only information that we need to write down the pure spinor equations explicitly is the knowledge of the complex structure form Ω and the Kähler form J. These have been derived in [144] and they are very simple in the basis G_i, related to g_i by equation (A.2):

$$\begin{aligned} J &= G_1 \wedge G_2 + G_3 \wedge g_4 + G_5 \wedge G_6\,, \\ \Omega &= (G_1 + iG_2) \wedge (G_3 + iG_4) \wedge (G_5 + iG_6)\,. \end{aligned} \quad (2.31)$$

In terms of the basis g_i these forms read

$$J = e^{-6p-x} d\tau \wedge g_5 - e^x g_1 \wedge g_4 + e^x g_2 \wedge g_3; \quad (2.32)$$

$$\begin{aligned} \Omega = e^{-3p+\frac{x}{2}} \Big[&- ie^y d\tau \wedge g_1 \wedge g_2 + d\tau \wedge g_1 \wedge g_3 + d\tau \wedge g_2 \wedge g_4 \\ &+ ie^{-y} d\tau \wedge g_3 \wedge g_4 + e^y g_1 \wedge g_2 \wedge g_5 + i g_1 \wedge g_3 \wedge g_5 \\ &+ i g_2 \wedge g_4 \wedge g_5 - e^{-y} g_3 \wedge g_4 \wedge g_5 \Big]\,. \end{aligned} \quad (2.33)$$

From this, we recall that we can obtain the pure spinors $\Phi_\pm$ as follows:

$$\Phi_- = -i\Omega\,, \qquad \Phi_+ = e^{-iJ}\,. \quad (2.34)$$

At this point we have all the ingredients to compute the pure spinor equations (2.12).

The final result for the first equation is the following

$$
\begin{aligned}
&e^{-3\tilde{A}+\phi}d_H\left[e^{3\tilde{A}-\phi}\Phi_-\right] = \qquad (2.35)\\
&= \frac{1}{2}e^{-3p+\frac{x}{2}}\Big[2id\tau\wedge g_1\wedge g_2\wedge g_3\wedge g_4\wedge g_5\left(f-k+e^{-y}f'-e^{y}k'\right)\\
&- d\tau\wedge(\wedge g_1\wedge g_3\wedge g_5+g_2\wedge g_4\wedge g_5)\left(2\cosh y+6p'-x'-6\tilde{A}'+2\phi'\right)\\
&- ie^{-y}d\tau\wedge g_3\wedge g_4\wedge g_5\left(2e^{y}+6p'-x'+2y'-6\tilde{A}'+2\phi'\right)\\
&- ie^{y}d\tau\wedge g_1\wedge g_2\wedge g_5\left(-2e^{-y}-6p'+x'+2y'+6\tilde{A}'-2\phi'\right)\Big] = 0\,,
\end{aligned}
$$

while for the Φ_+ equation we find

$$
\begin{aligned}
&e^{-3\tilde{A}+\phi}d_H\left[e^{3\tilde{A}-\phi}\Phi_+\right]+d\tilde{A}\wedge\bar{\Phi}_++e^{\phi}\star\lambda F = \qquad (2.36)\\
&= e^{-6p}d\tau\wedge g_1\wedge g_2\wedge g_3\wedge g_4\left[-2+e^{6p+2x}(4\tilde{A}'+2x'-\phi')\right]\\
&+ e^{-2x}d\tau\left[e^{\phi}P\left(f(2P-F)+Fk\right)+e^{2x}\left(4\tilde{A}'-\phi'\right)\right]\\
&+ \frac{1}{2}(g_1\wedge g_3+g_2\wedge g_4)\wedge g_5\left[f-k-2e^{\phi}F'\right]\\
&+ d\tau\wedge g_1\wedge g_2\left[e^{2y+\phi}(F-2P)-f'\right]\\
&+ d\tau\wedge g_3\wedge g_4\left[-e^{-2y+\phi}F+e^{-\phi}k'\right]\\
&+ ie^{-6p-x}d\tau\wedge(g_1\wedge g_4-g_2\wedge g_3)\left[-1+e^{6p+2x}(2\tilde{A}'+x'-\phi')\right] = 0\,.
\end{aligned}
$$

We note that the warp factor $\tilde{A}$ is related to the mode A of the PT Ansatz by $2\tilde{A} = 2A+2p-x$. From these expressions, we want to get a system of first-order equations for the first derivatives of the PT scalars ϕ^a. If we try to solve algebraically for $\dot{\phi}^a$ we see that there are too many independent equations, so we can rewrite the previous conditions as a system of eight differential equations plus an algebraic constraint. The latter comes from the top form in (2.35) (which is roughly $H\wedge\Omega$) and thus represents the $(0,3)$–flux. We can thus rewrite the pure spinor equations in the form

$$
\begin{aligned}
(0,3):\qquad & f - k + e^{-y} f' - e^{y} k' = 0\,, && (2.37)\\
\text{flow eqs}:\qquad & x' = \frac{1}{2} e^{-2x}\left(2e^{-6p} + f(2P - F) + kF\right)\,, && (2.38)\\
& y' = -\sinh y\,,\\
& p' = \frac{1}{6}\left(-2\cosh y + e^{-2x}(e^{-6p} + f(-2P+F) - kF)\right)\,,\\
& \tilde{A}' = \frac{1}{4} e^{-2x}\left(f(-2P+F) - kF\right)\,,\\
& f' = e^{2y}(F - 2P)\,,\\
& k' = -e^{2y} F\,,\\
& F' = \frac{1}{2}(f - k)\,,\\
& \phi' = 0\,.
\end{aligned}
$$

For simplicity we already used the last equation to set $e^{\phi} = 1$ in the above system, in order to avoid redefinition of the metric modes in passing from the string frame (in which we usually write the pure spinor equations) and the Einstein frame of the PT Ansatz. We are now interested in finding a regular solution of this system, which has the desired boundary conditions described above. Before solving the system, let us pause and comment about this result. We derived the system of supersymmetry equations in a rather unconventional way for the literature on gauge/gravity duality; we now describe a more standard way to derive the same result. Since the solution depends only on the radial variable, one can perform a reduction of the type IIB supergravity action and derive an effective one-dimensional dynamics for the scalars ϕ^a. This is a useful way to look at the problem and we will use it in the following. The result of this procedure is a Lagrangian of the type of eq. (2.3), where G_{ab} is a metric on a moduli space whose coordinates are the scalars ϕ^a. In the specific case of the PT Ansatz the result for G_{ab} is the following:

$$
\begin{aligned}
G_{ab}\phi'^a\,\phi'^b = e^{4p+4A}\Big[&x'^2 + \frac{1}{2}y'^2 + 6p'^2 - 6A'^2 + \frac{1}{4}\,\phi'^2\\
&+ \frac{1}{4}e^{-\Phi-2x}\left(e^{-2y}f'^2 + e^{2y}k'^2 + 2e^{2\Phi}\,F'^2\right)\Big]\,. \qquad (2.39)
\end{aligned}
$$

The scalar potential $V(\phi)$ has the following integrability property:

$$
V(\phi) = \frac{1}{8} G^{ab} \frac{\partial W}{\partial \phi^a}\,\frac{\partial W}{\partial \phi^b}\,. \qquad (2.40)
$$

The real function $W(\phi)$ is known as the superpotential, and this is basically one integration of the Hamilton-Jacobi equation for the one-dimensional system at hand [60]. In our case W is known:

$$
W_{KS}(\phi) = e^{4A-2p-2x} + e^{4A+4p}\cosh y + \frac{1}{2}e^{4A+4p-2x}\Big[f(2P-F) + kF\Big]\,. \qquad (2.41)
$$

From this superpotential we naturally derive gradient flow equations, which imply the equations of motion:

$$\phi'^a = \frac{1}{2} G^{ab} \frac{\partial W_{KS}(\phi)}{\partial \phi^b} \,. \tag{2.42}$$

While this is usually the form of the supersymmetry first-order system, with this procedure it is not clear the relation with supersymmetry. With the pure spinor result, we are now in the position to check if the system (2.42) is equivalent to the requirement of supersymmetry. It is easy to show that the flow equations are indeed the same as the system (2.38). However, supersymmetry imposes the additional algebraic constraint (2.37), so we conclude that there exist solutions of the flow equations (2.42) which break supersymmetry. This implies that W is not a true superpotential, but a "fake" one. Indeed, this result agrees with the five–dimensional gauged supergravity analysis of [97] (the fact that there exist solutions to (2.42) with susy–breaking $(0, 3)$–flux was originally noticed in [122]).

Let us now solve the first-order system (2.38). We will solve for the eight functions in the following order:

$$y, f, k, F, x, p, \tilde{A}, \phi. \tag{2.43}$$

The equation for $y(\tau)$ has a general solution of the form

$$y(\tau) = \log\left[\tanh\left(\frac{\tau}{2} + C_y\right)\right] \,. \tag{2.44}$$

In order to match the IR series $y(\tau) \sim \text{const} + \log \tau + \ldots$ we see that we can take $C_y = 0$. With this solution we see that the system for the flux modes f, k, F decouple; we thus need to solve:

$$\begin{aligned} f' &= \tanh^2\left(\frac{\tau}{2}\right)(F - 2P) \,, \\ k' &= \coth^2\left(\frac{\tau}{2}\right) F \,, \\ F' &= \frac{1}{2}(f - k) \,. \end{aligned} \tag{2.45}$$

The solution with regular boundary conditions is

$$f = -P \frac{(\tau \coth \tau - 1)(\cosh \tau - 1)}{\sinh \tau} \tag{2.46}$$

$$k = -P \frac{(\tau \coth \tau - 1)(\cosh \tau + 1)}{\sinh \tau} \tag{2.47}$$

$$F = P \frac{(\sinh \tau - \tau)}{\sinh \tau} \,. \tag{2.48}$$

With this solution we can now solve for the metric functions x and p. We can easily solve for the combination $x + 3p$, which is given by

$$x + 3p = \log\left[\frac{1}{2} \text{csch}\tau \sqrt{3 \sinh(2\tau) - 6\tau + C_x}\right] \,, \tag{2.49}$$

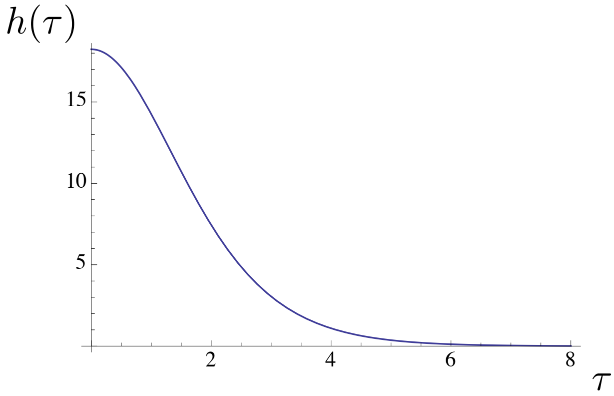


Figure 2.2: The warp factor of the Klebanov-Strassler solution.

and the regular boundary condition corresponds to $C_x = 0$. Recalling that the warp factor is defined as $h = e^{-4\tilde{A}}$, we have

$$h' = -e^{-2(x+2\tilde{A})}(f(2P-F)+kF)\,. \tag{2.50}$$

This can be solved by noticing that

$$x' + 2\tilde{A}' = e^{-2(3p+x)} = \frac{4\sinh^2\tau}{3(\sinh(2\tau)-2\tau)} \tag{2.51}$$

and thus

$$x + 2\tilde{A} = \int^\tau \frac{4\sinh^2 u}{3(\sinh(2u)-2u)}du = \frac{1}{3}\log\left(\sinh(2\tau)-2\tau\right) - \frac{7\sqrt{2}}{3}\,, \tag{2.52}$$

where the constant follows from the definition $h = e^{-4\tilde{A}}$. This expression gives the warp factor h in terms of a single integral:

$$h = h_0 - 16\,2^{2/3}P^2 \int^\tau \frac{(u\coth u - 1)\,(\sinh(2u)-2u)^{1/3}}{\sinh^2 u}du\,. \tag{2.53}$$

This integral cannot be evaluated analytically and a plot of its numerical values is shown in Figure 2.2. The constant $h_0 \sim 18.2373P^2$ is chosen to have $h(\infty) = 0$. In terms of this integral the metric mode x is then

$$x = \log\left[\frac{2^{-1/3}}{4}\sqrt{h}(\sinh(2\tau)-2\tau)^{1/3}\right]\,. \tag{2.54}$$

This concludes the derivation of the IR and UV regular solution of the first-order system (2.38), which was first derived by Klebanov and Strassler in [116]. It is easy to show that the constraint (2.37) is automatically satisfied for this solution, which is thus supersymmetric (this fact was first proven in [93]). We summarize here the Klebanov-Strassler solution we just derived, in terms of the function A that we will use in the following chapters. We also reintroduce the ϵ parameter of the conifold:

$$\begin{aligned}
e^x &= \frac{1}{4}h(\tau)^{1/2}\left(\tfrac{1}{2}\sinh(2\tau)-\tau\right)^{1/3}\,,\\
e^y &= \tanh(\tau/2)\,,\\
e^{6\,p} &= 24\frac{\left(\frac{1}{2}\sinh(2\,\tau)-\tau\right)^{1/3}}{h(\tau)\sinh^2\tau}\,,\\
e^{6\,A} &= \frac{\epsilon^4}{3\cdot 2^9}h(\tau)\left(\tfrac{1}{2}\sinh(2\tau)-\tau\right)^{2/3}\sinh^2\tau\,,\\
f &= -P\frac{(\tau\coth\tau-1)\,(\cosh\tau-1)}{\sinh\tau}\,,\\
k &= -P\frac{(\tau\coth\tau-1)\,(\cosh\tau+1)}{\sinh\tau}\,,\\
F &= P\frac{(\sinh\tau-\tau)}{\sinh\tau}\,,\\
\phi &= 0\,,\\
Q &= 0\,.
\end{aligned} \tag{2.55}$$

2.3 The baryonic branch of Klebanov-Strassler

In this section we would like to show the power of the generalized geometry techniques in deriving another cone-like solution of interest for the gauge/gravity correspondance: the dual of the baryonic branch of Klebanov-Strassler field theory. We will not need this solution in this thesis, although we will use a number of facts about the baryonic branch of the theory. We thus include this section for completeness. The original derivation of the solution is in [45] (BGMPZ in the following); here we will rederive it in a slightly different and simpler way, making use of the pure spinor equations.

As we discussed in the previous section, the Klebanov-Strassler solution has a $\mathbb{Z}_2$ symmetry which interchanges the two S^2's parametrized by (θ_1,ϕ_1) and (θ_2,ϕ_2). It was then suggested in [94] that the KS solution should lie at a $\mathbb{Z}_2$ symmetric point inside a moduli space of vacua which includes a baryonic branch, namely a locus where the global $U(1)_B$ symmetry is broken by a VEV for baryonic operators $\mathcal{B},\bar{\mathcal{B}}$

$$\begin{aligned}
\mathcal{B} &\sim \epsilon_{\alpha_1\ldots\alpha_{2M}}(A_1)^{\alpha_1}_1\ldots(A_1)^{\alpha_M}_M(A_2)^{\alpha_{M+1}}_{M+1}\ldots(A_2)^{\alpha_{2M}}_{2M}\,,\\
\bar{\mathcal{B}} &\sim \epsilon^{\alpha_1\ldots\alpha_{2M}}(B_1)^1_{\alpha_1}\ldots(B_1)^M_{\alpha_M}(B_2)^{M+1}_{\alpha_{M+1}}\ldots(B_2)^{2M}_{\alpha_{2M}}\,.
\end{aligned} \tag{2.56}$$

The spontaneous breaking of the baryonic $U(1)_B$ is associated to a massless Goldston boson which was identified in [94]. This boson should have a companion partner in a $\mathcal{N}=1$ chiral multiplet, the saxion. The VEV for this mode corresponds to a one parameter family of supersymmetric solutions. A VEV for the saxion corresponds to the breaking of the $\mathbb{Z}_2$ symmetry in the gravity background, thus in order to find this solution one should start by the most general $SU(2)\times SU(2)$ Ansatz for the metric and the fluxes. This was already written down in [144]. Based on this Ansatz, BGMPZ derived the first-order system of supersymmetric conditions by using G-structure techniques, essentially by matching the intrinsic torsion and the fluxes in each $SU(3)$ representation. Here we will show how to derive this system by using the same techniques of the previous chapter, namely by writing down the pure spinor equations.

The PT Ansatz [144] for the metric is very simple in the basis G_i:

$$ds^2 = e^{2A} ds_4^2 + \sum_{i=1}^{6} G_i^2, \tag{2.57}$$

while for the fluxes it is more concise in the original basis (e_i, ϵ_j) (we refer to Appendix A for the definitions):

$$\begin{aligned}
H &= h_2 \tilde{\epsilon}_3 \wedge (\epsilon_1\wedge e_1 + \epsilon_2 \wedge e_2) + d\tau \wedge \big[h_1'(\epsilon_1\wedge\epsilon_2 + e_1\wedge e_2),\\
&\quad + \chi'(e_1\wedge e_2 - \epsilon_1\wedge\epsilon_2) + h_2'(\epsilon_1\wedge e_2 - \epsilon_2\wedge e_1)\big];\\
F_3 &= P\big[\tilde{\epsilon}_3\wedge(\epsilon_1\wedge\epsilon_2 + e_1\wedge e_2 - b(\epsilon_1\wedge e_2 - \epsilon_2\wedge e_1)) + b' d\tau\wedge(\epsilon_1\wedge e_1 + \epsilon_2\wedge e_2)\big],\\
F_5 &= \mathcal{F}_5 + \star\mathcal{F}_5, \qquad \mathcal{F}_5 = K e_1\wedge e_2\wedge\epsilon_1\wedge\epsilon_2\wedge\epsilon_3.
\end{aligned}$$

We recall that a prime denotes derivative with respect to the radial direction τ. We also fix the following SU(3) structure:

$$J = (G_1\wedge G_2) + (G_3\wedge G_4) + (G_5\wedge G_6) \tag{2.58}$$

$$\Omega = (G_1 + iG_2)\wedge(G_3+iG_4)\wedge(G_5+iG_6). \tag{2.59}$$

The PT Ansatz thus consists of ten scalars ϕ^a:

$$\phi^a = (a, g, x, p, A, \phi, b, h_1, h_2, \chi)\,. \tag{2.60}$$

To obtain the $\mathbb{Z}_2$ symmetric Ansatz we used in the previous chapter one has to fix a particular relation between the metric modes a and g:

$$e^{2g} = 1 - a^2\,, \tag{2.61}$$

and set $\chi = 0$. The flux modes h_1, h_2 and b are related to the scalars f, k and F of previous section by

$$f(\tau) = h_1(\tau) - h_2(\tau) \tag{2.62}$$

$$
\begin{aligned}
k(\tau) &= h_1(\tau) + h_2(\tau) \\
b(\tau) &= \frac{F(\tau)}{P} - 1\,.
\end{aligned}
$$

The dynamics of the ten PT scalars is described by the type IIB supergravity action, which can be reduced to an effective one-dimensional problem of the type (2.3), with a moduli space metric given by:

$$
\begin{aligned}
\frac{1}{2}G_{ab}\dot{\phi}^a\dot{\phi}^b = e^{4A+2x-2\phi}\Big[&-\frac{1}{4}e^{-2g}\dot{a}^2 + 3\dot{A}^2 - \frac{1}{4}\dot{g}^2 + \frac{1}{4}\dot{x}^2 + \dot{\phi}^2 - 6\dot{A}\dot{p} + 3\dot{A}\dot{x} + 3\dot{A}\dot{x} \\
&- 3\dot{p}\dot{x} - 4\dot{A}\dot{\phi} + 3\dot{p}\dot{\phi} - \frac{3}{2}\dot{x}\dot{\phi}\Big] - \frac{1}{8}e^{4A}\Big[2P^2\dot{b}^2 + e^{-2\phi}\Big(e^{2g}(\dot{h}_1 - \dot{\chi})^2 \\
&+ 2(a\dot{h}_1 + \dot{h}_2 - a\dot{\chi})^2 + e^{-2g}((1+a^2)\dot{h}_1 + 2a\dot{h}_2 + (1-a^2)\dot{\chi})^2\Big)\Big]\,,
\end{aligned}
\tag{2.63}
$$

and an interaction potential:

$$
\begin{aligned}
V(\phi^a) = \frac{1}{8}e^{4A}\Big[&2e^{-2(g-x+\phi)}a^2 + e^{-2(g+6p+x+\phi)}\Big(1 + e^{4g} + 2(-1+e^{2g})a^2 + a^4\Big) \\
&- 4e^{-g-6p-2\phi}(1 + e^{2g} + a^2) + 2e^{-2\phi}h_2^2 + e^{-2x}(Q + 2P(h_1 + bh_2))^2 \\
&+ P^2\Big(e^{2g} + 2(a-b)^2 + e^{-2g}(1 + a^2 - 2ab)^2\Big)\Big]\,.
\end{aligned}
\tag{2.64}
$$

A superpotential W which satisfies the defining relation (2.40) is not known, even if there has recently been some attention to this problem (see for example [22, 97]). It has been suggested that a solution for this equation with the desired boundary conditions (as we will see, the solution should interpolate between the KS and the Maldacena-Nunez solution) does not exist [77]. A compromise was to find a function W that gives the potential V only after some algebraic constraints are imposed [49]. We will come back in a moment on these constraints. Now we want to follow the same procedure of the previous section, namely to write down the pure spinor equations and derive the corresponding system of first-order equations. This computation is more involved than in the KS case, but it is entirely algebraic and thus can be easily carried out with Mathematica. One important difference from the KS case is that we should be more general and allow for arbitrary phases $\theta_\pm(\tau)$ (functions of the radial variable τ) in the pure spinor definitions:

$$
\Phi_+ = e^{i\theta_+}e^{iJ}\,, \qquad \Phi_- = -ie^{\theta_-}\Omega\,. \tag{2.65}
$$

The reason is that the solution we seek should interpolate between different classes of $SU(3)$ structures (see for example [75] for interpolating G-structures). The KS solution derived in the previous section corresponds to $\theta_\pm = 0$ (which is known as the class B, see e.g. [45]). With this definition we can carry out the computation of the pure spinor equations; to avoid clutter we show the final result in Appendix C. These expressions contain the ten PT scalars ϕ^a and their first derivatives $\dot{\phi}^a$, as well as the SU(3) structure functions $\mathcal{A}$, $\mathcal{B}$, $\theta_\pm$. With some algebraic manipulations, one can show that the pure spinor equations are equivalent to the following set of first-order ODEs:

$$\begin{aligned}
a' &= -\frac{a\,C-1}{S}e^{g-2x-6p} + \frac{a(a-b)S}{bC-1}\,, \\
g' &= e^{-2g}\left[aS + (C-a)\left(-\frac{aC-1}{S}e^{g-2x-6p} + \frac{a(a-b)S}{bC-1}\right)\right]\,, \\
x' &= aSe^{-g-6p-2x} + \frac{b-C}{bC-1}h_2^2Se^{-2x}\,, \\
p' &= -\frac{e^{-2g}}{12S(bC-1)}\Big[e^{-2x-6p}(4(b-C)(-1-a^2+2aC)e^{6p}h_2^2S^2 \\
&\quad - 2a(bC-1)e^gS^2) - (4a+2b+2a^2b-2C-2a^2C-4abC \\
&\quad + 4aS^2+4bS^2+2a^2bS^2+2a^2CS^2-8abCS^2)\Big]\,, \\
A' &= \frac{b-C-b^2C+bC^2}{8S}e^{-2x+2\phi}\,, \\
\phi' &= -h_2S - \left(\frac{2a^2C-b(-1+a^2+2aC)}{bC-1}e^{-2g}h_2S\right)C\,, \\
b' &= \frac{2a^2C-b(-1+a^2+2aC)}{bC-1}e^{-2g}h_2S\,, \\
h_1' &= \frac{1-bC}{S}\,, \\
h_2' &= \frac{2a(b-C)(aC-1)}{bC-1}e^{-2g}h_2S\,, \\
\chi' &= \frac{(C-b)(aC-1)^2}{(bC-1)S}e^{-2g}\,,
\end{aligned} \tag{2.66}$$

together with two algebraic constraints:

$$\begin{aligned}
1 &= \frac{4e^{-2\phi}}{(bC-1)^2}\left[e^{-2g+2x}(aC-1)^2 + h_2S^2\right]\,, \\
h_2 &= -\frac{2ah_1}{1+e^{2g}+a^2}\,,
\end{aligned} \tag{2.67}$$

where we defined the combination C and S as:

$$C = \frac{1}{2a}\big(1+a^2+e^{2g}\big)\,, \tag{2.68}$$

$$S = \frac{1}{2a}\left[a^4+2a^2(e^{2g}-1)+(e^{2g}+1)^2\right]^{1/2}\,. \tag{2.69}$$

The $SU(3)$ structure functions are fixed by

$$\mathcal{A} = \frac{1}{S}\big(C-a\big)\,, \qquad \mathcal{B} = -\frac{e^g}{S}\,, \qquad e^{i\theta_+} = \frac{e^{g+\phi}P(bC-1)}{ie^x(aC-1)-e^gh_2S}\,, \qquad e^{i\theta_-} = 1\,. \tag{2.70}$$

One can show that the system (2.66), together with the two algebraic constraints, imply the second-order equations of motion for the PT scalars. When we try to solve the first-order equations, we find that a regular supersymmetric solution is determined by two coupled ODEs for the fields $a(\tau)$ and $v(\tau) = e^{6p+2x}$, while all other modes can be obtained from algebraic equations or simple decoupled ODEs. The solution cannot be determined analytically, but a numerical analysis has been performed in [45]. The end result is a family of supersymmetric solutions which interpolate between the KS solution of the previous section and the Maldacena-Nunez solution [128, 51] (MN), another $\mathcal{N} = 1$ background dual to a confining theory. MN corresponds to a phase $\theta_+ = \frac{\pi}{2}$ and it is known as a class C. For more details about this family of solutions and its relation to the baryonic branch of the KS gauge theory we refer to the original paper [45] (see also [68] for a detailed analysis of the moduli space of the gauge theory).

2.4 Metastable supersymmetry breaking

In the previous sections we reviewed the construction of the Klebanov-Strassler solution, which is a supergravity background dual to a confining $\mathcal{N} = 1$ gauge theory. This is clearly a good starting point to explore supersymmetry breaking in the gauge/gravity correspondence. This will be one of the main subjects of this thesis and we will discuss in great detail various solutions which break supersymmetry on the conifold. Here we want to introduce one particularly important mechanism, which plays a crucial role in the following chapters.

The idea is to add an ingredient which does not preserve the same supersymmetries of the Klebanov-Strassler solution. The simplest option is to add a stack of anti-D3 branes with worldvolume transverse to the conifold directions. Since the fluxes of the Klebanov-Strassler solution are BPS with D3 branes, a brane with opposite charge breaks supersymmetry down to $\mathcal{N} = 0$. Of course, supersymmetry is not there anymore to guarantee stability, and in general one expects this sort of configurations to be unstable. We will discuss in length about the stability in the following chapters. Here we review a simple analysis, due to Kachru, Pearson and Verlinde (KPV) [111], which leads to conjecture that for some range of parameters, the anti-D3 branes result in a metastable state, which is classically stable and decays via bubble nucleation.

The proposed mechanism is based on the fact that the anti-branes can polarize via the Myers effect [142] into a classically stable expanded five-brane, due to the presence of the background three-form fluxes. This conjecture is supported by a probe computation, in which one neglects the interaction of the anti-D3 branes with the background. There is however a quantum non-perturbative channel, which is mediated by an instanton, which causes the five-brane to be metastable, but extremely long-lived. If the five-brane is metastable, then the corresponding supergravity solution should be dual to a metastable non-supersymmetric state in the Klebanov-

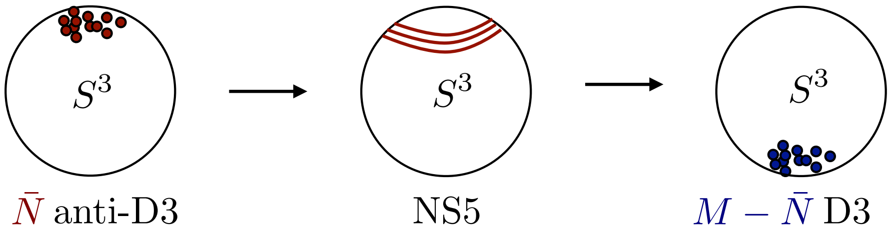


Figure 2.3: The KPV process.

Strassler theory. This is because of the intuition that the effects of supersymmetry breaking should be irrelevant in the UV, due to the IR redshift which comes from the warping. This holographic model of spontaneous supersymmetry breaking is extremely interesting, since in the gauge theory the identification of the metastable point is difficult due to strong coupling, and a field theory evidence is still lacking, despite various attempts to study the moduli space of the theory [68]. While Chapter 3 will be devoted to the construction of this supergravity solution, we now briefly review the KPV conjecture, following [111].

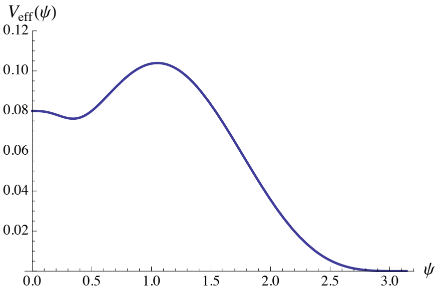
We start by placing a stack of $\bar{N}$ anti-D3 branes, extended along the $x_0, \ldots x_3$ directions, at a given point τ_0 along the radial direction of the deformed conifold. It is easy to compute the force which is acting on the anti-branes, due to the warping and the five-form flux. The DBI potential is proportional to $e^{4A+4p-2x}$, while the WZ term comes from the five-form flux and it is given by $Ke^{4A+4p-2x}$. We thus have

$$F_{D3_\pm} = F_{DBI} + F_{WZ} \sim \Big[kF + f(2P - F)\Big] e^{4A+4p-2x} \pm \partial_\tau e^{4A+4p-2x} \,, \tag{2.71}$$

where the $\pm$ means D3 and anti-D3 respectively. We see that a D3 brane is BPS in the Klebanov-Strassler solution, since the DBI and Wess-Zumino term cancel due to the supersymmetric equations (2.38). For anti-D3 branes, they sum up and they create a net attractive force which brings the anti-D3 branes toward $\tau = 0$, the tip of the cone. Thereby, we consider a situation in which the anti-D3 branes sit at the tip of the cone, which is topologically a three-sphere S^3 and we suppose that the branes sit at the north pole of the sphere. Since now we neglect the backreaction of the branes, the metric which surrounds them is that of the near-tip limit of the Klebanov-Strassler solution:

$$ds^2_{10} = dx_\mu dx_\mu + b_0^2 \left[\frac{1}{2} d\tau^2 + d\Omega_3^2 + \tau^2 d\Omega_2^2\right] \,, \tag{2.72}$$

where $d\Omega_3$ and $d\Omega_2$ are the metric on the round three and two spheres of the deformed conifold and $b_0 \sim 0.93266$. Since the three-sphere supports M units of the R-R three-form fluxes F_3, we want to check wether, in the probe limit, the anti-branes



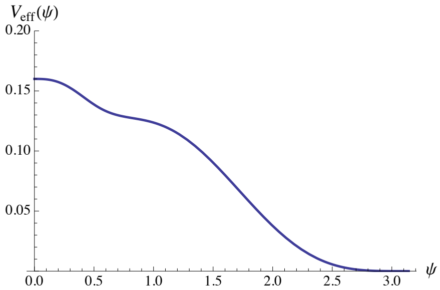


Figure 2.4: The effective potential $V_{eff}(\psi)$ for different values of $\bar{N}/M$. On the left: $\bar{N}/M = 0.04$ (we see the metastable minimum); on the right: $\bar{N}/M = 0.08$ (the minimum disappears).

can undergo a Myers effect [142] and polarize into an expanded brane. To check this, we need to compute the potential of the would-be expanded brane and look for some local minimum. Since the configuration breaks supersymmetry, we generically expect to find at most a metastable point (provided that a domain wall interpolating between this non-supersymmetric solution and one supersymmetric configuration with lower energy exists). We can think of various polarization channels: in general, one expects D5 and NS5 phases, together with obliques phases of (p,q)-branes [146, 160]. The KPV conjecture is about the existence of the NS5 channel, in which the anti-D3 branes polarize into a NS5 brane wrapping a two-sphere inside the S^3 at the tip, and sitting at particular value of a polar angle ψ. As we now show, this channel is present in the probe approximation. We consider the worldvolume action of the NS5 brane (we set $\alpha' = 1$):

$$S = \mu_5 \int_{\mathbb{R}^{3,1}\times S^2} \left[\det g_{\parallel} \det(g_{\perp} + 2\pi\mathcal{F})\right]^{1/2} + \mu_5 \int_{\mathbb{R}^{3,1}\times S^2} B_6 + C_4 \wedge 2\pi\mathcal{F}\,, \tag{2.73}$$

where $2\pi\mathcal{F} = 2\pi F_2 - C_2$ and the integral of F_2 over the S^2 gives the induced anti-D3 charge on the NS5 brane worldvolume:

$$\int_{S^2} F_2 = 2\pi\bar{N}\,. \tag{2.74}$$

We are interested in the dynamics of the NS5 brane, described by the time evolution of an angular variable $\psi(t)$, such that $\psi(t=0)$ is the north pole. This time evolution is generated by the following Hamiltonian H:

$$H = -\frac{A_0}{2\pi}(2\psi - \sin(2\psi)) + \left[A_0^2 V_{DBI}(\psi)^2 + P_\psi^2\right]^{1/2}, \tag{2.75}$$

where $P_\psi = \partial\mathcal{L}/\partial\psi$, $A_0 = 4\pi^2\mu_5 M/g_s$ and

$$V_{DBI}(\psi) = \frac{1}{\pi}\sqrt{b_0^4 \sin^4\psi + \left(\pi\frac{\bar{N}}{M} - \psi + \frac{1}{2}\sin(2\psi)\right)^2}\,. \tag{2.76}$$

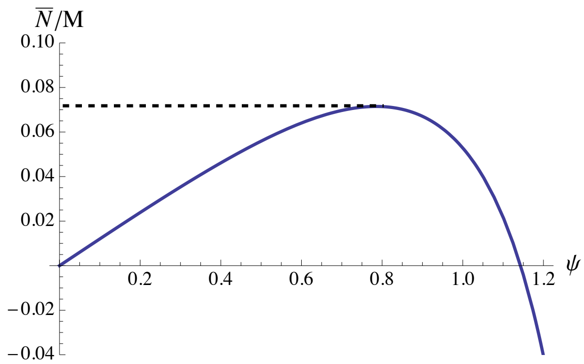


Figure 2.5: The values of $\bar{N}/M$ for which there is a minimum of the potential $V_{eff}(\psi)$. For $\bar{N}/M$ bigger then about 0.08 the potential has no minima.

We seek for a static solution, so we can use an effective potential derived by setting $P_\psi = 0$, obtaining

$$V_{eff}(\psi) = A_0 \left[\frac{1}{\pi} \sqrt{b_0^4 \sin^4 \psi + \left(\pi \frac{\bar{N}}{M} - \psi + \frac{1}{2} \sin(2\psi) \right)^2} - \frac{1}{2\pi} \Big(2\psi - \sin(2\psi) \Big) \right] . \tag{2.77}$$

It is useful to look at an expansion of this potential valid for small ψ. We are actually interested in the dependence of V on the two variables, ψ and $\bar{N}/M$. The region near the north pole is well approximated by a small ψ expansion:

$$V_{eff}(\psi, \bar{N}/M) \sim \frac{\bar{N}}{M} - \frac{4}{3\pi}\psi^3 + \frac{b_0^4}{2\pi(\bar{N}/M)}\psi^4 + \mathcal{O}(\psi^5) + \mathcal{O}\left((M/\bar{N})^2 \right) . \tag{2.78}$$

This expansion shows that the potential has a minimum at

$$\psi_0 = \frac{2\pi \bar{N}}{b_0^4 M} . \tag{2.79}$$

We should note that this approximation breaks down as ψ_0 becomes large, namely when the number of anti-D3 branes grows. By plotting the full potential we can see that the minimum is destroyed for large $\bar{N}/M$ (see Figure 2.4). Indeed, by requiring that $\partial_\psi V_{eff}(\psi) = 0$ one find the condition:

$$4\pi \frac{\bar{N}}{M} = 4\psi + (b_0^4 - 1)\sin(2\psi) - 2\tan\psi . \tag{2.80}$$

By plotting the right hand side (see Figure 2.5) we see that one can find a solution if $\bar{N}/M$ is less then a threshold value indicated by the dotted line. The precise value can be found numerically to be

$$\left(\frac{\bar{N}}{M} \right)^\star \sim 0.0714797 . \tag{2.81}$$

We note that this analysis is valid for a process in which the stack of $\bar{N}$ anti-D3 branes polarize into a single "giant graviton" NS5 brane. If $\bar{N}$ exceed the value (2.81) for fixed M the NS5 brane does not find an equilibrium configuration and rolls down to $\psi = \pi$. However, one can consider a process in which the anti-branes are divided into smaller subgroups, each of which does not exceed the threshold value for polarization: in this way one can end up with a state of multiple polarized NS5 branes. Let us focus now on a single NS5 brane, which finds a classically stable configuration at a given angle. From this point the decay towards the true vacuum at the south pole $\psi = \pi$ can only occur via non-perturbative effects. Indeed, a bubble of the supersymmetric vacuum can nucleate, surrounded by a spherical domain wall, which in our case is a NS5 brane wrapped on the S^3.

The solution for such domain wall was constructed by KPV in [111]. The computation of the decay rate reveals that one can tune the parameters in order to obtain an extremely long-lived metastable state. Here we skip the details of this computation. It is however easy to understand the result of this non-perturbative decay. For this it suffices to consider conservation of charge at infinity. We will discuss this in much more detail in Chapter 4, here we just note that the H-flux across the aforementioned domain wall jumps by one unit. Thus, if we start from a configuration for which

$$\frac{1}{4\pi^2}\int_{S^3} F_3 = M\,, \qquad \frac{1}{4\pi^2}\int_{S^2\times[0,\tau^\star]} H_3 = K\,, \tag{2.82}$$

where S^3 is the A-cycle of the manifold and $S^2 \times [0, \tau^\star]$ is the B-cycle (this would be correct in a compact space, here we consider a cutoff $\tau^\star$) we end up with a configuration for which the units of H-flux are $K - 1$. Then, charge conservation requires:

$$-\bar{N} + KM = Q_{\psi=\pi} + (K-1)M\ , \tag{2.83}$$

where $Q_{\psi=\pi}$ is the net D3 charge of the true vacuum. We thus get

$$Q_{\psi=\pi} = M - \bar{N}\,. \tag{2.84}$$

In the decay process the $\bar{N}$ units of anti-D3 charge are "eaten up" by the fluxes, but to compensate a number $M - \bar{N}$ of explicit D3 brane sources should materialize. This kind of process is known as brane/flux annihilation.

We will discuss in much more details the polarization mechanism in Chapter 5. For now we just note that the expansion of the effective potential (2.78), misses a ψ^2 term, which is expected when the full backreaction of the branes is taken into account, as in [146]. In a supersymmetric configuration, this term can easily be guessed from supersymmetry; a direct computation is much more involved, but gives the same result [73]. It was argued in [59] that in the present situation the potential, once full backreaction of the branes is taken into account, should behave in the same way as in the Polchinksi-Strassler solution [146]. However, it is not clear why this is the case, since supersymmetry breaking effects can allow for more general terms which can spoil the structure of the original PS setup. Indeed, in Chapter 5 we will prove that the current situation is rather different from the PS solution, since some of the phases, such as the D5 brane channel, are missing.

A final note is about the dual interpretation of the metastable state. In [111] some conjectures were made about the nature of the metastable state in the KS theory; the idea is that when $\bar{N} = 1$, and M is large, the non-supersymmetric minimum is closely related to the baryonic branch of the KS theory for $\bar{N} = 0$. The NS5 domain wall interpolating between the true and the false vaccum represents then a transition between the baryonic and the mesonic branches of the theory. Indeed the supersymmetric family of vacua with a number N of D3 branes on the deformed conifold is dual to the mesonic branch of the KS theory. While we refer to the original paper [111] for more details, it is important to keep in mind that a compelling evidence for the existence of such metastable states in the theory is lacking, the reason of which is mainly due to the difficulties in performing a strong coupling analysis.

We end this section by mentioning that there exists a vast literature on metastable supersymmetry breaking in string theory from different perspectives. A rather incomplete list of works on this subject includes [64, 65, 6, 72, 143, 160, 82, 63, 8, 9, 43, 1, 132, 123].

Chapter 3

Non–supersymmetric deformations of conifolds

In this chapter we introduce a computational technique to study non-supersymmetric deformations of supersymmetric flux compactifications of interest for the gauge/gravity duality. We will then solve analytically the equations governing the space of first–order deformations around the Klebanov-Strassler solution. We express the results in terms of at most three nested integrals. These are the simplest expressions for the space of $SU(2) \times SU(2) \times \mathbb{Z}_2$–invariant deformations. Among these solutions, we expect to find the putative solution for smeared anti–D3 branes that we will discuss in detail in the next chapter. We also explain why one cannot claim to identify this solution without fully relating the coefficients of the infrared and ultraviolet expansions of the deformation modes. This chapter is in part based on unpublished results an on the paper [20].

3.1 Motivation

As we discussed in the previous chapter, there are various motivations to study non-supersymmetric solutions based on conifold backgrounds. From the gauge/gravity point of view, it is interesting to have solutions corresponding to different models of supersymmetry breaking in the gauge theory. One can for example break supersymmetry explicitly by adding some mass terms, or look for mechanisms of spontaneous supersymmetry breaking. In phenomenology, this latter scenario is particularly important, since it is obtained by adding anti-D3 branes at the apex of the deformed conifold. Anti–D3 branes in Klebanov-Strassler (KS) throats [116] are a key ingredient in string model building and string cosmology, where they are used both for lifting AdS to de Sitter solutions [110], and to construct models of inflation using D3 branes moving in KS–like geometries [109]. Solving for a space of non-supersymmetric solutions is also useful to study how the pure spinor equations that we used to derive the KS solution are modified for a non-supersymmetric compactification, providing a first step in the exploration of general flux compactifications in string theory.

In this section we will discuss a very useful method, introduced in [41], that greatly simplifies the task of solving the second-order equations of motion for a cone-like compactification. We will also present an extension of the method of [41], which is more general and it will prove essential in the following chapters. The idea is to solve the equations perturbatively in a small supersymmetry breaking parameter around a given supersymmetric solution. By choosing the right variables, one can reformulate the second-order system as a set of two first-order ones, which decouple at any order of the expansion parameter. This formalism will also be useful to study the non-linear case directly, without relying on a perturbative scheme.

We then proceed to solve for linearized perturbations of the Klebanov-Strassler solution, inside the PT Ansatz that we discussed in the previous chapter. This set of solutions contains the backreaction of anti-D3 branes smeared at the tip of the deformed conifold. In the next chapter we will discuss the boundary conditions that permit to identify this solution and we will study the backreaction in great detail.

3.2 A first–order formalism

We consider a type IIB supergravity background which is a warped product of the form

$$ds_{10}^2 = e^{2\tilde{A}} ds_4^2 + ds_6^2\,, \tag{3.1}$$

where $ds_6^2 = g_{mn}\, dx^m dx^n$ is the line element of an internal manifold $\mathcal{M}_6$ with metric g_{mn}. We turn on a B field with field strength $H = dB$, RR fluxes F_1, F_3, F_5 and a running dilaton Φ[1]. We parametrize all the degrees of freedom by some scalars $\phi^a(x^m)$ which depend on the internal coordinates and which have values in a moduli space $\mathcal{M}$. Schematically,

$$\phi^a = \{g, h, \Phi, H, F_a\}\ . \tag{3.2}$$

We will use indices $a, b = 0, \ldots, n$ to indicate coordinates on moduli space and we will often use a coordinate free notation in which ϕ are maps $\phi : \mathcal{M}_6 \to \mathcal{M}$.

We are interested in the situation in which $\mathcal{M}_6$ has a cone-like structure, and the scalars ϕ^a only depends on the radial direction of the cone, denoted by τ. In this case we can perform a dimensional reduction of the type IIB supergravity action and obtain a one dimensional n–body dynamics over the radial direction, which play the role of time. The effective action is given by

$$\mathcal{S} = \int \mathcal{L}(\phi^a, \dot{\phi}^a)\, d\tau\,, \tag{3.3}$$

where

$$\mathcal{L} = \frac{1}{2}\, G_{ab}(\phi)\, \dot{\phi}^a\, \dot{\phi}^b + V(\phi)\,. \tag{3.4}$$

[1] In the following we call the dilaton Φ, in order to avoid confusion with the map ϕ.

A dot means derivative with respect to τ. The scalars ϕ^a are components of a map $\phi : \mathbb{R}^+ \to \mathcal{M}$, where $\mathcal{M}$ is the moduli space for this one–dimensional problem, which we assume a Riemannian manifold with metric G_{ab}. The equations of motion following from this action are the equations of a falling body in the potential V:

$$D\dot{\phi} = \nabla_{\dot{\phi}}\dot{\phi} = \mathrm{grad}V\,, \tag{3.5}$$

where D is the covariant derivative along the curve $\phi : \mathrm{R}^+ \to \mathcal{M}$, ∇ is the Levi-Civita connection on $\mathcal{M}$ and $(\mathrm{grad}V)^a = G^{ab}\partial_b V$. From the supersymmetry equations, we also know a first-order system which implies the equations of motion, namely functions $v^a(\phi)$ such that the supersymmetry conditions can be written as

$$\dot{\phi}^a = v^a(\phi)\,. \tag{3.6}$$

This is for example the case for the KS system (2.42). In this situation we also know one integration of the Hamilton-Jacobi equation, namely a superpotential W such that

$$V(\phi) = \frac{1}{8}G^{ab}\frac{\partial W}{\partial\phi^a}\frac{\partial W}{\partial\phi^b}\,, \tag{3.7}$$

and the functions $v(\phi)^a$ are given by

$$v^a(\phi) = (\mathrm{grad}W)^a\,. \tag{3.8}$$

These functions satisfy the equations of motion, namely

$$\nabla_v v = \mathrm{grad}V\,. \tag{3.9}$$

The knowledge of the vector field $v(\phi)$ is crucial to derive an efficient perturbative expansion scheme to study general non–supersymmetric solutions of the mechanical system described by the Lagrangian (3.4). The idea is to parametrize the supersymmetry breaking by introducing a new vector field ξ, defining:

$$\dot{\phi} = v + \xi\,. \tag{3.10}$$

Now we plug this definition into the equations of motion:

$$0 = \nabla_{v+\xi}(v+\xi) - \mathrm{grad}V = \nabla_v v - \mathrm{grad}V + \nabla_v\xi + \nabla_\xi v + \nabla_\xi\xi = \nabla_v\xi + \nabla_\xi v + \nabla_\xi\xi\,, \tag{3.11}$$

where we used the defining properties of $C^\infty(\mathcal{M})$– and $\mathbb{R}$–linearity of the connection ∇ and the fact that v solves the equations of motion (3.9). Thus we get the system:

$$\dot{\phi} = v + \xi \tag{3.12}$$
$$\nabla_v\xi = -\nabla_\xi(v+\xi)\,. \tag{3.13}$$

In component, this system is a set of $2n$ first–order ODEs which is equivalent to the set of n second–order equations of motion. This is a particularly nice parametrization since the scalar potential V disappear. As we will now describe, equations (3.12), (3.13) can be conveniently solved in a perturbative expansion around the supersymmetric solution.

We introduce a small supersymmetry breaking parameter γ and we expand the fields v and ξ in a power series in γ:

$$v = v_0 + \sum_{j=1}^{J-1} \gamma^j v_j + \mathcal{O}(\gamma^J)\,, \tag{3.14}$$

$$\xi = \sum_{j=1}^{J-1} \gamma^j \xi_j + \mathcal{O}(\gamma^J)\,. \tag{3.15}$$

We note that $\xi_0 = 0$ since the field ξ parametrizes the supersymmetry breaking. From equation (3.13) we get:

$$\nabla_{v_0+v_1+v_2+\dots}(\xi_1+\xi_2+\dots)+\nabla_{\xi_1+\xi_2+\dots}(v_0+v_1+v_2+\dots)+\nabla_{\xi_1+\xi_2+\dots}(\xi_1+\xi_2+\dots) = 0\,. \tag{3.16}$$

As it is clear from this expression, the fact that $\xi_0 = 0$ imply that at a given order j in the perturbative expansion, there cannot be any term which couples v_j to ξ_j. This means that at the jth–order, equation (3.13) decouples from (3.12), and only the modes ϕ_l, ξ_l with $l < j$ enter in the equations as source terms. Once the solution of the system for the ξ_j modes is known, one plugs the result in (3.12) and solves this equation at order j. This procedure clearly simplifies the task of solving the full equations of motion since at every order in γ one has to solve two systems of decoupled first–order ODEs. Here we are particularly interested in the linearized order. By writing

$$\nabla_\xi v = \xi^c\,\partial_c v^a + \xi^c\,\Gamma^a_{cb} v^b\,, \qquad \nabla_{v+\xi}\xi = (v^c+\xi^c)\nabla_c\,\xi^a = \dot{\xi}^a + (v^c+\xi^c)\,\Gamma^a_{cb}\xi^c\,, \tag{3.17}$$

where $\dot{\xi}^a = \dot{\phi}^c\partial_c\xi^a = (v^c+\xi^c)\partial_c\xi^a$, and the connection is derived from the metric on moduli space G_{ab}:

$$\Gamma^a_{bc} = \frac{1}{2}G^{ad}\Big[\partial_b G_{cd} + \partial_c G_{db} - \partial_d G_{bc}\Big]\,, \tag{3.18}$$

one can write the equations, at first-order in γ, as follows:

$$\frac{d\phi^a_1(\tau)}{d\tau} = \frac{\partial v^a(\phi_0)}{\partial\phi^c}\,\phi^c_1(\tau) + \xi^a(\tau)\,, \tag{3.19}$$

$$\frac{d\xi^a(\tau)}{d\tau} = -\left[\frac{\partial v^a(\phi_0)}{\partial\phi^c} + 2\,\Gamma^a_{bc}(\phi_0)\,v^b(\phi_0)\right]\xi^c(\tau)\,. \tag{3.20}$$

We write $\phi_a = \phi^a_0 + \gamma\phi^a_1 + \mathcal{O}(\gamma^2)$, and in the following sections we will assume that ϕ^a_0 are the scalars of a given supersymmetric regular cone-like solution, such as the Klebanov-Strassler one (reviewed in section 2.2).

3.2.1 The Borokhov–Gubser method

If the functions v^a come from a superpotential W, namely can be written as in (3.8), equations (3.19)-(3.20) have a simpler form in terms of the inverse modes ξ_a. The system can be rewritten as follows:

$$\frac{d\xi_a(\tau)}{d\tau} + \xi_b(\tau)\, M^b{}_a(\phi_0) = 0\,, \tag{3.21}$$

$$\frac{d\phi_1^a(\tau)}{d\tau} - M^a{}_b(\phi_0)\, \phi_1^b(\tau) = G^{ab}(\phi_0)\, \xi_b(\tau)\,, \tag{3.22}$$

where

$$M^b{}_d \equiv \frac{1}{2}\frac{\partial}{\partial \phi^d}\left(G^{bc}\frac{\partial W}{\partial \phi^c}\right)\,. \tag{3.23}$$

In this form these equations were first derived in [41]. Since in the following we will perturb around supersymmetric solutions for which a superpotential is known, we will use this form of the system. However we remark that from the pure spinor approach one does not need to know a superpotential in order to apply this method.

We note that in addition to the equations of motion (6.8), the functions ξ_a should additionally satisfy a zero–energy condition coming from Einstein's equations:

$$G_{ab}(\phi)\frac{d\phi^a}{d\tau}\frac{d\phi^b}{d\tau} - V(\phi) = 0\,. \tag{3.24}$$

It easy to show from the definition (3.22) that this amount to set:

$$\xi_a\, \frac{d\phi_0^a}{d\tau} = 0\,. \tag{3.25}$$

In the following section we specify the equations above to study perturbations around the Klebanov-Strassler (KS) solution. We write the expansion of the fields ϕ^a $(a = 1, ..., n)$ as follows

$$\phi^a = \phi_0^a + \phi_1^a(X) + \mathcal{O}(X^2)\,, \tag{3.26}$$

where X represents the set of perturbation parameters, ϕ_1^a is linear in them, and ϕ_0^a are the functions in the Klebanov–Strassler solution, written explicitly in (2.55).

3.3 Analytic solutions

In this section we apply the method described above to the Papadopoulos and Tseytlin Ansatz, we derive the two set of first-order equations for linearized deformation modes ϕ^a and their conjugate momenta ξ^a and we then solve these systems in closed form, thus providing analytic formulae for the full set of $SU(2)\times SU(2)\times \mathbb{Z}_2$–invariant deformation space around the Klebanov–Strassler solution. The method follows that of [25], but here we present numerous analytical improvements.

For sake of clarity, let us recall the metric and the fluxes of our Ansatz, that we already discussed in chapter 2.2. We use the Ansatz for the supergravity background fields proposed by Papadopoulos and Tseytlin (PT) [144], which is the most general Ansatz consistent with the $SU(2)\times SU(2)\times \mathbb{Z}_2$–symmetry of the Klebanov–Strassler background:

$$ds_{10}^2 = e^{2A+2p-x} ds_{1,3}^2 + e^{-6\,p-x} d\tau^2 + e^{x+y}\left(g_1^2+g_2^2\right) + e^{x-y}\left(g_3^2+g_4^2\right) + e^{-6p-x} g_5^2\,, \tag{3.27}$$

where all the functions depend on the variable τ. The fluxes and dilaton are

$$\begin{aligned} H_3 &= \tfrac{1}{2}\,(k-f)\, g_5\wedge(g_1\wedge g_3+g_2\wedge g_4) + d\tau\wedge\left(f' g_1\wedge g_2 + k' g_3\wedge g_4\right)\,,\\ F_3 &= F g_1\wedge g_2\wedge g_5 + (2P-F)\, g_3\wedge g_4\wedge g_5 + F' d\tau\wedge(g_1\wedge g_3+g_2\wedge g_4)\,, \qquad (3.28)\\ F_5 &= \mathcal{F}_5 + *\mathcal{F}_5\,,\qquad \mathcal{F}_5 = [kF + f\,(2P-F)]\, g_1\wedge g_2\wedge g_3\wedge g_4\wedge g_5\,,\\ \Phi &= \Phi(\tau)\,,\qquad C_0 = 0\,, \qquad (3.29)\end{aligned}$$

where P is a constant while f, k and F are functions of τ and a prime denotes a derivative with respect to τ. We will denote the set of functions ϕ^a, $a = 1, ..., 8$ of the PT Ansatz in the following order

$$\phi^a = (x, y, p, A, f, k, F, \Phi)\,. \tag{3.30}$$

The field–space metric in (3.4) is

$$\begin{aligned} G_{ab}\,\phi'^a\phi'^b = e^{4p+4A}\Big[&x'^2 + \frac{1}{2}y'^2 + 6p'^2 - 6A'^2 + \frac{1}{4}\Phi'^2\\ &+ \frac{1}{4}e^{-\Phi-2x}\left(e^{-2y}f'^2 + e^{2y}k'^2 + 2e^{2\Phi}F'^2\right)\Big] \end{aligned} \tag{3.31}$$

and the superpotential is given by

$$W(\phi) = e^{4A-2p-2x} + e^{4A+4p}\cosh y + \frac{1}{2}e^{4A+4p-2x}\,\left(f(2P-F)+kF\right)\,. \tag{3.32}$$

The background fields are given by the Klebanov–Strassler solution [116], summarized in (2.55).

3.3.1 $\tilde{\xi}_a$ equations

We start by writing down the first-order system of eight ODEs (3.21) for the modes ξ_a. As in [25] we shift to a slightly more convenient basis $\tilde{\xi}_a$, defined as

$$\tilde{\xi}_a \equiv (3\xi_1-\xi_3+\xi_4, \xi_2, -3\xi_1+2\xi_3-\xi_4, -3\xi_1+\xi_3-2\xi_4, \xi_5+\xi_6, \xi_5-\xi_6, \xi_7, \xi_8)\,. \tag{3.33}$$

The equations in the order in which we solve them, are[2]

$$\tilde{\xi}_1' = e^{-2x_0}\left[2Pf_0 - F_0\,(f_0-k_0)\right]\tilde{\xi}_1 \tag{3.34}$$

[2] We have accounted for two misprints in the published version of [25].

$$\tilde{\xi}_4' = -e^{-2x_0}\left[2Pf_0 - F_0\left(f_0 - k_0\right)\right]\tilde{\xi}_1 \tag{3.35}$$

$$\tilde{\xi}_5' = -\frac{1}{3}Pe^{-2\,x_0}\tilde{\xi}_1 \tag{3.36}$$

$$\tilde{\xi}_6' = -\tilde{\xi}_7 - \frac{1}{3}e^{-2\,x_0}\left(P - F_0\right)\tilde{\xi}_1 \tag{3.37}$$

$$\tilde{\xi}_7' = -\sinh(2y_0)\tilde{\xi}_5 - \cosh(2y_0)\tilde{\xi}_6 + \frac{1}{6}e^{-2x_0}\left(f_0 - k_0\right)\tilde{\xi}_1 \tag{3.38}$$

$$\tilde{\xi}_8' = \left(Pe^{2\,y_0} - \sinh(2y_0)F_0\right)\tilde{\xi}_5 + \left(Pe^{2y_0} - \cosh(2y_0)F_0\right)\tilde{\xi}_6 + \frac{1}{2}\left(f_0 - k_0\right)\tilde{\xi}_7 \tag{3.39}$$

$$\tilde{\xi}_3' = 3e^{-2x_0-6p_0}\tilde{\xi}_3 + \left[5e^{-2x_0-6p_0} - e^{-2x_0}\,\left(2Pf_0 - F_0\left(f_0 - k_0\right)\right)\right]\tilde{\xi}_1 \tag{3.40}$$

$$\begin{aligned}\tilde{\xi}_2' &= \tilde{\xi}_2\cosh y_0 + \frac{1}{3}\sinh y_0\left(2\tilde{\xi}_1 + \tilde{\xi}_3 + \tilde{\xi}_4\right)\\ &\quad + 2\left[\left(Pe^{2y_0} - \cosh(2y_0)F_0\right)\tilde{\xi}_5 + \left(Pe^{2y_0} - \sinh(2y_0)F_0\right)\tilde{\xi}_6\right]\,.\end{aligned} \tag{3.41}$$

The key development we present here is to solve for all the $\tilde{\xi}_a$ in terms of two simple integrals, one of which is the KS warp factor:

$$h(\tau) = h_0 - 32P^2\int_0^\tau \frac{u\coth u - 1}{\sinh^2 u}\left(\cosh u\sinh u - u\right)^{1/3}\,du\,, \tag{3.42}$$

$$j(\tau) = \int^\tau \frac{du}{\left(\cosh u\sinh u - u\right)^{2/3}}\,, \tag{3.43}$$

with $h_0 = 18.2373\,P^2$ a numerical constant.

In solving the system of $\tilde{\xi}$ equations, we make the following key observations. In the equations for $\tilde{\xi}_1$ and $\tilde{\xi}_4$, we note that

$$e^{-2x_0}[2Pf_0 - F_0\left(f_0 - k_0\right)] = \frac{h'}{h}\,. \tag{3.44}$$

This implies

$$\tilde{\xi}_1 = X_1 h(\tau)\,,\qquad \tilde{\xi}_4 = -X_1 h(\tau) + X_4\,. \tag{3.45}$$

To obtain $\tilde{\xi}_8$ we use the relations (which can be easily derived from the first-order flow equations for the KS fields (2.38))

$$Pe^{2y_0} - \sinh(2y_0)F_0 = -\frac{1}{2}\left(f_0 + k_0\right)'\,, \tag{3.46}$$

$$Pe^{2y_0} - \cosh(2y_0)F_0 = -\frac{1}{2}\left(f_0 - k_0\right)'\,, \tag{3.47}$$

which yields

$$\tilde{\xi}_8' = -\frac{1}{2}\left(f_0 + k_0\right)'\tilde{\xi}_5 - \frac{1}{2}\left(f_0 - k_0\right)'\tilde{\xi}_6 + \frac{1}{2}\left(f_0 - k_0\right)\tilde{\xi}_7\,. \tag{3.48}$$

Integrating by parts and using (3.37) and (3.45), we get

$$\tilde{\xi}_8' = -\frac{1}{2}\Big(\left(f_0 + k_0\right)\tilde{\xi}_5\Big)' - \frac{1}{2}\Big(\left(f_0 - k_0\right)\tilde{\xi}_6\Big)' - \frac{1}{6}X_1 h'\,. \tag{3.49}$$

This easily integrates to

$$\tilde{\xi}_8 = -\frac{1}{2}\left(f_0 + k_0\right)\tilde{\xi}_5 - \frac{1}{2}\left(f_0 - k_0\right)\tilde{\xi}_6 - \frac{1}{6}X_1 h(\tau) + X_8\,. \tag{3.50}$$

For $\tilde{\xi}_3$ we observe that, from (2.51)

$$e^{-2x_0-6p_0} = \frac{4}{3}\frac{\sinh^2\tau}{\sinh 2\tau - 2\tau} = \frac{1}{3}\frac{\tilde{\xi}'_{3,h}}{\tilde{\xi}_{3,h}}\,, \tag{3.51}$$

where $\tilde{\xi}_{3,h}$ is the solution to the homogeneous equation, namely

$$\tilde{\xi}_{3,h} = \sinh 2\tau - 2\tau\,. \tag{3.52}$$

As a result,

$$\begin{aligned}\tilde{\xi}'_3 &= 3e^{-2x_0-6p_0}\tilde{\xi}_3 + \left[5e^{-2x_0-6p_0} - e^{-2x_0}\left(2Pf_0 - F_0\left(f_0 - k_0\right)\right)\right]\tilde{\xi}_1 \\ &= 3e^{-2x_0-6p_0}\tilde{\xi}_3 + X_1\Big(\frac{5}{3}\frac{h\tilde{\xi}'_{3,h}}{\tilde{\xi}_{3,h}} - h'\Big)\,. \end{aligned} \tag{3.53}$$

Henceforth,

$$\tilde{\xi}_3 = -\frac{5}{3}X_1 h(\tau) + \frac{2}{3}X_1\tilde{\xi}_{3,h}\int^\tau du\frac{h'}{\tilde{\xi}_{3,h}} + X_3\tilde{\xi}_{3,h}\,. \tag{3.54}$$

The above observations allow us to write the full solution in terms of two simple integrals. We collect here the full solution

$$\tilde{\xi}_1 = X_1 h(\tau)\,, \tag{3.55}$$

$$\begin{aligned}\tilde{\xi}_3 = &-\frac{5}{3}X_1 h(\tau) - \frac{32}{3}P^2 X_1 \mathrm{csch}^2\tau\left(\sinh\tau\cosh\tau - \tau\right)^{4/3} \\ &- \frac{128}{9}P^2 X_1\left(\sinh\tau\cosh\tau - \tau\right)j(\tau) + 2X_3\left(\cosh\tau\sinh\tau - \tau\right)\,,\end{aligned} \tag{3.56}$$

$$\tilde{\xi}_4 = -X_1 h(\tau) + X_4\,, \tag{3.57}$$

$$\tilde{\xi}_5 = -\frac{16P}{3}X_1 j(\tau) + X_5\,, \tag{3.58}$$

$$\tilde{\xi}_6 = -\frac{1}{\sinh\tau}\lambda_6(\tau) - \frac{\cosh\tau\sinh\tau - \tau}{2\sinh\tau}\lambda_7(\tau)\,, \tag{3.59}$$

$$\tilde{\xi}_7 = -\frac{\cosh\tau}{\sinh^2\tau}\lambda_6(\tau) + \frac{-3 + \cosh 2\tau + 2\tau\coth\tau}{4\sinh\tau}\lambda_7(\tau)\,, \tag{3.60}$$

$$\tilde{\xi}_8 = P\left(\tau\coth\tau - 1\right)\coth\tau\,\tilde{\xi}_5 - P\frac{\tau\coth\tau - 1}{\sinh\tau}\tilde{\xi}_6 - \frac{1}{6}X_1 h(\tau) + X_8\,, \tag{3.61}$$

where

$$\lambda_6(\tau) = X_6 + \frac{1}{2}\left(-\tau + \coth\tau - \tau\coth^2\tau\right)\tilde{\xi}_5(\tau) + \frac{1}{6}\frac{X_1}{P}h(\tau)\,, \tag{3.62}$$

$$\lambda_7(\tau) = X_7 - \text{csch}^2\tau\,\tilde{\xi}_5(\tau) + \frac{16}{3}PX_1\text{csch}^2\tau\,(\cosh\tau\sinh\tau - \tau)^{1/3} + \frac{64}{9}PX_1 j(\tau)\,. \tag{3.63}$$

Finally, $\tilde{\xi}_2$ can be obtained through the zero–energy condition (3.25) or just by direct integration of (3.41). The latter will introduce another integration constant, X_2, that one could then determine as some combination of the other ones via the zero–energy condition. We find

$$\begin{aligned}
\tilde{\xi}_2 = &-\frac{2}{3}X_3\tau\cosh\tau + \frac{1}{3}X_4\cosh\tau + PX_6\text{csch}\tau\left(\coth\tau - \tau\text{csch}^2\tau\right)\\
&+ PX_5\text{csch}\tau\left(1 - 2\tau\text{coth}\tau + \tau^2\text{csch}^2\tau\right) + X_2\sinh\tau\\
&+ \frac{1}{2}PX_7\left(-2\tau\coth^3\tau + \text{csch}^2\tau + \tau^2\text{csch}^4\tau\right)\sinh\tau\\
&- \frac{1}{108}X_1\Bigg[3\text{csch}^3\tau\,h(\tau)\,(6\tau - 5\sinh 2\tau + \sinh 4\tau)\\
&+ 2P^2\text{csch}^5\tau\Big(-15 + 24\tau^2 + 16\cosh 2\tau - \cosh 4\tau - 32\tau\sinh 2\tau + 4\tau\sinh 4\tau\Big)\\
&\times\left[4\sinh^2\tau j(\tau) - 6\,(\cosh\tau\sinh\tau - \tau)^{1/3}\right]\Bigg]\,.
\end{aligned} \tag{3.64}$$

The zero–energy condition then amounts to

$$X_2 - \frac{2}{3}X_3 - PX_5 - \frac{3}{2}PX_7 = 0\,. \tag{3.65}$$

3.3.2 $\tilde{\phi}^a$ equations

The analytic expressions we obtain for the eight $\tilde{\phi}^a$ modes are all double integrals, except for $\tilde{\phi}^4$ where we obtain a triple integral. This is a considerable improvement over all previous works. Depending on the reader's taste, the expressions may appear somewhat cumbersome but they will be crucial for explicit numerical computations that we will discuss in the next chapter.

To solve the system of ϕ^a, we also use a shifted basis [25]

$$\tilde{\phi}_a = (x - 2p - 5A, y, x + 3p, x - 2p - 2A, f, k, F, \Phi)\,. \tag{3.66}$$

The system of equations for the $\tilde{\phi}^a$ modes is (in the order in which we actually solve them)

$$\tilde{\phi}_8' = -4e^{-4A_0-4p_0}\tilde{\xi}_8\,, \tag{3.67}$$

$$\tilde{\phi}_2' = -\cosh y_0\tilde{\phi}_2 - 2e^{-4A_0-4p_0}\tilde{\xi}_2\,, \tag{3.68}$$

$$\tilde{\phi}_3' = -3e^{-6p_0-2x_0}\tilde{\phi}_3 - \sinh y_0\,\tilde{\phi}_2 - \frac{1}{6}e^{-4A_0-4p_0}\left(9\tilde{\xi}_1 + 5\tilde{\xi}_3 + 2\tilde{\xi}_4\right)\,, \tag{3.69}$$

$$\tilde{\phi}_1' = 2e^{-6p_0-2x_0}\tilde{\phi}_3 - \sinh y_0\,\tilde{\phi}_2 + \frac{1}{6}e^{-4A_0-4p_0}\left(\tilde{\xi}_1 + 3\tilde{\xi}_4\right)\,, \tag{3.70}$$

$$\tilde{\phi}_5' = e^{2y_0}\,(F_0 - 2P)\left(2\tilde{\phi}_2 + \tilde{\phi}_8\right) + e^{2y_0}\tilde{\phi}_7 - 2e^{-4A_0-4p_0+2x_0+2y_0}\left(\tilde{\xi}_5 + \tilde{\xi}_6\right)\,, \quad (3.71)$$

$$\tilde{\phi}_6' = e^{-2y_0}\left[F_0\left(2\tilde{\phi}_2 - \tilde{\phi}_8\right) - \tilde{\phi}_7\right] - 2e^{-4A_0-4p_0+2x_0-2y_0}\left(\tilde{\xi}_5 - \tilde{\xi}_6\right)\,, \quad (3.72)$$

$$\tilde{\phi}_7' = \frac{1}{2}\left(\tilde{\phi}_5 - \tilde{\phi}_6 + (k_0 - f_0)\,\tilde{\phi}_8\right) - 2e^{-4A_0-4p_0+2x_0}\,\tilde{\xi}_7\,, \quad (3.73)$$

$$\begin{aligned}\tilde{\phi}_4' = &\frac{1}{5}e^{-2x_0}\left[f_0\,(2P - F_0) + k_0F_0\right]\left(2\tilde{\phi}_1 - 2\tilde{\phi}_3 - 5\tilde{\phi}_4\right) + \frac{1}{2}e^{-2x_0}\,(2P - F_0)\,\tilde{\phi}_5 \\ &+ \frac{1}{2}e^{-2x_0}F_0\tilde{\phi}_6 + \frac{1}{2}e^{-2x_0}\,(k_0 - f_0)\,\tilde{\phi}_7 - \frac{1}{3}e^{-4A_0-4p_0}\tilde{\xi}_1\,. \end{aligned} \quad (3.74)$$

The expressions for the $\tilde{\xi}_a$ modes are given in equations (3.55)–(3.61). We will now show the solutions of this system. For clarity, we omit all the lengthy derivations of the results. As in [25], we use the Lagrange method of variation of parameters to obtain a solution in terms of integrals. We then performs various integrations by part in order to reduce the number of nested integrations and to pick a nice basis of integrals.

The general procedure is the following. Given a system of first-order ODEs

$$f_i' = \mathbf{M}_i^j\, f_j + b_i\,, \quad (3.75)$$

where $i = 1, \ldots, n$ and f_i, $\mathbf{M}_j^i$, b_i are functions of τ, we first solve the homogeneous equations, to find solutions $f^j_{(h),i}$. To obtain a solution of the inhomogeneous system, we write a linear combination of the homogeneous solutions, promoting the coefficients to functions of τ. Then we have

$$f_i = \sum_{j=1}^{n} f^j_{(h),i}\Lambda_j(\tau)\,, \qquad \Lambda(\tau) = \int (f^{-1}_{(h)})^i_j\, b_i\,. \quad (3.76)$$

For $n = 1$ we simply get

$$f = f_{(h)}\int^\tau \frac{b(u)}{f_{(h)}(u)}\,du\,, \quad f_{(h)} = \exp\int M\,. \quad (3.77)$$

The integration constant depends on the limit of the integration in Λ.

The $\tilde{\phi}^8$ solution

By directly integrating (3.67) and a little bit of massaging, we arrive at

$$\begin{aligned}\tilde{\phi}_8 =& Y_8 - 64\,X_8\,j(\tau) + \frac{X_7}{P}\,h(\tau) \\ &- 64\,P\,X_6\int^\tau \frac{(u\,\coth u - 1)}{\sinh^2 u\,(\cosh u\,\sinh u - u)^{2/3}}\,du \\ &+ \frac{2}{P}\,h(\tau)\,\tilde{\xi}_5(\tau) + \frac{16}{3}\,X_1\,\mathrm{csch}^2\tau\,(\cosh\tau\,\sinh\tau - \tau)^{1/3}\,h(\tau) \\ &+ \frac{64}{9}\,X_1\,h(\tau)\,j(\tau) + \frac{64}{3}\,X_1\int^\tau \frac{\left(\sinh^2 u + 1 - u\,\coth u\right)}{\sinh^2 u\,(\cosh u\,\sinh u - u)^{2/3}}\,h(u)\,du\,. \end{aligned} \quad (3.78)$$

The $\tilde{\phi}^2$ solution

The solution to equation (3.68) is given by

$$\tilde{\phi}_2 = \mathrm{csch}\tau\,\Lambda_2(\tau)\,, \tag{3.79}$$

where

$$\begin{aligned}
\Lambda_2(\tau) =& Y_2 - 16\,P\,X_7 \int^\tau \frac{\left(-2\,u\,\coth^3 u + \mathrm{csch}^2 u + u^2\,\mathrm{csch}^4 u\right)\,\sinh^2 u}{(\cosh u\,\sinh u - u)^{2/3}}\,du \\
& - 32\,P\,X_6 \int^\tau \frac{\coth u - u\,\mathrm{csch}^2 u}{(\cosh u\,\sinh u - u)^{2/3}}\,du - 32\,P\,X_5 \int^\tau \frac{1 - 2\,u\,\coth u + u^2\,\mathrm{csch}^2 u}{(\cosh u\,\sinh u - u)^{2/3}}\,du \\
& - \frac{32}{3}\,X_4 \int^\tau \frac{\cosh u\,\sinh u}{(\cosh u\,\sinh u - u)^{2/3}}\,du + \frac{64}{3}\,X_3 \int^\tau \frac{u\,\cosh u\,\sinh u}{(\cosh u\,\sinh u - u)^{2/3}}\,du \\
& - 48\,X_2\,(\cosh\tau\,\sinh\tau - \tau)^{1/3} + \frac{8}{9}\,X_1 \int^\tau \frac{6\,u - 5\,\sinh 2\,u + \sinh 4\,u}{\sinh^2 u\,(\cosh u\,\sinh u - u)^{2/3}}\,h(u)\,du \\
& - \frac{32}{9}\,P^2\,X_1 \int^\tau \frac{-15 + 24\,u^2 + 16\,\cosh 2\,u - \cosh 4\,u - 32\,u\,\sinh 2\,u + 4\,u\,\sinh 4\,u}{\sinh^4 u\,(\cosh u\,\sinh u - u)^{1/3}}\,du \\
& + \frac{64}{27}\,P^2\,X_1 \int^\tau \frac{-15 + 24\,u^2 + 16\,\cosh 2\,u - \cosh 4\,u - 32\,u\,\sinh 2\,u + 4\,u\,\sinh 4\,u}{\sinh^2 u\,(\cosh u\,\sinh u - u)^{2/3}}\,j(u)\,du\,.
\end{aligned} \tag{3.80}$$

The $\tilde{\phi}^3$ solution

Equation (3.69) is solved by

$$\tilde{\phi}_3(\tau) = \frac{1}{\sinh 2\,\tau - 2\,\tau}\,\Lambda_3(\tau)\,, \tag{3.81}$$

where Λ_3 is specified as

$$\begin{aligned}
\Lambda_3 =& Y_3 - \frac{32}{3}\,X_4 \int^\tau (\cosh u\,\sinh u - u)^{1/3}\,du - \frac{112}{3}\,X_1 \int^\tau (\cosh u\,\sinh u - u)^{1/3}\,h(u)\,du \\
& - \frac{80}{3} \int^\tau (\cosh u\,\sinh u - u)^{1/3}\,\tilde{\xi}_3(u)\,du + 2\,\tau\,\coth\tau\,\Lambda_2(\tau) - 2 \int^\tau u\,\coth u\,\Lambda_2'(u)\,du\,.
\end{aligned} \tag{3.82}$$

Expanding and simplifying this expression, it becomes

$$\begin{aligned}
\Lambda_3 =& Y_3 + 32\,P\,X_7 \int^\tau \frac{u\,\cosh u\,\left(-2\,u\,\coth^3 u + \mathrm{csch}^2 u + u^2\,\mathrm{csch}^4 u\right)\,\sinh u}{(\cosh u\,\sinh u - u)^{2/3}}\,du \\
& + 64\,P\,X_6 \int^\tau \frac{u\,\coth u\,\left(\coth u - u\,\mathrm{csch}^2 u\right)}{(\cosh u\,\sinh u - u)^{2/3}}\,du
\end{aligned}$$

$$
\begin{aligned}
&+ 64\, P\, X_5 \int^\tau \frac{u\, \coth u\, \left(1 - 2\, u\, \coth u + u^2\, \mathrm{csch}^2 u\right)}{(\cosh u\, \sinh u - u)^{2/3}}\, du \\
&+ \frac{32}{3}\, X_4 \left\{ 2 \int^\tau \frac{u\, \cosh^2 u}{(\cosh u\, \sinh u - u)^{2/3}}\, du - \int^\tau (\cosh u\, \sinh u - u)^{1/3}\, du \right\} \\
&- \frac{32}{3}\, X_3 \left\{ 5 \int^\tau (\cosh u\, \sinh u - u)^{4/3}\, du + 4 \int^\tau \frac{u^2\, \cosh^2 u}{(\cosh u\, \sinh u - u)^{2/3}}\, du \right\} \\
&+ 2\, \tau\, \coth \tau\, \Lambda_2(\tau) + 64\, X_2 \int^\tau \frac{u\, \cosh u\, \sinh u}{(\cosh u\, \sinh u - u)^{2/3}}\, du \\
&+ \frac{64}{9}\, X_1 \int^\tau (\cosh u\, \sinh u - u)^{1/3}\, h(u)\, du \\
&+ \frac{10240}{27}\, P^2\, X_1 \int^\tau (\cosh u\, \sinh u - u)^{4/3}\, j(u)\, du \\
&+ \frac{2560}{9}\, P^2\, X_1 \int^\tau \mathrm{csch}^2 u\, (\cosh u\, \sinh u - u)^{5/3}\, du \\
&- \frac{16}{9}\, X_1 \int^\tau \frac{u\, \coth u\, \mathrm{csch}^2 u\, (6\, u - 5\, \sinh 2\, u + \sinh 4\, u)}{(\cosh u\, \sinh u - u)^{2/3}}\, h(u)\, du \\
&+ \frac{64}{9}\, P^2\, X_1 \int^\tau \frac{u\, \coth u\, \left(-15 + 24\, u^2 + 16\, \cosh 2\, u - \cosh 4\, u - 32\, u\, \sinh 2\, u + 4\, u\, \sinh 4\, u\right)}{\sinh^4 u\, (\cosh u\, \sinh u - u)^{1/3}}\, du \\
&- \frac{128}{27}\, P^2\, X_1 \int^\tau \left(-15 + 24\, u^2 + 16\, \cosh 2\, u - \cosh 4\, u - 32\, u\, \sinh 2\, u + 4\, u\, \sinh 4\, u\right) \\
&\times \frac{u\, \coth u\, \mathrm{csch}^2 u}{(\cosh u\, \sinh u - u)^{2/3}}\, j(u)\, du\,. \qquad (3.83)
\end{aligned}
$$

The $\tilde{\phi}^1$ solution

Next comes $\tilde{\phi}_1$ which we express concisely in terms of Λ_2 and $\tilde{\phi}_3$:

$$
\begin{aligned}
\tilde{\phi}_1 = &Y_1 + \frac{40}{9}\, X_4\, j(\tau) - \frac{2}{3}\, \tilde{\phi}_3(\tau) - \frac{160}{9}\, X_3 \int^\tau (\cosh u\, \sinh u - u)^{1/3}\, du \\
&+ \frac{5}{3} \int \coth u\, \Lambda_2'(u)\, du - \frac{5}{3} \coth \tau\, \Lambda_2(\tau) + \frac{2560}{27}\, P^2\, X_1 \int^\tau \mathrm{csch}^2 u\, (\cosh u\, \sinh u - u)^{2/3}\, du \\
&+ \frac{10240}{81}\, P^2\, X_1 \int^\tau (\cosh u\, \sinh u - u)^{1/3}\, j(u)\, du - \frac{80}{27}\, X_1 \int^\tau \frac{h(u)}{(\cosh u\, \sinh u - u)^{2/3}}\, du\,.
\end{aligned}
\qquad (3.84)
$$

The $(\tilde{\phi}^5, \tilde{\phi}^6, \tilde{\phi}^7)$ solutions

The fields $\tilde{\phi}_{5,6,7}$ are determined by a system of coupled ordinary differential equations. The homogeneous solutions are easily found and then we apply the Lagrange method of variation of parameters to find the following expressions

$$\tilde{\phi}_5 = \frac{1}{2}\,\mathrm{sech}^2(\tau/2)\;[\tau + 2\,\tau\,\cosh\tau - (2+\cosh\tau)\,\sinh\tau]\;\Lambda_5(\tau) + \frac{1}{1+\cosh\tau}\,\Lambda_6(\tau) + \Lambda_7(\tau)\,,$$
$$\tilde{\phi}_6 = \left[\tau\,\left(2 - \frac{1}{1-\cosh\tau}\right) - \coth(\tau/2) + \sinh\tau\right]\,\Lambda_5(\tau) + \frac{1}{1-\cosh\tau}\,\Lambda_6(\tau) + \Lambda_7(\tau)\,,$$
$$\tilde{\phi}_7 = (-\cosh\tau + \tau\,\mathrm{csch}\tau)\;\Lambda_5(\tau) - \mathrm{csch}\tau\,\Lambda_6(\tau)\,, \tag{3.85}$$

where

$$\begin{aligned}\Lambda_5 =& Y_5 - \frac{1}{2}\,P\;(\tau\,\coth\tau - 1)\;\mathrm{csch}^2\tau\,\tilde{\phi}_8(\tau) - 32\,P\int^\tau \frac{(u\,\coth u - 1)\;\mathrm{csch}^2 u}{(\cosh u\,\sinh u - u)^{2/3}}\,\tilde{\xi}_8(u)\,du\\
&+ \frac{1}{4}\,X_7\int^\tau \mathrm{csch}^4 u\;[2\,u\;(2+\cosh 2\,u) - 3\,\sinh 2\,u]\;h(u)\,du - X_6\int^\tau \frac{2+\cosh 2\,u}{\sinh^4 u}\,h(u)\,du\\
&+ \int^\tau \mathrm{csch}^2 u\;\left[-3\,\coth u + u\,\left(2 + 3\,\mathrm{csch}^2 u\right)\right]\;h(u)\,\tilde{\xi}_5(u)\,du - \frac{1}{2}\,P\,\frac{\cosh\tau\,\sinh\tau - \tau}{\sinh^4\tau}\,\Lambda_2(\tau)\\
&+ \frac{1}{2}\,P\int^\tau \mathrm{csch}^4 u\;(\cosh u\,\sinh u - u)\;\Lambda_2'(u)\,du - \frac{X_1}{6\,P}\int^\tau (2+\cosh 2\,u)\;\mathrm{csch}^4 u\,h^2(u)\,du\\
&+ \frac{16}{9}\,P\,X_1\int^\tau \mathrm{csch}^4 u\;[2\,u\;(2+\cosh 2\,u) - 3\,\sinh 2\,u]\;j(u)\,h(u)\,du\\
&+ \frac{4}{3}\,P\,X_1\int^\tau \mathrm{csch}^6 u\;(\cosh u\,\sinh u - u)^{1/3}\;[2\,u\;(2+\cosh 2\,u) - 3\,\sinh 2\,u]\;h(u)\,du\,,\end{aligned} \tag{3.86}$$

$$\begin{aligned}\Lambda_6 =& Y_6 - \frac{1}{2}\,P\;\left[-\tau + \coth\tau + \tau\;(-2 + \tau\,\coth\tau)\;\mathrm{csch}^2\tau\right]\;\tilde{\phi}_8(\tau)\\
&- 32\,P\int^\tau \frac{\left[-u + \coth u + u\;(-2 + u\,\coth u)\;\mathrm{csch}^2 u\right]}{(\cosh u\,\sinh u - u)^{2/3}}\,\tilde{\xi}_8(u)\,du\\
&+ \frac{1}{2}\,X_7\int^\tau \left[\cosh 2\,u + \mathrm{csch}^2 u\,\left(3 + 2\,u^2 - 6\,u\,\coth u + 3\,u^2\,\mathrm{csch}^2 u\right)\right]\;h(u)\,du\\
&+ X_6\int^\tau \mathrm{csch}^2 u\;\left[3\,\coth u - u\,\left(2 + 3\,\mathrm{csch}^2 u\right)\right]\;h(u)\,du\\
&+ \int^\tau \left[1 + \left(3 + 2\,u^2 - 6\,u\,\coth u\right)\,\mathrm{csch}^2 u + 3\,u^2\,\mathrm{csch}^4 u\right]\;h(u)\,\tilde{\xi}_5(u)\,du\\
&- \frac{1}{2}\,P\;\left[2\,\coth^2\tau\;(-1 + \tau\,\coth\tau) + \mathrm{csch}^2\tau - \tau^2\,\mathrm{csch}^4\tau\right]\Lambda_2(\tau)\\
&+ \frac{1}{2}\,P\int^\tau \left[2\,\coth^2 u\;(-1 + u\,\coth u) + \mathrm{csch}^2 u - u^2\,\mathrm{csch}^4 u\right]\;\Lambda_2'(u)\,du\\
&+ X_1\int^\tau \left\{\frac{\mathrm{csch}^4 u\;[-2\,u\,(2+\cosh 2\,u) + 3\,\sinh 2\,u]}{12\,P}h(u)\; + \frac{1}{36}\,P\,\mathrm{csch}^6 u\right.\\
&\times\left[8\,j(u)\,\sinh^2 u + 6\;(\cosh u\,\sinh u - u)^{1/3}\right]\left[-28 + 32\,u^2 + \left(31 + 16\,u^2\right)\,\cosh 2\,u\right.\end{aligned}$$

$$- 4\,\cosh 4\,u + \cosh 6\,u - 48\,u\,\sinh 2\,u\Big]\Big\}\,h(u)\,du \tag{3.87}$$

and

$$\begin{aligned}
\Lambda_7 =& Y_7 + P\,\left[-\tau + \coth\tau + \tau\,\left(-2 + \tau\,\coth\tau\right)\,\mathrm{csch}^2\tau\right]\,\tilde{\phi}_8(\tau) \\
&+ 64\,P \int^\tau \frac{\left[-u + \coth u + u\,\left(-2 + u\,\coth u\right)\,\mathrm{csch}^2 u\right]}{\left(\cosh u\,\sinh u - u\right)^{2/3}}\,\tilde{\xi}_8(u)\,du \\
&+ X_7 \int^\tau \left[-1 + \left(-3 - 2\,u^2 + 6\,u\,\coth u\right)\,\mathrm{csch}^2 u - 3\,u^2\,\mathrm{csch}^4 u\right]\,h(u)\,du \\
&+ X_6 \int^\tau \mathrm{csch}^4 u\,\left[2\,u\,\left(2 + \cosh 2\,u\right) - 3\,\sinh 2\,u\right]\,h(u)\,du \\
&+ \int^\tau \left[-2 - 2\,\mathrm{csch}^2 u\,\left(3 + 2\,u^2 - 6\,u\,\coth u + 3\,u^2\,\mathrm{csch}^2 u\right)\right]\,h(u)\,\tilde{\xi}_5(u)\,du \\
&- P\,\mathrm{csch}^2\tau\,\left(1 - 2\,\tau\,\coth\tau + \tau^2\,\mathrm{csch}^2\tau\right)\,\Lambda_2(\tau) \\
&+ P \int^\tau \mathrm{csch}^2 u\,\left(1 - 2\,u\,\coth u + u^2\,\mathrm{csch}^2 u\right)\,\Lambda_2'(u)\,du \\
&+ X_1 \int^\tau \Bigg\{\frac{\mathrm{csch}^4 u\,\left[2\,u\,\left(2 + \cosh 2\,u\right) - 3\,\sinh 2\,u\right]}{6\,P}\,h(u) - \frac{1}{9}\,P\,\mathrm{csch}^6 u \\
&\times \left[8\,j(u)\,\sinh^2 u + 6\,\left(\cosh u\,\sinh u - u\right)^{1/3}\right] \\
&\times \left[-9 + 16\,u^2 + 8\,\left(1 + u^2\right)\,\cosh 2\,u + \cosh 4\,u - 24\,u\,\sinh 2\,u\right]\Bigg\}\,h(u)\,du\,.
\end{aligned} \tag{3.88}$$

The $\tilde{\phi}^4$ solution

While all the $\tilde{\phi}^a$ modes so far have been double integrals, we obtain for $\tilde{\phi}_4$ a triple integral expression

$$\begin{aligned}
\tilde{\phi}_4(\tau) =& \frac{1}{h(\tau)}\Bigg\{Y_4 - \frac{16}{3}\,X_1 \int^\tau \frac{h(u)^2}{\left(\cosh u\,\sinh u - u\right)^{2/3}}\,du + 32\,P \int^\tau \frac{\left(u\,\coth u - 1\right)\,\mathrm{csch}^2 u\,\Lambda_6(u)}{\left(\cosh u\,\sinh u - u\right)^{2/3}}\,du \\
&+ 16\,P \int^\tau \frac{\Lambda_7(u)}{\left(\cosh u\,\sinh u - u\right)^{2/3}}\,du + \frac{32}{5}\,P \int^\tau \left(u\,\coth u - 1\right)\,\mathrm{csch}^2 u\,\left(\cosh u\,\sinh u - u\right)^{1/3} \\
&\times \left[5\,\Lambda_5(u) + 2\,P\,\left(-\tilde{\phi}_1(u) + \tilde{\phi}_3(u)\right)\right]\,du\Bigg\}\,.
\end{aligned} \tag{3.89}$$

It may be possible that this expression can also be reduced to double integrals, but we could not find any obvious way to do it and we will not need a simpler expressions in the following. This completes the solution to the system.

3.4 Boundary conditions and anti–D3 branes

The deformation space we have solved for is a fifteen–dimensional linear space (one of the X's can be eliminated through the zero–energy condition (3.65)) and contains numerous solutions, out of which one can fish out the possible solution for backreacted anti–D3 branes by imposing appropriate boundary conditions.

The strategy for doing this is explained in detail in [25]: one should eliminate all integration constants that give divergent fields in the IR or non-normalizable modes in the UV. Furthermore, to argue that the solution corresponds to D3 branes one should set the divergence in the warp factor perturbation (given by $\tilde{\phi}_4$) to be commensurate with the divergence coming from the five–form. This should fix all the integration constants in terms of the would–be anti–D3 charge. Nevertheless, to obtain the precise values of these constants, one needs to relate the infrared and ultraviolet expansion parameters of the modes we presented in this chapter, and this can only be done by numerical integration. This analysis will be the subject of the next chapter.

Chapter 4

Anti-D3 branes on the Klebanov-Strassler geometry

In this chapter we study the full numerical solution for the 15-dimensional space of linearized deformations of the Klebanov-Strassler background which preserve the $SU(2) \times SU(2) \times \mathbb{Z}_2$ symmetries, which we constructed in closed form in the previous chapter. We identify within this space the solution corresponding to anti-D3 branes, modulo the presence of a certain "subleading" singularity in the infrared. All the 15 integration constants of this solution are fixed in terms of the number of anti-D3 branes, and the solution differs in the UV from the supersymmetric solution into which it is supposed to decay by a mode corresponding to a rescaling of the field theory coordinates. Deciding whether two solutions that differ in the UV by a rescaling mode are dual to the same theory is involved even for supersymmetric Klebanov-Strassler solutions, and we explain in detail some of the subtleties associated to this. We then discuss in some detail the infrared singularity of the anti-D3 solution, and we present a physical argument which shows that such singularity is not due to the linearized approximation. This chapter is based on [21] and [134].

4.1 Introduction

As we discussed in the first chapters, antibranes in warped deformed conifold backgrounds [116] are a staple ingredient of string phenomenology and cosmology constructions, being essentially the only generic method for lifting AdS solutions with stabilized moduli, to dS solutions, and thus give rise to a landscape of dS vacua of string theory [110]. These configurations are also believed to be dual to non-supersymmetric metastable states in the KS confining gauge theory [111, 58].

In the previous chapter we constructed the full space of first-order $SU(2) \times SU(2) \times \mathbb{Z}_2$-invariant deformations around the KS background. We now want to use these solutions to establish whether a solution corresponding to anti-D3 branes in this background exists, whether it has the properties one expects from the brane-probe analysis of [111], and whether it is dual to a metastable vacuum of the dual boundary theory. The underlying philosophy of this investigation is that one cannot decide a-priori that a metastable anti-D3 brane solution must exist, and then accept whatever boundary conditions are necessary in order for this to happen, but rather one should start from a set of physical infrared and ultraviolet boundary conditions, and ask whether a solution compatible with these boundary conditions exists or not.

Partial results about this investigation, which we will review in the following, where obtained in [25, 19], and they can be summarized in the following points:

- The force on a probe D3 brane in the first-order perturbed background depends only on *one* of the 16 integration constants, and this constant must be nonzero if the solution is to correspond to antibranes [25]. Furthermore, the full functional expression of this force can be calculated [19], and matches exactly the expression one obtains from considering the action of probe anti-D3 branes in a background with backreacted D3 branes à la KKLMMT [109].

- The putative solution for anti-D3 branes smeared on the three-sphere at the tip of the KS solution is expected to have a singularity in the five-form and warp factor, coming from the physical brane sources. Besides this, the solution linear in $\bar{N}/M$ has three-form RR and NS-NS field strengths that diverge at the tip, but are subleading with respect to the five-form and warp factor. This subleading singularity is proportional to the coefficient of the brane-attracting mode of the solution.

As explained in [25], if the singularity is not physical, then the backreaction of anti-D3 branes in the KS solution gives rise to a large deformation of this solution, which cannot be captured in perturbation theory, much like when one tries to construct metastable vacua using type IIA brane engineering [16]. On the other hand, if the singularity is physical, then our technology produces the full first-order backreacted solution corresponding to antibranes in the KS background, as well as all first-order deformation of the KS solution by non-normalizable $SU(2) \times SU(2) \times \mathbb{Z}_2$-invariant modes, corresponding to all the relevant and irrelevant deformations of the dual field theory.

In the following chapters we will discuss in detail about the nature of the singularity and about its possible resolution in string theory. Before coming to that, in this chapter we ask whether inside the 15-dimensional space of parameters that characterize our first-order solution one can identify a solution that has the correct physics to correspond to anti-D3 branes in the KS geometry, subleading singularity aside. Identifying this solution inside the 15-dimensional space is simpler than finding a needle in a haystack, but not by far: One has to throw away divergent terms both in the UV and in the IR expansion [25], and to impose the correct D-brane boundary conditions on the divergence of the warp factor and electric field in the infrared.

Those conditions yield algebraic relations between the various integration constants that appear in the UV or IR expansions of the fields; however, the integration constant that appears in the UV expansion of a given field, say the dilaton, is not the same as the one that appears in its IR expansion, but differs by highly nontrivial combination of the other integration constants. Hence, even if we impose all the physical boundary conditions in the UV and in the IR, we are far from being done, because the UV conditions are expressed using the UV integration constants, and the IR conditions are expressed using the IR integration constants, and it is possible that upon translating the UV conditions into IR variables one may have the unpleasant surprise that these conditions are incompatible. Hence, in order to establish whether there is an antibrane solution, to correctly identify it inside the 15-dimensional space of first-order deformations, and to establish whether this solution is dual or not to a metastable vacuum of a supersymmetric field theory, it is crucial to relate the UV and IR solutions, which is the main subject of the following sections.

Before unveiling those results, we would like to point out that identifying whether two asymptotically-KS supergravity solutions are dual to vacua of the same field theory is not as straightforward as it might seem, even for supersymmetric solutions, essentially because, besides the seven normalizable and seven non-normalizable deformations, there exists another deformation corresponding to rescaling the four-dimensional coordinates. Of course, if two solutions differ by non-normalizable deformations, they clearly are dual to two different field theories; however, as we will explain in section 4.4, two solutions that differ by a rescaling of the field theory coordiates, though technically the same, may or may not belong to the same theory. Hence, using purely UV data one cannot distinguish asymptotically-KS supersymmetric solutions that we expect [68] to be dual to different field theories, unless one introduces extra assumptions about the infrared of the solutions, or about their bulk behavior.

Anticipating our results, we compute the unique solution that has the correct infrared and ultraviolet divergences (modulo the subleading singularity) to describe anti-D3 branes in the KS background. All the parameters of this solutions can be determined in terms of the number of antibranes.

We then discuss in detail the nature of the infrared singularity, and we will argue that, even if the linearized approximation we use breaks down near the source, the singularity is not an artifact of the perturbation scheme, and will be present in the full non-linear solution as well. We will come back on this point with more technical details in the next chapter.

In order to simplify the reading of this chapter, we include a brief overview. In section 4.2 we recall very briefly the results of the previous chapter and of some previous papers on the subject [25, 19]. In section 4.3 we explain the procedure we use to relate the UV and the IR integration constants, and illustrate with more details how this procedure can be implemented for one of the perturbation modes.

We also give the relations between the UV and IR integration constants of the other modes; the derivation of all these relations is left for appendix D. In section 4.4 we present the different criteria for distinguishing supersymmetric asymptotically-KS solutions, and in section 4.5 we identify the solution for anti-D3 branes inside the space of solutions. Section 4.6 is devoted to the relation between our solution and the one obtained in [58] by perturbing around the Klebanov-Tseytlin (KT) solution, and to the identification within our space of solutions to perturbation of the KS solution by non-normalizable modes dual to gaugino masses. In section 4.7 we study in more detail the infrared singularity of the anti-D3 solution, and we show that both the ISD and AISD fluxes $G_\pm$ are singular. We present an argument which shows that this singularity is not an artifact of perturbation theory and we discuss the relation with other works on anti-brane backreaction.

4.2 Setup

We wish to construct the backreacted solution corresponding to $\bar{N}$ anti-D3 branes smeared on the S^3 at the tip of the warped deformed conifold. Since the smearing preserves the symmetries of the background solution, we use the Ansatz proposed by Papadopoulos and Tseytlin [144], which is the most general one (with vanishing RR axion C_0) that preserves the $SU(2)\times SU(2)\times \mathbb{Z}_2$-symmetry of the Klebanov-Strassler solution (KS). The metric and the fluxes are given in (3.27), (3.28).

The fields from this Ansatz are collectively denoted ϕ^a, $a = 1, ..., 8$. In the previous chapter we studied and fully determined the solution space of first-order non-supersymmetric deformations of the supersymmetric Klebanov-Strassler theory,

$$\phi^a = \phi^a_0 + \phi^a_1(Z) + \mathcal{O}(Z^2)\,. \tag{4.1}$$

The background fields ϕ^a_0 are given by the Klebanov-Strassler solution without mobile D3-branes, summarized in (2.55). We reproduce them here for the reader's convenience:

$$\begin{aligned}
e^x &= \frac{1}{4} h(\tau)^{1/2} \left(\tfrac{1}{2}\sinh(2\tau) - \tau\right)^{1/3}\,,\\
e^y &= \tanh(\tau/2)\,,\\
e^{6\,p} &= 24 \frac{\left(\tfrac{1}{2}\sinh(2\,\tau) - \tau\right)^{1/3}}{h(\tau)\sinh^2\tau}\,,\\
e^{6\,A} &= \frac{\varepsilon_0^4}{3\cdot 2^9} h(\tau) \left(\tfrac{1}{2}\sinh(2\tau) - \tau\right)^{2/3} \sinh^2\tau\,,\\
f &= -P \frac{(\tau\coth\tau - 1)\,(\cosh\tau - 1)}{\sinh\tau}\,,\\
k &= -P \frac{(\tau\coth\tau - 1)\,(\cosh\tau + 1)}{\sinh\tau}\,,
\end{aligned} \tag{4.2}$$

$$
\begin{aligned}
F &= P\frac{(\sinh\tau-\tau)}{\sinh\tau}\,,\\
\Phi &= 0\,,\\
Q &= 0\ .
\end{aligned}
$$

where ε_0 is the deformation parameter of the conifold, related to the confinement scale of the dual gauge theory. Of significance are also the warp factor h and the Green's function j for this background:

$$
h(\tau) = 32\,P^2\int_\tau^\infty \frac{u\,\coth u-1}{\sinh^2 u}\,(\cosh u\,\sinh u-u)^{1/3}\,du\,, \tag{4.3}
$$

$$
j(\tau) = -\int_\tau^\infty \frac{du}{(\cosh u\,\sinh u-u)^{2/3}}\,. \tag{4.4}
$$

Note that the last equality in (4.2) implies we are taking $g_s = 1$. Furthermore, the dimensionful constant P is related to the quantized dimensionless units of flux M entering in the rank of the gauge groups of the dual field theory (see section 4.4.2) by

$$
P = \frac{1}{4}\,M\,\alpha'\ , \tag{4.5}
$$

So as to avoid extra clutter, in what follows we take $\alpha' = 1$, and $\varepsilon_0 = 1$.

In the previous chapter we described how, by using a method due to Borokhov and Gubser [41], finding linearized deformations away from a supersymmetric solution can be reduced to solving two sets of first-order ordinary differential equations in the radial variable τ, instead of second-order differential equations. Out of those two sets, the first one forms a closed system for the variables ξ_a that can be thought of as "conjugate momenta" for the perturbations ϕ_1^a of the fields entering our Ansatz (3.27), (3.28). The integration constants associated to that first system are labelled X_a, and are non-zero for a non-supersymmetric solution. The integrations constants from the second system of coupled 1st-order ODE's are denoted Y_a.

For the problem of present interest, i.e. the backreaction of anti-D3's on KS, there is one relation between the constants X_a that has to be obeyed on the whole space of solutions. Namely, the zero-energy condition

$$
6X_2 - 4X_3 - 6PX_5 - 9PX_7 = 0\ . \tag{4.6}
$$

Another integration constant, Y_1 as it happens, looks naively like it can be gauged away by a rescaling of the four-dimensional coordinates but as we will see later plays a crucial role in the physics. We are therefore left with fifteen meaningful integration constants.

Out of those fifteen parameters, the one called X_1 plays a key role. Indeed, the force exterted on a probe D3-brane is directly proportional to it and does not depend on any other integration constant [25]. Its expression was found in [19] and is given by

$$
\begin{aligned}
F_{D3_+} &= \frac{2}{3}\, e^{-2x_0} \xi_1 \\
&= \frac{2}{3}\, e^{-2x_0}\, X_1\, h(\tau)\,, \\
&= \frac{32}{3}\, \frac{2^{2/3}\, X_1}{(\sinh 2\,\tau - 2\,\tau)^{2/3}}\,.
\end{aligned}
\tag{4.7}
$$

One can also use the conventions of [76] to describe the same result for a first order expansion around any warped Calabi-Yau background with ISD flux. Here the derivative of the DBI and WZ actions for D3-branes are respectively proportional to the warp factor $e^{4\,\tilde{A}}$ and the four-form RR potential $C_4 = \alpha\, dx^0 \wedge ... \wedge dx^3$, where in the language of (3.27) and (3.28), we have

$$
\tilde{A} = A + p - \frac{x}{2}\,, \qquad \alpha' = -e^{4\,A + 4\,p - 4\,x} \left[\frac{\pi\, Q}{4} + k\, F + f\ (2\,P - F) \right]\,. \tag{4.8}
$$

The force is found to be

$$
F_{D3_\pm} = \Phi'_\mp\,, \quad \text{where } \Phi_\pm = e^{4\,\tilde{A}} \pm \alpha\,, \tag{4.9}
$$

and by $D3_-$ we mean $\overline{D3}$-branes. The combinations $\Phi_\pm$ are sourced by $D3_\pm$ respectively, and by $|G_\pm|^2$ [59, 67] where $G_\pm = G_3 \mp i * G_3$ and $G_3 = F_3 + ie^{-\phi} H_3$. We refer to section 4.7 for more detail about this notation.

4.3 Numerical integration

In the previous chapter, we found that the fully analytic generic solution to the most general first-order deformation of the Klebanov-Strassler background involves at most two nested integrals of the form

$$
\int^\tau h(u)\, f(u)\, du\ , \quad \text{or} \quad \int^\tau j(u)\, f(u)\, du\,, \tag{4.10}
$$

where $f(\tau)$ is a certain combination of hyperbolic functions. Expressions for the warp factor $h(\tau)$ of the KS background and its Green's function $j(\tau)$ are provided in (4.3) and (4.4).

Let us illustrate this with the result for $\tilde{\phi}_8$, corresponding to shifts in the dilaton, whose expressions is given in (3.78) and we reproduce it here:

$$
\begin{aligned}
\tilde{\phi}_8 =& Y_8 - 64\, X_8\, j(\tau) + \frac{X_7}{P}\, h(\tau) \\
&- 64\, P\, X_6 \int_1^\tau \frac{(u\, \coth u - 1)}{\sinh^2 u\ (\cosh u\, \sinh u - u)^{2/3}}\, du
\end{aligned}
$$

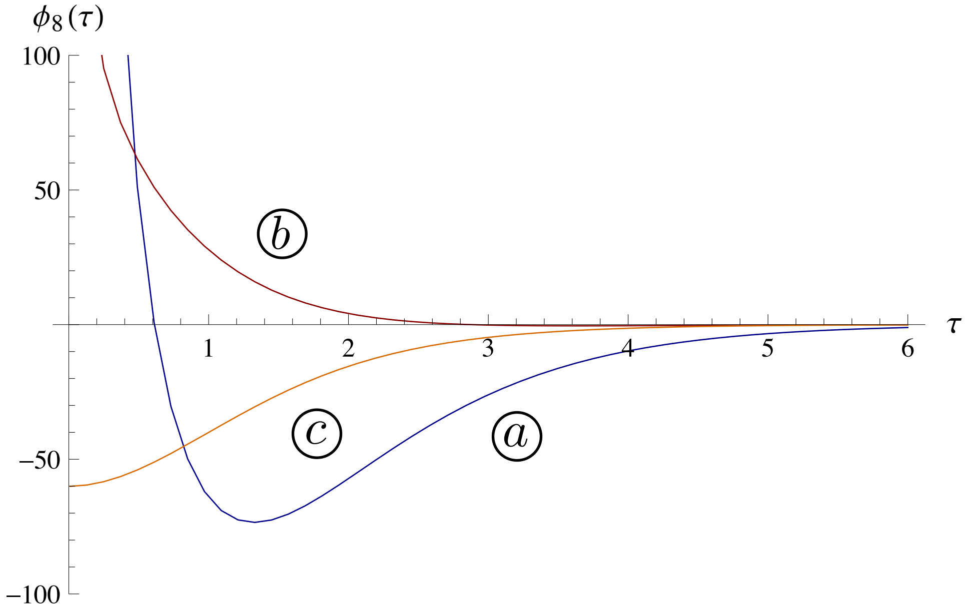


Figure 4.1: The profile of the field $\tilde{\phi}_8$ corresponding to a shift of the dilaton, for the following choices of integration constants (with e.g. $P = 1$). Blue, also labelled ⓐ: $X_1 = 1, X_5 = -\frac{15}{2}, X_6 = X_7 = 5, X_8 = 2, Y_8 = -88.05$; Red ⓑ: $X_1 = X_6 = X_7 = 1, X_5 = -\frac{7}{6}, X_8 = 1.8, Y_8 = -111.5$; Yellow ⓒ: $X_1 = X_7 = 2, X_5 = -\frac{7}{6}, X_6 = 8.608, X_8 = -0.843, Y_8 = -133.9$. In each case, Y_8 is fixed so as to ensure that $\tilde{\phi}_8(\infty) = 0$.

$$+\,2\,\frac{X_5}{P}\,h(\tau) + \frac{16}{3}\,X_1\,\mathrm{csch}^2\tau\;(\cosh\tau\,\sinh\tau-\tau)^{1/3}\;h(\tau)$$
$$+\,\frac{64}{9}\,X_1\,h(\tau)\,j(\tau) - \frac{32}{9}\,X_1\int_1^\tau \frac{\left(\sinh^2 u + 1 - u\,\coth u\right)}{\sinh^2 u\;(\cosh u\,\sinh u - u)^{2/3}}\,h(u)\,du\,. \qquad (4.11)$$

We have chosen to integrate in the domain $[1, \tau]$, given that many of the integrands (like the one from the last term above) are infrared-divergent. Once the limits of integration are fixed, the constant Y_8 in (4.11) is defined unambiguously. The profile for $\tilde{\phi}_8$ is given in Figure 4.1.

The infrared and ultraviolet behaviors of the modes are given in Appendix D. Some of the integration constants appearing in the infrared expansions (like Y_3^{IR} or Y_6^{IR}) correspond to unphysical divergences of various fields, and we will set them to zero. Other constants (like Y_7^{IR} or X_1) correspond to physical divergences in the warp factor and in the RR five-form field strength coming from the presence of smeared anti-D3 branes, and we need to keep them in the final solution. We will explain this procedure when we construct the antibrane solution in section 4.5.

In order to stress out how the integration constants X_a and Y^a are paired into normalizable and non–normalizable modes we also remind the reader of the UV behaviors of those modes [25], which one can also extract from the expansions in Appendix D:

dim Δ	non-norm/norm	integration constants
8	r^4/r^{-8}	Y_4/X_1
7	r^3/r^{-7}	Y_5/X_6
6	r^2/r^{-6}	X_3/Y_3
5	r/r^{-5}	$---$
4	r^0/r^{-4}	$Y_7, Y_8, Y_1/X_5, X_4, X_8$
3	r^{-1}/r^{-3}	$X_2, X_7/Y_6, Y_2$
2	r^{-2}/r^{-2}	$---$

Table 4.1: The UV behavior of all sixteen modes for the $SU(2) \times SU(2) \times \mathbb{Z}_2$-symmetric deformation Ansatz around the Klebanov-Strassler solution.

4.3.1 Relating the IR and UV integration constants

Given that ultimately we will have to impose boundary conditions on the generic analytic solution to the full space of first order deformations around KS, we should look at the IR and UV behavior of the modes $\tilde{\phi}_a$. Moreover, it is not enough to consider the expansions shown in Appendix D. The zeroth-order terms in the expansions collected in that Appendix include arbitrary integration constants coming from indefinite integrations, which are generically denoted as Y_a^{IR}, Y_a^{UV}. In order to determine how the Y_a^{IR}'s are related to the Y_a^{UV}'s and thus to connect the IR and UV regions, we have to perform a numerical integration that will fix Y_a^{UV} as follows:

$$Y_a^{UV} = Y_a^{IR} + \sum_{b=1}^{8} \mathbf{N}_a{}^b \, X_b \,, \tag{4.12}$$

where $\mathbf{N}$ is a matrix of numerical coefficients arising out of evalutions of the single and double integrals appearing in the analytic solutions for the $\tilde{\phi}_a$ modes.

All in all, following the procedure we have just outlined, the relations between all[1] the Y_a^{UV} and Y_a^{IR} that we have derived are as follows:

$$\begin{pmatrix} Y_8^{UV} \\ Y_2^{UV} \\ Y_3^{UV} \\ Y_1^{UV} \\ Y_5^{UV} \\ Y_6^{UV} \\ Y_7^{UV} \end{pmatrix} = \begin{pmatrix} Y_8^{IR} \\ Y_2^{IR} \\ Y_3^{IR} - 2\,Y_2^{IR} \\ Y_1^{IR} - \frac{5}{3}\,Y_2^{IR} \\ Y_5^{IR} + \frac{P}{6}\,Y_8^{IR} \\ Y_6^{IR} + \frac{3\,P}{2}\,Y_2^{IR} - \frac{P}{2}\,Y_8^{IR} \\ Y_7^{IR} - P\,Y_2^{IR} + P\,Y_8^{IR} \end{pmatrix} + \mathbf{N}\cdot \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \end{pmatrix}, \tag{4.13}$$

with the matrix $\mathbf{N}$

$$\mathbf{N} = \begin{pmatrix} -235.3\,P^2 & 0 & 0 & 0 & -36.47\,P & 35.71\,P & -18.24\,P & 53.56 \\ -3.870\,P^2 & 0 & 83.34 & 7.791 & 83.34\,P & -12.37\,P & 166.7\,P & 0 \\ 93.63\,P^2 & 250.0 & 206.7 & 93.84 & -284.0P & 61.22\,P & -243.8\,P & 0 \\ -123.8\,P^2 & -40.25 & 70.31 & -1.827 & 22.93\,P & 35.50\,P & 71.33\,P & 0 \\ -165.9\,P^3 & -20.16\,P & 19.52\,P & 1.488\,P & 14.08\,P^2 & 11.90\,P^2 & 36.32\,P^2 & 17.85\,P \\ 100.6\,P^3 & -166.7\,P & 81.27\,P & -46.06\,P & 221.4\,P^2 & -48.57\,P^2 & 265.8\,P^2 & -8.545\,P \\ -225.8\,P^3 & 83.34\,P & -94.65\,P & 16.52\,P & -158.9\,P^2 & 35.92\,P^2 & -221.4\,P^2 & 17.09\,P \end{pmatrix}.$$

The above relations (4.13) depend at an intermediary stage on our results for the relation between the integration constants Y_a that appear in the analytic solution derived in section 3.3.2 and the constants Y_a^{IR} that appear in the IR expansions (D.3)–(D.10), obtained via the method summarized at the beginning of this section and further expanded upon in the next subsection. We provide them here as a matter of having accessible intermediate results:

$$\begin{pmatrix} Y_8^{IR} \\ Y_2^{IR} \\ Y_3^{IR} \\ Y_1^{IR} \\ Y_5^{IR} \\ Y_6^{IR} \\ Y_7^{IR} \end{pmatrix} = \begin{pmatrix} Y_8 \\ Y_2 \\ 2\,Y_2 + Y_3 \\ Y_1 \\ Y_5 - \frac{P}{6}\,Y_8 \\ -\frac{P}{2}\,Y_2 + Y_6 \\ P\,Y_2 + Y_7 \end{pmatrix} + \mathbf{M}_{(Y^{IR},Y)}\cdot \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \end{pmatrix}, \tag{4.14}$$

[1] Except Y_4, which is far more difficult to get and will not be needed for our following analysis in any case.

$$\mathbf{M}_{(Y^{IR},Y)} =$$

$$\begin{pmatrix}
352.6\,P^2 & 0 & 0 & 0 & 36.47\,P & -41.56\,P & 18.24\,P & -53.56 \\
25.86\,P^2 & 0 & -33.23 & 3.918 & -38.81\,P & -3.432\,P & -69.25\,P & 0 \\
-18.62\,P^2 & -99.69 & 15.54 & -0.9673 & 7.797\,P & -7.959\,P & 15.92\,P & 0 \\
144.4\,P^2 & 98.79 & -67.47 & 5.146 & -81.34\,P & -44.35\,P & -153.9\,P & 0 \\
92.62\,P^3 & 12.26\,P & -9.501\,P & -4.435\,P & -16.54\,P^2 & -18.03\,P^2 & -22.52\,P^2 & -11.85\,P \\
8.129\,P^3 & 24.44\,P & -1.632\,P & 1.147\,P & -4.773\,P^2 & 2.180\,P^2 & -11.20\,P^2 & -3.979\,P \\
-1.307\,P^3 & -38.81\,P & -4.754\,P & 3.491\,P & 1.749\,P^2 & 3.599\,P^2 & -6.256\,P^2 & 7.959\,P
\end{pmatrix} .$$

Analogously, the link between the parameters Y_a^{UV} and Y_a can similarly be obtained from the UV/IR relation (4.13).

4.3.2 An illustration of the procedure

As an example making this procedure plainer to the reader, we show how we relate Y_8^{UV} and Y_8^{IR}. This is a three-stage procedure:
(i) first, we relate Y_8^{IR} and the parameter Y_8 appearing in (4.11);
(ii) we next obtain the relation between Y_8^{UV} and Y_8;
(iii) finally, using results from the above steps, we get Y_8^{UV} in terms of Y_8^{IR}.

In order to implement step (i) above and relate Y_8^{IR} to Y_8, we expand the integrands entering the IR expansion of the solution to the $\tilde{\phi}_8$ equation up to a certain power in τ. We then evaluate the indefinite integral and call Y_8^{IR} the constant term in $\tilde{\phi}_8$. The first few terms in those expansions are given by (D.3), which we provide here for convenience:

$$\tilde{\phi}_8^{IR} = \frac{1}{\tau}\Big(\frac{32}{3}\left(\frac{2}{3}\right)^{1/3}(3PX_6 - h_0X_1) + 32\cdot 2^{1/3}\cdot 3^{2/3}\,X_8\Big) + Y_8^{IR} + \mathcal{O}(\tau)\ . \quad (4.15)$$

We now have to match (4.15) at some small τ with the numerical value of $\tilde{\phi}_8$ that we obtain by performing the integrals in (4.11) numerically. Since the expansions for the integrands are good up to $\tau > 1$, we did choose to match at $\tau = 1$, where the integrals that enter the solutions for the $\tilde{\phi}$'s are zero by definition. Evaluating numerically (4.11) at $\tau = 1$, we find

$$\tilde{\phi}_8(\tau=1) = Y_8 + 84.0493\,P^2\,X_1 + 28.5159\,P\,X_5 + 14.2579\,P\,X_7 + 41.2221\,X_8\,, \quad (4.16)$$

while from the IR expansion of $\tilde{\phi}_8$ (4.15), we have

$$\begin{aligned}\tilde{\phi}_8^{IR}(1) = Y_8^{IR} &- 268.524\,P^2\,X_1 - 7.9588\,P\,X_5 + 41.5621\,P\,X_6 \\ &- 3.97940\,P\,X_7 + 94.786\,X_8\,. \end{aligned} \quad (4.17)$$

Comparing the above two results, (4.16) and (4.17), we finally obtain the end-result of step (i) above:

$$Y_8^{IR} = Y_8 + 352.574\,P^2\,X_1 + 36.4747\,P\,X_5 - 41.5621\,P\,X_6$$

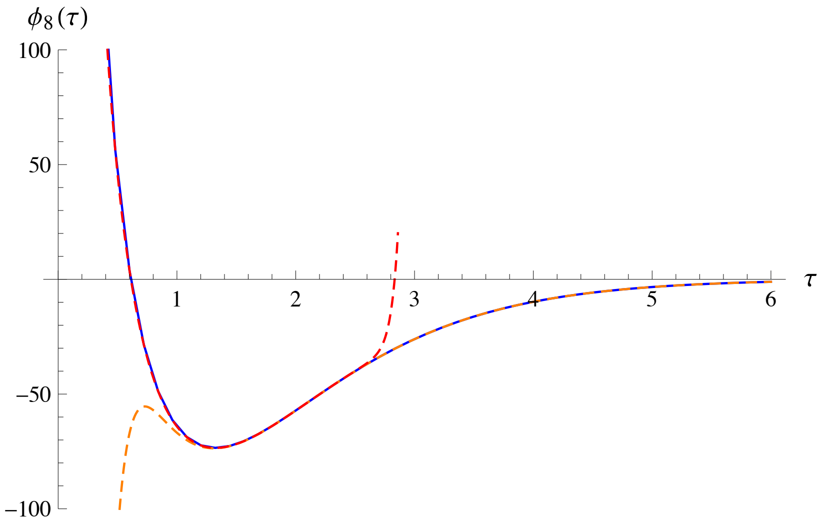


Figure 4.2: The numerical solution for the field $\tilde{\phi}_8$ for $X_1 = 1, X_5 = -\frac{15}{2}, X_6 = 5, X_7 = 5, X_8 = 2, Y_8 = -88.05, P = 1$ (underlying blue solid line). The red and orange dashed lines correspond respectively to the IR and UV expansions.

$$+\,18.2373\,P\,X_7 - 53.5642\,X_8\,. \tag{4.18}$$

With this relation at hand, we can furthermore make sure, as one more consistency test, that the numerical integrals and the series agree at small τ. The result is shown on Figure 4.2.

We go through the same recipe for the UV and compare the value of the UV series of the integrands with the value of $\tilde{\phi}_8$ that we have obtained by performing the integrals numerically[2] at $\tau = 15$. When the dust settles down, we find the following relation between Y_8^{UV} and Y_8:

$$Y_8^{UV} = Y_8 + 117.318\,P^2\,X_1 - 5.85263\,P\,X_6\,. \tag{4.19}$$

As one extra check, inserting the above result in the UV expansions, we can verify that the UV series approximates well our numerical results at large τ. This can also be see on Figure 4.2.

Note that for $\tilde{\phi}_8$ there is a rather large range of overlap between its IR and UV series expansions. So, with hindsight, for this particular mode, we could have avoided going through tedious numerical work. On the other hand, for most of the other $\tilde{\phi}^a$ fields, the overlap is much narrower. Therefore, in order to attain satisfactory precision in relating the IR and UV integration constants, we have opted for a careful numerical analysis.

[2]With as much precision as desired. Here, for both IR and UV expansions, we have settled for 20 orders of WorkingPrecision using Mathematica. The UV series expansions were derived up to order 15.

4.4 Asymptotically KS solutions and their field theory interpretation

Having found the full 15-dimensional space of perturbative solutions around the KS background, we would now like to develop the machinery that will allow us to identify whether the antibrane solution is in the same theory as the supersymmetric background into which it is conjectured to decay [111]. However, as mentioned in the introduction, distinguishing between asymptotically-KS solutions and arguing which background is dual to which field theory using only UV data is not trivial even for supersymmetric solutions, essentially because of the existence of the scale deformation Y_1, which equivalently can be traded for the ε parameter that characterizes the size of the deformed conifold before the warping.

If two solutions differ by non-normalizable deformations, they are dual to two different field theories. However, our fifteen-dimensional deformation space has the peculiarity that there are seven pairs of normalizable/non-normalizable modes and then one extra mode Y_1. The putative partner to Y_1 is eliminated by the zero-energy condition and it may seem that Y_1 itself is a gauge artifact which can be removed by rescaling the four-dimensional space-time coordinates. As we will mention in more detail below, while for a single vacuum this is true, if there are two isolated vacua in the same theory then there remains a dimensionless number (essentially the ratio of the confinement scales) which can be attributed to Y_1.

One can inquire whether two solutions that have the same non-normalizable modes but two different ε's, hence two different scale deformations, are dual to the same field theory. The answer is not clear, because one can change ε and at the same time change also the number of mobile branes, keeping the total charge at infinity constant. Changing ε changes the volume of the space, and since the space has charge dissolved in flux, one also changes the total charge; one can compensate for this change by introducing or taking away mobile branes.

Hence, a vacuum with no mobile branes and one value for ε has exactly the same UV data as a vacuum with one mobile brane and another value of ε, or a vacuum with, say, 17 mobile branes and yet another value of ε. Clearly these solutions cannot be all dual to vacua of the same KS field theory. On the other hand, a background with M mobile branes (where M is the amount of RR three-form flux on the KS three-cycle) and a certain value of ε and another one with no mobile branes were argued in [68] to be dual respectively to the mesonic vacuum and the baryonic vacuum of the same $SU(kM) \times SU(kM + M)$ theory. Hence, even in the supersymmetric theory, one cannot decide whether two vacua with different scales and different amounts of mobile branes are in the same theory by simply examining their UV data.

In this section, we discuss the supersymmetric KS situation in detail, and argue that in order to be able to use UV data to distinguish between two supersymmetric asymptotically-KS solutions that should *not* be dual to the same theory, one must introduce an additional criterion. The most obvious choice is requiring that the value of the NSNS B_2 field that wraps the S^2 which shrinks to zero size at the conifold tip must be zero, and can only jump by integral periods. After all, the S^2 is topologically trivial, and if the integral of B_2 is nonzero, one can stay at a fixed radius, consider a very small closed fundamental string at the north pole and take it around the S^2 to the south pole; during this process its world-sheet action will pick up a phase proportional to the B_2 integral. If one now brings back the string to the north pole, the string will interfere destructively with itself unless the integral of B_2 on S^2 is an integer[3]. This argument is similar to that ruling out Dirac strings, and in principle should also hold in the presence of D3 or anti-D3 branes.

A second possible criterion is requiring that the integrals of the H_3 from the origin to a certain holographic screen differ by an integer amount for two solutions in the same theory, or equivalently that the difference in the number of Seiberg duality cascades between two solutions dual to vacua of the same theory has to be integer-valued. This criterion has a clear physical justification for compact settings, where the KS throat is seen as the zoom-in of a compact CY, and where the three-cycle wrapped by H_3 that appears non-compact from a KS perspective is in fact embedded into a compact CY three-cycle. However, for a non-compact KS solution this criterion is very hard to justify from a holographic perspective, because it involves integrals over the whole bulk.

Since neither the first nor the second criterion satisfy a "hard core holography" point of view, according to which *all* the data of the boundary theory must be readable from the UV of solution that is regular in the bulk, one can also try to use the analysis of [68] to reverse-engineer such a criterion. This third criterion boils down to imposing that two solutions must satisfy equation (4.50) (and its equivalents for higher mesonic vacua) in order to describe vacua of the same theory. This criterion, if correct, would allow one to distinguish between vacua with various numbers of mobile branes without introducing any extra IR boundary conditions, and using only UV data. However, it certainly begs for a more physical explanation.

Of course, another possibility is that the holography is just not refined-enough to distinguish between these different theories, especially because we are dealing with cascading solutions that are not asymptotically AdS, cannot be thought of as the near-horizon of any brane, and have an infinite charge unless one imposes an UV cut-off.

In this section, we will use the first criterion, and give a holographic recipe for distinguishing between asymptotically-KS vacua that have different numbers of mobile branes.

[3] We thank Nick Warner for this argument.

4.4.1 Maxwell charge, Page charge and mobile D3-branes

For a supergravity solution with non-trivial Wess-Zumino terms one can generally define three different types of charges [131, 3], which we review in this section. The D3-Page charge, specialized to the KS background is

$$\mathcal{Q}_{D3}^{Page} \;=\; \frac{1}{(4\,\pi^2)^2}\int_{T^{1,1}}\big(\mathcal{F}_5 - B_2\wedge F_3\big)\,. \tag{4.20}$$

This is conserved and is independent of the radius at which it is evaluated. In string theory it must also be quantized. If we shift B_2 by a small gauge transformation $B_2 \to B_2 + d\Lambda_1$ for some one-form Λ_1, the charge stays invariant. In principle there are two independent ways to generate a non-zero, integer-valued $\mathcal{Q}_{D3}^{Page}$ starting from the smooth KS background:

$$\begin{aligned} \mathcal{F}_5 &\;\to\; \mathcal{F}_5 + 27\,Q\pi\,\mathrm{vol}_{T^{1,1}}\,, && (4.21)\\ B_2 &\;\to\; B_2 + \frac{p}{M}\,\pi\,\omega_2\,, && (4.22)\\ \Rightarrow\; \mathcal{Q}_{D3}^{Page} &\;=\; Q - p && (4.23) \end{aligned}$$

where $(Q,p)\in\mathbb{Z}^2$, M is related to P by (4.5) and

$$\begin{aligned} \mathrm{vol}_{T^{1,1}} =& \frac{1}{108}\, g_1\wedge g_2\wedge g_3\wedge g_4\wedge g_5\\ \omega_2 =& \frac{1}{2}(g_1\wedge g_2 + g_3\wedge g_4)\,. \end{aligned} \tag{4.24}$$

Having $Q\neq 0$ generates a singularity in both the warp factor and $*\mathcal{F}_5$, which one must interpret as due to Q D3 branes smeared on the tip of the deformed conifold. On the other hand, the meaning of the singularity due to $p\neq 0$ is more subtle, and if one imposes as an IR regularity condition that the B_2 field at the KS tip be zero or an integer mod M, then $\mathcal{Q}_{D3}^{Page} = Q$ measures the number (modulo M) of mobile BPS D3-branes in any particular KS background.

The Maxwell D3-charge is

$$\mathcal{Q}_{D3}^{Max} = \frac{1}{(4\pi^2)^2}\int_{T^{1,1}_{r_c}}\mathcal{F}_5\,, \tag{4.25}$$

where the integral is performed on a Gaussian surface at the UV cut-off $r = r_c$. There are two physically distinct contributions to the Maxwell charge, from mobile branes (q_b) and from charge dissolved in flux (q_f):

$$\begin{aligned} \mathcal{Q}_{D3}^{Max} &\;=\; q_b + q_f\,, && (4.26)\\ q_b &\;=\; \frac{1}{(4\,\pi^2)^2}\int_{T^{1,1}_0} F_5\,, && (4.27)\\ q_f &\;=\; \frac{1}{(4\,\pi^2)^2}\Big(\int_{T^{1,1}_{r_c}} F_5 - \int_{T^{1,1}_0} F_5\Big) = \frac{1}{(4\,\pi^2)^2}\int_{M_6} H_3\wedge F_3\,. && (4.28) \end{aligned}$$

The Maxwell charge depends on the scale at which it is measured, but if we fix a holographic screen, we expect physical processes to preserve its value at the screen. In particular, for a given scale, it must be the same if two solutions are to describe different vacua of the same theory. Using the Ansatz (3.28), this is

$$\mathcal{Q}_{D3}^{Max} = Q + \frac{4}{\pi}\,\left[(k-f)\,F + 2\,P\,f\right]\,. \tag{4.29}$$

Note that if we set $\int_{S^2} B_2 = 0$ at the tip (i.e. requiring $f(\tau = 0) = 0$), then we have $Q = q_b = \mathcal{Q}_{D3}^{Page}$ modulo M, while the second term in (4.29) gives the flux contribution to the Maxwell charge.

4.4.2 A dictionary for the charges

Our purpose is to establish using only UV data at a holographic screen whether two asymptotically-KS solutions describe vacua of the same theory. Any particular KS field theory is defined at a scale Λ_c through a gauge group $SU(N_1) \times SU(N_2)$ and the associated gauge couplings (g_1, g_2). The UV data of the supergravity theory consists of $\mathcal{Q}_{D5}^{Max}(= M)$, $\mathcal{Q}_{D3}^{Max}$, $\int_{S^2} B_2$, Φ, and the "standard lore" dictionary between the supergravity UV data and the field theory is

$$\begin{aligned}
N_1 &= \mathcal{Q}_{D3}^{Max} + \mathcal{Q}_{D5}^{Max}\,, && (4.30)\\
N_2 &= \mathcal{Q}_{D3}^{Max}\,, && (4.31)\\
\frac{4\,\pi^2}{g_1^2} + \frac{4\,\pi^2}{g_2^2} &= \pi g_s^{-1}\, e^{-\Phi}\,, && (4.32)\\
\left[\frac{4\,\pi^2}{g_1^2} - \frac{4\,\pi^2}{g_2^2}\right] g_s\, e^{\Phi} &= \left[\frac{1}{2\,\pi\,\alpha'}\int_{S^2} B_2 - \pi\right] \mathrm{mod}\,(2\,\pi)\,, && (4.33)
\end{aligned}$$

as reviewed in [101]. We can also trade the integral of B_2 for $\mathcal{Q}_{D3}^{Page}$ using

$$\int_{S^2_{r_c}} B_2 = (\mathcal{Q}_{D3}^{Max} - \mathcal{Q}_{D3}^{Page})/\mathcal{Q}_{D5}^{Max} = q_f/\mathcal{Q}_{D5}^{Max} + \int_{S^2_0} B_2\,. \tag{4.34}$$

As we will see shortly, this dictionary is in fact more involved.

All this data is defined in the supergravity solution at some UV cut-off r_c related to the field theory scale Λ_c. To obtain this relation, we change to a radial coordinate r such that the metric on the transverse six-dimensional space asymptotes to a warped conical metric:

$$ds_{10}^2 = h^{-1/2}\, ds_{1,3}^2 + h^{1/2}\, ds_6^2\,, \tag{4.35}$$

with

$$ds_6^2 \;\sim\; dr^2 + r^2\, ds^2_{T^{1,1}}\,, \quad r >> 1\,.$$

For any KS background (4.2), this r coordinate is related to the deformed-conifold τ coordinate via

$$r^2 \;=\; \frac{3}{2^{5/3}}\,\varepsilon_0^{4/3}\,e^{2\tau/3}\,. \tag{4.36}$$

The field theory cut-off Λ_c should then be identified with the holographic cut-off r_c. Note that from the point of view of the τ coordinate, the parameter ε only enters the function A from the Ansatz, and changing it corresponds to a rescaling of the four-dimensional metric (see (4.2)).

We now run into the first puzzle, which can be expressed on the supergravity side alone. According to the dictionary above, since the field theory gauge group ranks depend only on $\mathcal{Q}_{D3}^{Max}$ but not on $\mathcal{Q}_{D3}^{Page}$ or q_b, one can see from equation (4.26) that the duals to solutions with different q_b and q_f but the same $\mathcal{Q}_{D3}^{Max}$ have the same charges and should be dual to the same field theory. This is achieved by shortening the domain of integration in (4.28), which lowers q_f, and by increasing q_b to compensate this. Hence, the only UV holographic data that will be different between, say, a solution with no mobile branes and a solution with one mobile brane will be the integral of B_2 on the S^2. However, this difference is not gauge-invariant, and if one does not impose any infrared boundary condition on B_2, we can see from (4.34) that this value is arbitrary, and hence nothing in the UV will distinguish between a solution with one mobile brane and one with no mobile brane; we expect this to be incorrect.

One way to remedy this is to impose an IR boundary condition, namely that the integral of B_2 on the shrunken S^2 at the tip be gauge-equivalent to zero. If so, then two solutions with different numbers of mobile branes and different q_f will have different B fields in the UV, and will correspond to different theories. The only situation when the UV fields will be the same is when the number of mobile branes differs by multiples of M, when indeed we expect these solutions to correspond to different vacua of the same theory [68]. In the next subsection we will illustrate this in detail using our perturbation theory machinery.

The second quandary has to do with the field theory interpretation of two solutions that have the same $\mathcal{Q}_{D3}^{Max}$ but different numbers of mobile branes. If one is to take a holographic screen at r_c and use the dictionary (4.30,4.31,4.32,4.33), a solution with $p < M$ mobile branes and one with none will be dual to two field theories that have the same ranks of the gauge group at the same cutoff, but differ only in the coupling constant. Furthermore, a solution that has $\mathcal{Q}_{D3}^{Max} = M + 1$ at a holographic screen at r_c will have $\mathcal{Q}_{D3}^{Max} = M$ at a holographic screen placed further down in the infrared; this would appear to imply that a theory with rank $SU(2M + 1) \times SU(M + 1)$ at some energy flows at lower energies to a theory with rank $SU(2M)\times SU(M)$, then $SU(2M-1)\times SU(M-1)$, which is definitely incorrect.

A partial solution to this puzzle is given by a comment in [101], where it was noted that one cannot relate the UV supergravity data to field theory data at an arbitrary UV holographic screen. The dictionary (4.30,4.31,4.32,4.33) can only be used at special values of r_c, given by the requirement that from the infrared up to that scale the number of duality cascades is an integer, or alternatively, that the value of q_f is a multiple of M. This is a stronger requirement than demanding that the ranks of the putative dual gauge groups are integer-valued. We will call for convenience the holographic screens at which one can define the dictionary "K-screens."

However, this cannot be the whole story. As we can see from equation (4.34), this restriction alongside the requirement that B_2 be zero at the tip imply that the value of the B_2 integral at the K-screen is a multiple of M, and hence the two field theory coupling constants will have the same values at any K-screen. Thus, at those screens (which are the only places where the field theory has an approximate Lagrangian description), the right-hand side of equation (4.33) is always equal to π, and the coupling constant of one of the gauge group always becomes infinite. Conversely, out of the set of possible field theory data defined at a scale Λ_c via the 4 parameters N_1, N_2, g_1 and g_2, the KS supergravity solutions would only describe field theories that belong to a codimension-one subspace, and hence not the most generic field theory.

In order to avoid the above-mentioned problems, equations (4.32) and (4.33) should be used to obtain the values of the coupling constants as a function of the corresponding energy Λ_c. However, the ranks of the gauge groups given in equations (4.30),(4.31) must be read from the K-screen right above it. Those equations then provide the ranks of the gauge groups both at the scale corresponding to r_c and at the scale corresponding to the K-screen above. The ranks do not change when one changes the position of the holographic screen by decreasing r_c, unless one crosses another K-screen, which corresponds to a Seiberg duality in the dual theory.

One can also ask how can a holographist tell, using purely UV data, where the K-screen lies. The answer is given by (4.33) – the screen is at the location above r_c where the B_2 integral is gauge equivalent to zero. Hence, if the B_2 integral at the tip is zero, this dictionary gives a way to relate all 4 parameters of the field theory to the four parameters of the supergravity solution, using UV data alone.

4.4.3 Baryonic and mesonic branches

When the ranks of the two gauge groups are

$$N_1 = (k+1)\,M\,, \quad N_2 = k\,M\,, \quad k \in \mathbb{Z} \tag{4.37}$$

the theory has two classically disconnected supersymmetric moduli spaces, the baryonic and mesonic branches [68]. For more general (N_1, N_2) the mesonic branch is supersymmetric while the baryonic branch is lifted. It is instructive to use the dictionary above together with the infrared boundary condition for B_2 to demonstrate in the supergravity perturbation theory framework we have developed that when they exist, both the baryonic and mesonic branches are indeed different vacua of the same theory.

As mentioned in section (4.4.1), if one imposes $\int_{S^2} B_2 = 0$ modulo M at the tip, then the function f shoud go to zero at the origin. On the other hand, we have from (D.7) in Appendix D that

$$\tilde{\phi}_5(\tau = 0) = f(\tau = 0) + \frac{\pi\, Q}{2\, M} = Y_7^{IR}\,, \tag{4.38}$$

where we have set $Y_6^{IR} = 0$ since this mode diverges as $1/\tau^3$, and we have used the relation between P and M from (4.5). This implies that in our perturbation theory

$$\mathcal{Q}_{D3}^{Page} = Q = \frac{2}{\pi}\, M\, Y_7^{IR}\,. \tag{4.39}$$

Setting this equal to an integer multiple of $-M$, leads to[4]

$$\begin{aligned} \mathcal{Q}_{D3}^{Page} &= -\ell\, M\,, & (4.40)\\ \Rightarrow Y_7^{IR} &= -\frac{\pi}{2}\,\ell\,. & (4.41) \end{aligned}$$

Physically this corresponds to adding $\ell\, M > 0$ mobile D3-branes smeared on the tip of the KS solution and for each $\ell \in \mathbb{Z}$ this provides the bulk dual to the ℓ-th mesonic branch. Let us note for later use that from (D.10), Appendix D, we get that the warp factor at the tip is

$$\tau\, \tilde{\phi}_4(0) = -6\, \frac{M}{h_0} \left(\frac{2}{3}\right)^{\frac{1}{3}} Y_7^{IR} = \frac{3}{h_0} \left(\frac{2}{3}\right)^{\frac{1}{3}} \pi\, |Q|\,. \tag{4.42}$$

To compare the Maxwell charges of the baryonic and mesonic branches, we must demand that they are defined at the same scale Λ_c. To do so we must address the fact that the constant ε_0 appearing in (4.36) is not gauge invariant and can be set to one by rescaling the space-time coordinates x_μ. As such one would normally fix the gauge and eliminate this constant. Indeed, ε_0 is dimensionful and just serves to fix the units which may as well be set to unity. However the ratio between the value of ε_0 in two different KS vacua, such as the mesonic and baryonic branches, is dimensionless and physically relevant.

This is similar to the familiar domain wall solution from one AdS vacuum to another. In either vacuum the AdS radius sets the units in which all other dimensionful numbers are measured but the ratio of the two radii is related to the ratio of central charges and is physically meaningful. Having said this, it is important to establish that in our Ansatz the rescaling of x_μ is done by the constant shift in A, given in the UV by

$$A = \frac{1}{3}\,(\tilde{\phi}_4 - \tilde{\phi}_1) = -\frac{1}{5}\, Y_1^{UV} + \mathcal{O}(1/\tau)\,, \tag{4.43}$$

[4]In our conventions the KS background has negative D3 charge.

where we have preemptively used the UV boundary conditions (4.62) introduced below. So, allowing for just Y_7 and Y_1 to be non-zero, we can find the supergravity solution of the mesonic branch as a perturbation of the baryonic branch. Using (4.29) and (4.2), along with (D.16)-(D.18), we find that in our perturbation theory the zeroth- and first-order Maxwell charge at a particular radius $r_c >> 1$, is[5]

$$\mathcal{Q}^{Max}_{D3} = -\frac{8\,P^2}{\pi}\,(\tau - 1) + \frac{8\,P}{\pi}\,Y_7^{UV} + \mathcal{O}\left(e^{-\tau/3}\right)\,. \tag{4.44}$$

Using an expansion of ε

$$\varepsilon = \varepsilon_0\left(1 + \frac{\varepsilon_1}{\varepsilon_0} + \mathcal{O}(Z^2)\right)\,, \tag{4.45}$$

where ε_0 denotes that of the baryonic branch, it is apparent that if we want to stay at a fixed r_c, then (4.36) requires at first order

$$\delta\tau = -2\,\frac{\varepsilon_1}{\varepsilon_0}\,. \tag{4.46}$$

Demanding that $\mathcal{Q}^{Max}_{D3}$ at r_c is equal for the baryonic and mesonic vacua, yields the relation

$$\frac{\varepsilon_1}{\varepsilon_0} = -\frac{Y_7^{UV}}{2\,P}\,. \tag{4.47}$$

Using (4.13) and the fact that $X_a = 0$, we have $Y_7^{UV} = Y_7^{IR}$. Then, referring to (4.41), we have

$$\frac{\varepsilon_1}{\varepsilon_0} = \frac{\ell\,\pi}{M}\,, \tag{4.48}$$

which is the first-order approximation to the known result $\varepsilon_\ell = \varepsilon_0\,e^{\ell\,\pi/M}$ [68, 67].

Now, we can find the value of the other integration constant, Y_1. Using the way that ε enters into the PT Ansatz through A, equation (4.2) and the UV expansions of Section (D.2) for $A = (\tilde{\phi}_4 - \tilde{\phi}_1)/3$ we get

$$\frac{\varepsilon_1}{\varepsilon_0} = -\frac{3\,Y_1^{UV}}{10}\,. \tag{4.49}$$

Combining this with (4.47) results in an expression for Y_1 in terms of Y_7:

$$Y_1^{UV} = \frac{5}{3\,P}\,Y_7^{UV}\,. \tag{4.50}$$

The relations obtained in this subsection can also be used to formulate the second and the third criteria for distinguishing between asymptotically-KS solutions.

[5] See footnote (4).

4.5 Finding the anti-D3 brane solution

We can now summarize the necessary ingredients for identifying the candidate supergravity solution describing the backreaction of anti-D3 branes. Firstly, we must eliminate unphysical IR singularities. For many modes this is entirely unambiguous, for other modes this can be somewhat subtle and as such we will discuss each mode as it arises. Secondly, we demand that the UV asymptotics are the same as for the original KS solution which we are perturbing around.

In total, we have sixteen integration constants but the seven physical modes (dual to seven gauge invariant operators) account for just fourteen of these. In addition, one is accounted for by the zero energy condition (6.11), which we use to eliminate X_5:

$$X_5 = \frac{1}{P}\left(X_2 - \frac{2}{3}X_3\right) - \frac{3}{2}X_7 \ . \tag{4.51}$$

The zero-energy condition is necessary to completely fix the reparameterization invariance of the radial coordinate (see [95] for a very explicit description of this). The final mode corresponds to the rescaling of x_μ and for reasons discussed above this is an important physical constant which is given again by (4.49). It was pointed out in the revised version of [67] that the two vacua of the Klebanov-Strassler theory necessarily have different values of ε. With our technology we are able to in fact compute the precise ratio of ε in the two different vacua.

The reader who is more interested in the end-process and in seeing or using our solution than in the boundary conditions we imposed to pick it out of the full parameter space of first-order deformations around the Klebanov-Strassler background can directly proceed to Section 4.5.3.

4.5.1 IR boundary conditions

We impose that the divergences in the IR for all the fields are zero, except for $\tilde{\phi}_4$ and $\sqrt{\mathcal{F}_5^2}$, the warp factor and 5-form flux along the brane, which should go respectively like $1/\tau$ and $1/\tau^2$ due to the anti-D3-brane sources. The latter means that $\tilde{\phi}_5$ should go to a constant.

From the divergent term in $\tilde{\phi}_8$ appearing in equation (D.3) of Appendix D, one finds the first relation among X's and Y's parameters that must be enforced:

$$X_8 = \frac{1}{9}\left(h_0 X_1 - 3\,P\,X_6\right) \ . \tag{4.52}$$

From the divergent terms in $\tilde{\phi}_2$ we get upon using (4.51) that

$$Y_2^{IR} = 0\ , \quad X_6 = \frac{h_0 X_1 - 3\,X_4}{6\,P}\ . \tag{4.53}$$

Out of the divergent terms in $\tilde{\phi}_3$ we set (after using (4.51) and (4.53))

$$Y_3^{IR} = 0\ , \quad X_4 = \frac{2}{3}h_0 X_1\ . \tag{4.54}$$

Note that the $\log\tau/\tau$ term is automatically zero once we take into account (4.53). Finally, the divergent term in $\tilde{\phi}_6$ requires

$$Y_6^{IR} = 0\,. \tag{4.55}$$

Likewise, the other piece is zero upon using (4.53), (4.54).

In summary, out of requiring IR regularity in all fields apart from the warp factor, we have obtained the following relations

$$Y_2^{IR} = Y_3^{IR} = Y_6^{IR} = 0\ , \quad X_4 = \frac{2}{3}h_0 X_1\ , \quad X_6 = -\frac{h_0}{6P}X_1\ , \quad X_8 = \frac{1}{6}h_0 X_1\ . \tag{4.56}$$

They are part of the relations that pick out of the full space of first order KS deformations the candidate solution describing the dual to a metastable state, taking into account the backreaction of anti-branes onto the zeroth order background. Let us move on and impose the remaining IR boundary conditions.

We will now impose that there are $\bar{N}$ anti-D3 sources at the tip. The IR regularity conditions (4.56) yields

$$\tilde{\phi}_5(0) = Y_7^{IR}\ , \tag{4.57}$$

as in the supersymmetric case described in Section 4.4.3, equation (4.38). We require $Q = \bar{N}$ (cf. footnote 4), which results in

$$Y_7^{IR} = \frac{\pi}{8\,P}\,\bar{N}\ , \tag{4.58}$$

where we have used (4.39) and (4.5). On the other hand, the warp factor is such that

$$\tau\,\tilde{\phi}_4(0) = \quad 8\left(\frac{2}{3}\right)^{\frac{1}{3}}\left(h_0\,X_1 - \frac{3\,P}{h_0}\,Y_7^{IR}\right)\ . \tag{4.59}$$

It ensues from requiring this exhibits the expected behavior for regular 3-branes (given in (4.42)) that

$$X_1 = \frac{3\,\pi}{4\,h_0^2}\,\bar{N}\ . \tag{4.60}$$

Before moving on to discussing UV boundary conditions in the subsequent section, we note that inserting (4.60) in (4.7) leads to the following expression for the force exerted on a D3-brane probing this backreacted supersymmetry-breaking solution:

$$F_{D3} = \frac{8\,\pi}{h_0^2}\,\frac{2^{2/3}\,\bar{N}}{(\sinh 2\,\tau - 2\,\tau)^{2/3}}\ . \tag{4.61}$$

This is precisely equal to the force on a probe anti-D3 brane exerted by $\bar{N}$ D3-branes that is computed in KKLMMT [109]. This provides further support that our IR boundary conditions are the right ones for anti-D3 branes.

4.5.2 UV boundary conditions

As part of our UV boundary conditions, we impose the absence of non-normalizable modes (we will come back to discussing this point in section 4.6.2). Requiring no divergent terms in $\tilde{\phi}_3$, $\tilde{\phi}_4$ as well as $\tilde{\phi}_5$, $\tilde{\phi}_6$ and $\tilde{\phi}_7$ implies

$$Y_4^{UV} = 0\ , \qquad X_3 = 0\ , \qquad Y_5^{UV} = 0\ . \tag{4.62}$$

Requiring no $e^{-\tau/3} \sim 1/r$ terms in $\tilde{\phi}_2$, and using (4.62) then determines

$$X_7 = 0\ , \qquad X_2 = -\frac{2}{9}\, h_0\, X_1\ . \tag{4.63}$$

Besides, we do not want to turn on the non-normalizable mode that shifts the dilaton, which would correspond in the gauge theory to changing the sum of the coupling constants for the gauge group. Hence, we must enforce that

$$Y_8^{UV} = 0\ . \tag{4.64}$$

From (4.62) and (4.64), we see that the Maxwell charge in the UV is the same as in Section 4.4.3, equation (4.44). We should demand that at a given bulk radial slice r, this is the same as the Maxwell charge for the supersymmetric vacuum, which is in the (first) mesonic branch and has $M - \bar{N} = 4\,P - \bar{N}$ D3-branes at the bottom. Keeping in mind that ε is allowed to differ in the two vacua, which using (4.36) implies that the Maxwell charges have to be evaluated at different τ, we require that[6]

$$\mathcal{Q}_{D3}^{Max} = -\frac{8\,P^2}{\pi}\,(\tau_0 + \delta\tau_{ms} - 1) + \frac{8\,P}{\pi}\,Y_7^{UV} \tag{4.65}$$

$$\stackrel{!}{=} -\frac{8\,P^2}{\pi}\,(\tau_0 + \delta\tau_1 - 1) - 4\,P + \bar{N}\,. \tag{4.66}$$

Here $\delta\tau_1$ corresponds to the cut-off associated to the first mesonic branch. It is given by

$$\delta\tau_1 = -\frac{\pi}{2\,P}\,, \tag{4.67}$$

where we have used[7] (4.46) and (4.48) for $\ell = 1$. We therefore have

$$\frac{16\,P^2}{\pi}\,\frac{\varepsilon_{ms}}{\varepsilon_0} + \frac{8\,P}{\pi}\,Y_7^{UV} = \bar{N}\,. \tag{4.68}$$

Using (4.49) to relate the change in ε to Y_1^{UV} leads to

$$-\frac{8\,P^2}{\pi}\,\frac{3}{5}\,Y_1^{UV} + \frac{8\,P}{\pi}\,Y_7^{UV} - \bar{N} = 0\,. \tag{4.69}$$

[6] See Figure 4.4 below.

[7] Recall that $P = \frac{1}{4}\,M\,\alpha'$. For convenience we have fixed $\alpha' = 1$ throughout.

Note that if Y_7^{UV} were equal to Y_7^{IR}, the latter being given in (4.58), it would ensue that $Y_1^{UV} = 0$ and no change in ε would be necessary. However, consequent on inserting all our boundary conditions apart from the one associated to Y_1 in (4.13), one finds

$$\frac{8\,P}{\pi}\,Y_7^{UV} = \frac{8\,P}{\pi}\,5.64178\,Y_7^{IR} = 5.64178\,\bar{N}\;. \tag{4.70}$$

The shift in ε can be tuned to cancel the difference in the first-order Maxwell charge $\mathcal{Q}^{Max}$ between the anti-D3 and the supersymmetric solution.

4.5.3 The perturbative solution for anti-D3 branes in KS

In summary, from the IR and the UV boundary conditions, all the integration constants turn out to be expressed in terms of the number $\bar{N}$ of anti-D3's at the tip of the throat. As a reminder, $h_0 = h(\tau = 0)$ denotes the zeroth order warp factor of the Klebanov-Strassler solution (4.3) evaluated at the tip. Below we collect the outcome of the analysis from the previous two subsections:

$$\begin{aligned}
X_1 &= \frac{3\,\pi}{4\,h_0^2}\,\bar{N}\,, & Y_1^{UV} &= \frac{3.03804}{P^2}\,\bar{N}\,, & Y_1^{IR} &= \frac{4.33971}{P^2}\,\bar{N}\,,\\
X_2 &= -\frac{\pi}{6\,h_0}\,\bar{N} & Y_2^{IR} &= 0\,, & Y_2^{UV} &= -\frac{1.48261}{P^2}\,\bar{N}\,,\\
X_3 &= 0\;, & Y_3^{IR} &= 0\;, & Y_3^{UV} &= \frac{8.40238}{P^2}\,\bar{N}\,,\\
X_4 &= \frac{\pi}{2\,h_0}\,\bar{N}\;, & Y_4^{UV} &= 0\,, & & \\
X_5 &= -\frac{\pi}{6\,P\,h_0}\,\bar{N}\;, & Y_5^{UV} &= 0\,, & Y_5^{IR} &= \frac{0.70514}{P}\,\bar{N}\,,\\
X_6 &= -\frac{\pi}{8\,P\,h_0}\,\bar{N}\;, & Y_6^{IR} &= 0\,, & Y_6^{UV} &= -\frac{4.08244}{P}\,\bar{N}\,,\\
X_7 &= 0\;, & Y_7^{IR} &= \frac{\pi}{8\,P}\,\bar{N}\,, & Y_7^{UV} &= \frac{2.21552}{P}\,\bar{N}\,,\\
X_8 &= \frac{\pi}{8\,h_0}\,\bar{N}\;, & Y_8^{UV} &= 0\,, & Y_8^{IR} &= \frac{0.234935}{P^2}\,\bar{N}\,.
\end{aligned} \tag{4.71}$$

All the constants in the leftmost and middle columns, with the exception of Y_1^{UV}, have been obtained by directly imposing boundary conditions in either the IR or UV. From there on, Y_1^{UV} was obtained from Y_7^{UV} via (4.69). Finally, the rightmost column was derived from the numerical integration which is tabulated in (4.13). We have not computed the value of Y_4^{IR} as it is more involved than the others and we do not need it, but in principle it can be done through numerical integration of the analytic solution (3.89).

It is interesting to observe the profile of the first-order perturbation to the Maxwell D3 charge $\mathcal{Q}_{D3}^{Max}$, given in Figure 4.3 for $\bar{N} = 1$ (see footnote 4). Note that it does not increase monotonically.

On Figure 4.4 we have plotted the total Maxwell D3 charge (i.e. the zeroth- plus first-order contributions) for the anti-D3-brane solution, alongside the Maxwell charge of the supersymmetric vacuum (4.66), the latter belonging to the first mesonic branch. For the purpose of illustrating equations (4.65)-(4.66), we also plot the "would-be supersymmetric vacuum" in the baryonic branch, that we use as a reference to measure the difference in UV cut-off, $\delta\tau$. This branch obviously does not exist for $\bar{N} \neq 0$, but it is instructive to use it as yardstick.

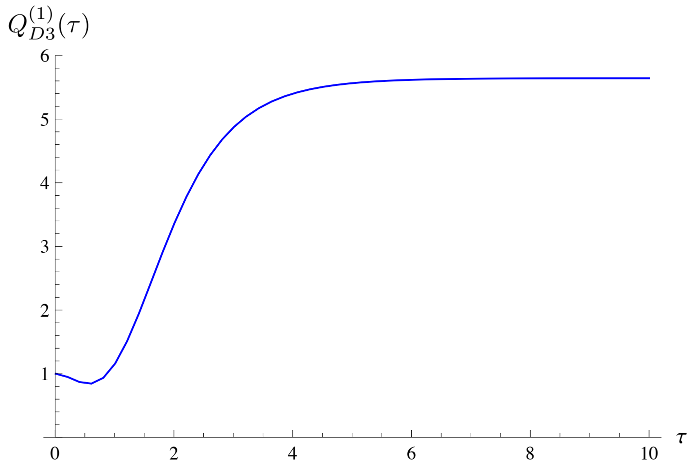


Figure 4.3: The profile of the first-order Maxwell charge for the anti-D3 solution, setting $\bar{N} = 1$.

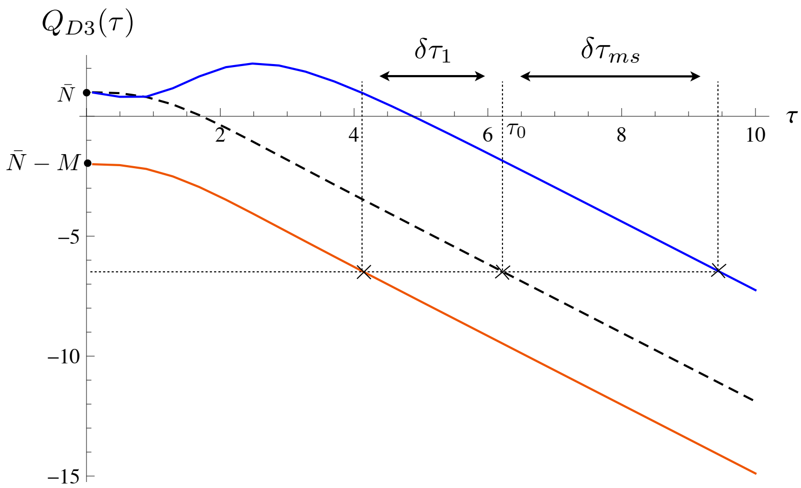


Figure 4.4: Total Maxwell charge for the anti-D3 solution (blue), for the supersymmetric vacuum from the first mesonic branch (red) and for the "would-be supersymmetric vacuum in the baryonic branch" (black dashed line), fixing $\bar{N} = 1, M = 3\,(P = \frac{3}{4})$.

4.5.4 Asymptotics of the solution

The Green's function for the KS background diverges in the IR (D.1), and we denote the constant in its series expansion around $\tau = 0$ as j_0, Eq. (D.2). The IR and UV series expansions of the solution in terms of h_0, j_0 and $X_1 = \frac{3\pi}{4h_0^2}\bar{N}$ are as follows.

Behavior in the infrared

In the IR the solution behaves as

$$\tilde{\phi}_8 = 33.1634\, P^2 X_1 - \frac{512}{3}\left(\frac{2}{3}\right)^{2/3} P^2 X_1\, \tau + \left[\frac{64}{27}\left(\frac{2}{3}\right)^{1/3} h_0\, P^2 X_1 + \frac{512}{27}\left(\frac{2}{3}\right)^{1/3} j_0\, P^2 X_1\right]\tau^2 + \mathcal{O}(\tau^3)\,, \tag{4.72}$$

$$\tilde{\phi}_2 = -128\left(\frac{2}{3}\right)^{\frac{2}{3}} P^2 X_1\, \tau + \frac{128}{81}\left(\frac{2}{3}\right)^{\frac{1}{3}}\left(h_0 + 16\, P^2 j_0\right) X_1\, \tau^2 + \mathcal{O}(\tau^3)\,, \tag{4.73}$$

$$\tilde{\phi}_3 = -\frac{224}{3}\left(\frac{2}{3}\right)^{\frac{2}{3}} P^2 X_1\, \tau + \frac{128}{405}\left(\frac{2}{3}\right)^{\frac{1}{3}}\left(h_0 + 136\, P^2 j_0\right) X_1\, \tau^2 + \mathcal{O}(\tau^3)\,, \tag{4.74}$$

$$\tilde{\phi}_1 = 612.592\, P^2 X_1 - \frac{704}{3}\left(\frac{2}{3}\right)^{\frac{2}{3}} P^2 X_1\, \tau + \frac{64}{405}\left(\frac{2}{3}\right)^{\frac{1}{3}}\left(7\, h_0 + 352\, P^2 j_0\right) X_1\, \tau^2 + \mathcal{O}(\tau^3)\,, \tag{4.75}$$

$$\tilde{\phi}_5 = \frac{1}{6} h_0^2\, P X_1 - 4\left(\frac{2}{3}\right)^{\frac{1}{3}} h_0\, P X_1\, \tau^2 + \mathcal{O}(\tau^3)\,, \tag{4.76}$$

$$\tilde{\phi}_6 = \frac{1}{6} h_0^2\, P X_1 - \frac{16}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}} h_0\, P X_1 + \frac{2}{81}\left(\frac{4\, h_0^2}{P} - 160\, h_0\, j_0\, P + 10451.6 P^3\right) X_1\, \tau + \left(\frac{4}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}} P\, h_0 - \frac{1280}{9}\left(\frac{2}{3}\right)^{\frac{2}{3}} P^3\right) X_1\, \tau^2 + \mathcal{O}(\tau^3)\,, \tag{4.77}$$

$$\tilde{\phi}_7 = \frac{8}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}} h_0\, P X_1\, \tau - 83.769\, P^3 X_1\, \tau^2 + \mathcal{O}(\tau^3)\,, \tag{4.78}$$

$$\tilde{\phi}_4 = \left(4\left(\frac{2}{3}\right)^{\frac{1}{3}} h_0\, X_1\right)\frac{1}{\tau} + Y_4^{IR} + \left(\frac{8}{15}\left(\frac{2}{3}\right)^{\frac{1}{3}} h_0\, X_1 - \frac{64}{3}\left(\frac{2}{3}\right)^{\frac{2}{3}} P^2 X_1\right)\tau + \mathcal{O}(\tau^2)\,, \tag{4.79}$$

UV behavior of the solution

As for the ultra-violet behavior of the solution, it is described by the following UV series expansions:

$$\tilde{\phi}_8 = -\frac{64}{3}\, 2^{1/3}\, e^{-4\tau/3}\, h_0\, X_1\, (\tau - 1) - 288\, 2^{2/3}\, e^{-8\tau/3}\, P^2 X_1 + \mathcal{O}(e^{-10\tau/3})\,, \tag{4.80}$$

$$\tilde{\phi}_2 = -418.571\, e^{-\tau}\, P^2\, X_1 + \frac{16}{3}\, 2^{1/3}\, e^{-7\tau/3}\, h_0\, X_1\, (1+8\tau) + \mathcal{O}(e^{-3\tau})\,, \tag{4.81}$$

$$\tilde{\phi}_3 = -\frac{32}{3}\, 2^{1/3}\, e^{-4\tau/3}\, h_0\, X_1 + 2\, e^{-2\tau}\, (1186.08 - 418.571\,\tau)\, P^2\, X_1 - \frac{1152}{5}\, 2^{2/3}\, e^{-8\tau/3}\, P^2\, X_1 + \mathcal{O}(e^{-10\tau/3})\,, \tag{4.82}$$

$$\tilde{\phi}_1 = 428.85\, P^2\, X_1 + \frac{8}{3}\, 2^{1/3}\, e^{-4\tau/3}\, h_0\, X_1 - \frac{2}{3}\, e^{-2\tau}\, (1325.73 - 837.143\,\tau)\, P^2\, X_1 + \frac{24}{5}\, 2^{2/3}\, e^{-8\tau/3}\, P^2\, (29 + 40\,\tau)\, X_1 + \mathcal{O}(e^{-10\tau/3})\,, \tag{4.83}$$

$$\tilde{\phi}_5 = 312.743\, P^3\, X_1 + e^{-\tau}\, (-1361.84 + 418.571\,\tau)\, P^3\, X_1 - 4\, 2^{1/3}\, e^{-4\tau/3}\, h_0\, P\, X_1\, (1 + 8\,\tau) + 2\, e^{-2\tau}\, (1361.84 - 837.143\,\tau)\, P^3\, X_1 + \mathcal{O}(e^{-7\tau/3})\,, \tag{4.84}$$

$$\tilde{\phi}_6 = 312.743\, P^3\, X_1 + e^{-\tau}\, (1361.84 - 418.571\,\tau)\, P^3\, X_1 - 4\, 2^{1/3}\, e^{-4\tau/3}\, h_0\, P\, X_1 (1 + 8\,\tau) + 2\, e^{-2\tau}\, (1361.84 - 837.143\,\tau)\, P^3\, X_1 + \mathcal{O}(e^{-7\tau/3})\,, \tag{4.85}$$

$$\tilde{\phi}_7 = e^{-\tau}\, (943.269 - 418.571\,\tau)\, P^3\, X_1 - \frac{4}{125}\, 2^{1/3}\, e^{-7\tau/3}\, h_0\, P\, (1199 + 80\,\tau\, (1 + 10\,\tau))\, X_1 + \mathcal{O}(e^{-11\tau/3})\,, \tag{4.86}$$

$$\tilde{\phi}_4 = 171.54 P^3 X_1 + \frac{4\, 2^{1/3}\, e^{-4\tau/3}\, h_0\, (7 + 32\,\tau)\, X_1}{3\, (4\,\tau - 1)} - \frac{625.486 P^2 X_1}{(4\,\tau - 1)} + \mathcal{O}(e^{-2\tau})\,. \tag{4.87}$$

4.6 Additional comments

Having solved for the full space of linearized perturbations around the Klebanov-Strassler background, we now discuss other solutions that we easily obtain as a by-product of our analysis, as well as other possible interpretations of our results.

4.6.1 Relation to previous works

The first attempt to construct the a linearized antibrane solution in the UV region alone was [58], which studied several of the $SU(2) \times SU(2) \times \mathbb{Z}_2$-invariant modes around the Klebanov–Tseytlin (KT) background [118]. Since the KT solution is a subset of the parametrization (3.27)–(3.28) given by

$$y(\tau) = 0, \quad k(\tau) = f(\tau), \quad F(\tau) = P\,, \tag{4.88}$$

in our setup we can understand the perturbations around KT as solutions of a reduced system of first-order differential equations in the Borokhov–Gubser formalism. The "backreacted" KT solution contains some integration constants that cannot be fixed by infrared boundary conditions, and hence we cannot relate them to the constant X_1, which is proportional to $\bar{N}$.

We can directly compare the UV expansion of our full KS solution (4.80)-(4.87) to the perturbed KT solution of [58] and we find the following crucial discrepancy:

The correct UV expansion has terms of order $\mathcal{O}(r^{-3})$ in (4.81,4.84,4.85,4.86) while the first non-trivial terms in the solution of [58] are at $\mathcal{O}(r^{-4})$.

In hindsight this is not so surprising, since [58] only considered a subset of the modes, and furthermore, the KT solution precisely agrees with the UV limit of the KS solution only at leading order. At subleading order the KT solution has an ambiguity which can be fixed to agree with the UV limit of the KS solution but then the lower-order perturbation theory around each solution quantitatively differs. For this reason, we conclude that one cannot derive the correct UV expansion for the anti-brane solution by starting with the KT geometry. Another problematic issue with the Ansatz made in [58] is that, as we have explicitly demonstrated in this work, the anti-D3-branes turns on modes which are outside of the truncation, so it is not consistent to restrict oneself to this subset of mode.

4.6.2 Gaugino masses

As an additional outcome of our analysis, we can easily identify other interesting solutions that correspond to different deformations of the dual gauge theory. In particular, we can construct a solution in which the non-normalizable UV modes X_2 and X_7 are turned on. They decay as $1/r$, and are associated to operators of dimension $\Delta = 3$, which correspond to deformations by gaugino mass terms for each of the gauge groups, $\mathrm{Tr}(\lambda_1\lambda_1 \pm \lambda_2\lambda_2)$. We will identify a one-parameter subfamily for which $\mathcal{Q}_{D3}^{Max}$ approaches the same constant value in the IR and in the UV, and therefore for which the parameter ε does not need to be modified.

The boundary conditions we have to impose are exactly the same as before, except that now we do not require (4.63). Relaxing these, we find that the leading terms in the IR expansions are not modified, and the value of $\tilde{\phi}_5$ at the origin is still given by (4.57), together with the relations (4.58),(4.60)

$$\tilde{\phi}_5(0) = Y_7^{IR} = \frac{h_0^2}{6P} X_1. \tag{4.89}$$

By using the UV/IR relation (4.13) we get that in the UV

$$\tilde{\phi}_5(\infty) = Y_7^{UV} = 154.299\, P^3\, X_1 - 19.5477\, P\,(2X_2 + PX_7)\,. \tag{4.90}$$

Imposing $\tilde{\phi}_5(0) = \tilde{\phi}_5(\infty)$, we thus see that for the family of solutions

$$2X_2 + PX_7 = 5.05767\, P^2\, X_1 \tag{4.91}$$

we get that the first order Maxwell D3 charge at infinity is the same as that of the supersymmetric vacuum with the same ε as for the original KS background. The profile of the perturbation to the D3-brane Maxwell charge is shown in Figure 4.5, where it is plotted as a function of $\bar{N}$ using the condition from equation (4.91).

We also note that by setting $X_1 = 0$, i.e. requiring that no anti-D3 brane be present at the origin, we obtain a family of solutions parametrized by the constants

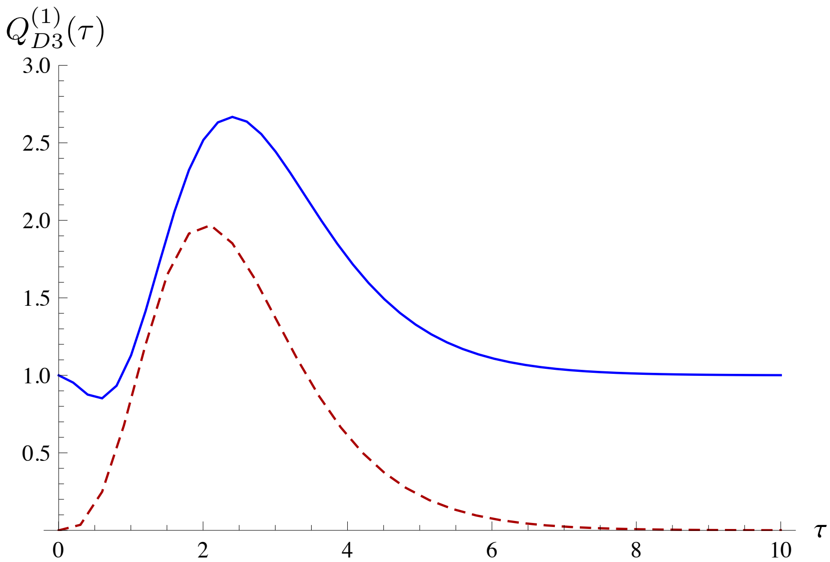


Figure 4.5: The profile of the first order Maxwell charge for the solution with gaugino masses turned on, satisfying the constraint (4.91) (blue solid line). The plot is for $\bar{N} = 1$ and $X_7 = 1/(24\, 2^{1/3} P^3)$. The red dashed curve is the profile for $\bar{N} = 0$.

X_2 and X_7 which in the dual gauge theory describe soft supersymmetry breaking due to gaugino mass terms. This solution encompasses the one built in [122], which corresponds[8] to the family $X_2 = PX_7$.

4.6.3 Other UV boundary conditions

In section 4.5 we have identified the anti-D3 backreacted solution using one of the three criteria to distinguish asymptotically-KS supersymmetric solutions that we have put forth in section 4.4. The resulting solution has a different scale parameter Y_1 than its supersymmetric counterpart, and if the criterion that the NSNS B_2 field be zero at the KS tip is the correct one, then, putting aside concerns about the subleading singularity and about backreaction, the anti-D3 perturbative solution we have constructed describes a metastable state of a supersymmetric KS field theory, and would be the first metastable solution constructed in supergravity.

However, we can also ask whether this result holds if one imposes the other criteria, or if one insists, perhaps with a view towards embedding the KS solution in a compact setting, that the UV scale parameter Y_1 be the same as in the supersymmetric theory. It is not hard to see that if one imposes the criterion that the H_3

[8]The constant X in [122] is then related to X_7 by $X = -\frac{1}{2}X_7$ and their parameter μ is such that $\mu = 48\, 2^{1/3} P X_7$.

integral only jumps by integer units, one finds again that Y_1^{UV} has to change; the anti-D3 solution is identical to the one we have written down above, and would be dual also to a metastable field theory vacuum.

If one on the other hand imposes the criterion that two vacua of the same theory must have a Y_1^{UV} related to Y_7^{UV} as in equation (4.50) (which also distinguishes between various supersymmetric KS vacua), or imposes the requirement that the UV scale must be the same as in the supersymmetric theory, then the resulting solution will have a different IR Maxwell charge than the one inferred from the UV data (essentially because antibranes give rise to negative charge dissolved in flux in their vicinity, as shown in Figure 4, and if one cannot make the throat longer to compensate for this, this charge will be visible at infinity). As a result, the relation between the force on a probe D3 brane and the anti-D3 charge of the background will not be the one of [109]. If one then insists that this relation does not receive corrections at first order in the number of antibranes, as suggested by the no-screening results of [19], then the anti-D3 solution must have a nontrivial $1/r$ mode turned on, of the type presented in the previous subsection, such that the contribution to the charge dissolved in flux from the antibranes is canceled by the contribution from the X_2 and X_7 modes. The value of this non-normalizable relevant perturbation can be easily read off from our analysis. Interestingly enough, such modes were argued in [12] to be present when a KS solution is embedded in a stabilized flux compactification, and it would be interesting to see if the relation between the anti-D3 charge and the strength of this mode that we find here has any relevance to this analysis.

4.7 Flux singularities

In this section we analyze in more details the infrared limit of the anti-D3 solution. Let us recapitulate our findings. The boundary conditions that we imposed in the near–brane region are those consistent with smeared anti–branes at the tip: singular warp factor and five–form flux, coming from the anti–D3 source with equal mass and charge, and regularity in all other modes. This requirement fixes half of the sixteen integration constants of the general linearized deformation around the conifold in terms of a physical quantity: the number $\bar{N}$ of anti–D3 branes at the tip. In particular, this fixes the value of the mode X_1 that gives rise to the force felt by a probe D3 brane in the backreacted geometry

$$F_{D3} = \frac{8\,2^{2/3}\,\pi\,\bar{N}}{h_0^2}\,j'(\tau)\,, \tag{4.92}$$

where $j(\tau)$ is the Green's function at the linear order, defined in (4.4). This agrees with the computation à la KKLMMT [109] and is a nice check that the boundary conditions are the correct ones to describe anti–branes. However, we will now show that once all those requirements are fulfilled, one finds that remnant nonzero perturbations to three–form fluxes near $\tau = 0$ cause the energy density of such fluxes to diverge [140, 25]. We stress that this is purely an infrared phenomenon, in the sense that the presence of the singularity is insensitive to the UV boundary conditions.

In particular we will show that the perturbative solution contains a singular imaginary self–dual (ISD) and anti-imaginary self-dual (AISD) fluxes and we will provide physical intuition of why these singularities are expected to be present in the full non-linear solution. This is supported by the construction of the fully backreacted solution for anti–D6 branes in a flux background [37, 38, 39], where a singularity in the H-flux is unavoidable. This solution can be thought as a toy model for ours: by T-dualizing it three times along the D6 worldvolume one obtains a solution for anti–D3 branes in $\mathbb{R}^3 \times T^3$, which is an increasingly better approximation of the KS near–tip region as the S^3 radius grows or as the number of anti–D3 branes becomes ever more smaller than the number of fractional branes.

A we will discuss in detail, this result suggests that at least some components of the singular ISD and AISD fluxes, already visible in the linearized solution, will persist in the fully backreacted regime. In the next Chapter we will verify this conjecture by the explicit computation of the non-linear backreaction in the near-brane region. We will then address the question of possible string theory resolutions of the singularity.

4.7.1 ISD and AISD fluxes

By using the linearized solutions obtained in the previous sections, we will compute the GKP modes $\Phi_\pm$, $G_\pm$, defined in (4.8)[9]

$$G_\pm = \star_6 G_3 \pm i G_3 \,, \qquad \Phi_\pm = e^{4\tilde{A}} \pm \alpha \,, \tag{4.93}$$

where $G_\pm$ are the ISD and IASD parts of the three–form flux, α is the RR 4–form and $e^{-4\tilde{A}}$ is the warp factor. The dynamics of these modes is described by the equation of motion [76]

$$(d + i\frac{d\tau}{\mathrm{Im}\ \tau} \wedge \mathrm{Re}\ )(\Phi_- G_+ + \Phi_+ G_-) = 0 \,. \tag{4.94}$$

In our Ansatz we have $\tau = e^{-\phi}$ since $C_0 = 0$[10]. As explained in the previous section, we linearize the problem by expanding in the parameter $\gamma = \bar{N}/M$:

$$G_\pm = G^0_\pm + G^1_\pm(\gamma) + \mathcal{O}(\gamma^2) \,, \tag{4.95}$$

$$\Phi_\pm = \Phi^0_\pm + \Phi^1_\pm(\gamma) + \mathcal{O}(\gamma^2) \,. \tag{4.96}$$

For the Klebanov–Strassler background we have

$$\Phi^0_- = G^0_- = 0, \tag{4.97}$$

[9]The modes $\Phi_\pm$ have nothing to do with the pure spinors introduced in section 2.1. Since this notation is standard and we never use these objects together, we keep the same name for both.

[10]The axion/dilaton τ in equation (4.94) should not be confused with the radial direction of the conifold.

while

$$\Phi_+^0 = \frac{2}{h(\tau)}\,, \tag{4.98}$$

$$\begin{aligned} G_+^0 &= (f_0 - k_0)\,(g_1\wedge g_3\wedge g_5 + g_2\wedge g_4\wedge g_5 + ig_1\wedge g_3\wedge g_6 + ig_2\wedge g_4\wedge g_6) \\ &\quad + 2i(2P - F_0)\,g_3\wedge g_4\wedge g_5 + 2iF_0\,g_1\wedge g_2\wedge g_5 \\ &\quad + 2e^{2y_0}(2P - F_0)\,g_1\wedge g_2\wedge g_6 + 2e^{-2y_0}F_0\,g_3\wedge g_4\wedge g_6\,, \end{aligned} \tag{4.99}$$

where the function $h(\tau)$ is the KS warp factor defined in (4.3), while the other KS functions are given in (4.2). At the linear order in γ, by using the expansions for the flux modes we find the fluxes:

$$\begin{aligned} G_-^1 = 2e^{-4A_0}\Big[&\left(ig_1\wedge g_2\wedge g_5 - e^{-2y_0}g_3\wedge g_4\wedge g_6\right)\left(\tilde\xi_5 - \tilde\xi_6\right) \\ &- \left(e^{2y_0}g_1\wedge g_2\wedge g_6 - ig_3\wedge g_4\wedge g_5\right)\left(\tilde\xi_5 + \tilde\xi_6\right) \\ &- \left(g_1\wedge g_3\wedge g_5 + g_2\wedge g_4\wedge g_5 - ig_1\wedge g_3\wedge g_6 - ig_2\wedge g_4\wedge g_6\right)\tilde\xi_7\Big]\,, \end{aligned} \tag{4.100}$$

$$\begin{aligned} G_+^1 = e^{-4A_0}\Big[&2ig_3\wedge g_4\wedge g_5\left(\tilde\xi_5 + \tilde\xi_6 - e^{4A_0}\tilde\phi_7\right) + 2ig_1\wedge g_2\wedge g_5\left(\tilde\xi_5 - \tilde\xi_6 + e^{4A_0}\tilde\phi_7\right) \\ &+ 2e^{2y_0}g_1\wedge g_2\wedge g_6\left(\tilde\xi_5 + \tilde\xi_6 + e^{4A_0}(4P\tilde\phi_2 - 2F_0\tilde\phi_2 - \tilde\phi_7)\right) \\ &+ 2e^{-2y_0}g_3\wedge g_4\wedge g_6\left(\tilde\xi_5 - \tilde\xi_6 - e^{4A_0}(2F_0\tilde\phi_2 - \tilde\phi_7)\right) \\ &- \left(g_1\wedge g_3\wedge g_5 + g_2\wedge g_4\wedge g_5 + ig_1\wedge g_3\wedge g_6 + ig_2\wedge g_4\wedge g_6\right)\cdot \\ &\cdot\left(2\tilde\xi_7 + e^{4A_0}(\tilde\phi_6 - \tilde\phi_5 + (f_0 - k_0)\tilde\phi_8)\right)\Big]\,. \end{aligned} \tag{4.101}$$

We recall that the modes $\tilde\xi^a$ and $\tilde\phi^a$ are respectively linear combinations of the conjugate–momenta ξ^a and the perturbations modes $\delta\phi^a$ (see (3.33) and (3.66) for their definition). By using the definition (4.93), we find the expressions for the $\Phi_\pm$ modes at the linearized level in terms of the modes $\tilde\xi^a$, $\tilde\phi^a$

$$\frac{d\Phi_-^1}{d\tau} = \frac{2}{3}e^{-2x_0}\tilde\xi_1\,, \tag{4.102}$$

$$\frac{d\Phi_+^1}{d\tau} = -\frac{2}{3}e^{-2x_0}\tilde\xi_1 + \frac{4\tilde\phi_4 h'(\tau) - 4h(\tau)\tilde\phi_4'}{h(\tau)^2}\,. \tag{4.103}$$

The first equation can be integrated by using the equation of motion for $\tilde\xi_5$ (6.23) and gives

$$\Phi_-^1 = -\frac{2}{P}\tilde\xi_5 + \text{const} = \frac{32}{3}X_1 j(\tau) + \text{const}\,, \tag{4.104}$$

where $P = M/4$ and from (4.60) X_1 is proportional to the number of anti–branes $\bar{N}$

$$X_1 = \frac{3\,\pi}{4\,h_0^2}\,\bar{N}\,, \tag{4.105}$$

where $h_0 = 18.2373\,P^2$. The equation for Φ_+^1 can be easily integrated to get

$$\Phi_+^1 = -\Big(\frac{32}{3}X_1 j(\tau) + \frac{4\,\tilde{\phi}_4}{h(\tau)}\Big) + \text{const}\,. \tag{4.106}$$

We note that G_-^1 and Φ_-^1 are parametrized by the modes $\tilde{\xi}^a$ only, and thus vanish if the perturbation is supersymmetric. One can check that the equation of motion (4.94) is equivalent to the equations for the modes $\tilde{\xi}_{5,6,7}$.

4.7.2 Infrared behavior

We now discuss the behavior of the three–form flux in the near–brane region, namely at small τ, and we will show that both the ISD and IASD modes are singular. The presence of a singularity in the IASD flux mode was first noticed in [140, 25]. An explanation of this behavior was given in [140, 67], where the singularity was interpreted as coming from the coupling of anti–D3 branes to the mode Φ_-, which is singular in the linearized solution, as we will show in (4.119). We remark that the G_+ mode also presents a singularity at linearized level and discuss the possible implications of this behavior.

We derived the infrared expansions for the perturbations modes, as well as the anti–D3 boundary conditions, in the previous sections. For the IASD flux G_- we only need the expansions for the scalars conjugate to the flux perturbation modes $\tilde{\xi}_{5,6,7}$ which are given by:

$$\begin{aligned}
\tilde{\xi}_5 &= \frac{1}{\tau}\left(8\left(\frac{2}{3}\right)^{1/3} PX_1\right) - \frac{2}{9}\,(h_0 + 24 j_0)\,PX_1 + \frac{16}{15}\left(\frac{2}{3}\right)^{1/3} PX_1\tau + \mathcal{O}(\tau^3)\,,\\
\tilde{\xi}_6 &= \frac{1}{\tau}\left(8\left(\frac{2}{3}\right)^{1/3} PX_1\right) - \frac{2}{9}\,(h_0 + 24 j_0)\,PX_1 + \frac{4}{5}\,2^{1/3}\,3^{2/3} PX_1\tau + \mathcal{O}(\tau^2)\,,\\
\tilde{\xi}_7 &= -\frac{2}{27}\,(h_0 - 40 j_0)\,PX_1\tau - \frac{4}{5}\,2^{1/3}\,2^{2/3} PX_1\tau^2 + \mathcal{O}(\tau^3)\,,
\end{aligned} \tag{4.107}$$

where $j_0 = 0.836941$. From them and (4.100) we get

$$G_-^1 = \frac{1}{\tau}\bigg(\frac{32}{3}\left(\frac{2}{3}\right)^{1/3} Ph_0X_1\bigg)\,(g_3\wedge g_4\wedge g_6 + 3ig_3\wedge g_4\wedge g_5) + \mathcal{O}(\tau^0)\,, \tag{4.108}$$

where X_1 is defined in (4.105). For the ISD flux G_+ we also need the expansions for the modes $\tilde{\phi}^a$ which can be found in appendix D. The final result is

$$G_+^1 = \frac{1}{\tau}\bigg(\frac{32}{3}\left(\frac{2}{3}\right)^{1/3} Ph_0X_1\bigg)\,(g_3\wedge g_4\wedge g_6 + 3ig_3\wedge g_4\wedge g_5) + \mathcal{O}(\tau^0)\,. \tag{4.109}$$

We note that G_+ shows the same kind of singularity as the G_- mode. However we remark that, as we can see from (4.101), two contributions enter in (4.109): one is from the $\tilde{\xi}^a$ modes, the other is from the $\tilde{\phi}^a$ terms and both give rise to the singularity. We are now going to rederive these results in a way that will makes clear their interpretation.

Let us introduce a set of functions $\lambda(\tau)_A$ that parametrize the breaking of the ISD condition

$$H_3 = -\sum_A \lambda(\tau)_A \, e^{\phi} \star F_3^A \,, \tag{4.110}$$

where the index A runs over the components of the three–forms. A straightforward calculation shows that the ISD and IASD fluxes are given by

$$G_\pm = \sum_A \Big[\big(1 \pm \lambda(\tau)_A\big) \star F_3^A + i\big(\pm 1 + \lambda(\tau)_A\big) F_3^A \Big] \,. \tag{4.111}$$

The functions $\lambda(\tau)_A$ can be obtained from the Ansatz (3.28). By expanding at first–order in $\gamma = \bar{N}/M$ around the Klebanov–Strassler solution (for which the fluxes are imaginary–self–dual), one finds the following non–vanishing components:

$$\lambda(\tau)^{345} = -\frac{e^{-2y-\phi} f'}{2P - F} = 1 + \frac{2e^{-4A_0}}{2P - F_0} (\tilde{\xi}_5 + \tilde{\xi}_6) + \mathcal{O}(\gamma^2) \,, \tag{4.112}$$

$$\lambda(\tau)^{125} = -\frac{e^{2y-\phi} k'}{F} = 1 + \frac{2e^{-4A_0}}{F_0} (\tilde{\xi}_5 - \tilde{\xi}_6) + \mathcal{O}(\gamma^2) \,, \tag{4.113}$$

$$\lambda(\tau)^{136} = \lambda(\tau)^{246} = \frac{e^{-\phi}(f-k)}{2F'} = 1 + \frac{4e^{-4A_0}}{f_0 - k_0} \tilde{\xi}_7 + \mathcal{O}(\gamma^2) \,. \tag{4.114}$$

Recall that the legs 1 and 2 are on the shrinking S^2, while legs 3, 4 and 5 are on the S^3. While these expressions are valid for the whole conifold, we need their near–tip behavior. The infrared expansions (4.107) yields

$$\lambda(\tau)^{345} \sim \frac{1}{\tau}\left(16\left(\frac{2}{3}\right)^{1/3} P h_0 X_1\right), \quad \lambda(\tau)^{125} \sim -\frac{1}{\tau}\left(16\left(\frac{2}{3}\right)^{1/3} P h_0 X_1\right), \tag{4.115}$$

while $\lambda(\tau)^{136} = \lambda(\tau)^{246} = \mathcal{O}(\tau)$. We thus see than only two components of $\lambda(\tau)_A$ are relevant for the infrared physics. We can now compute $G^1_\pm$ by expanding the expression (4.111) at first–order in γ. We find

$$G^1_+ = \sum_A \Big[2\,(\star F_3^A)^1 + \lambda(\tau)^1_A (\star F_3^A)^0 + i\lambda(\tau)^1_A (F_3^A)^0 \Big] \,, \tag{4.116}$$

$$G^1_- = \sum_A \Big[-\lambda(\tau)^1_A (\star F_3^A)^0 + i\lambda(\tau)^1_A (F_3^A)^0 \Big] \,, \tag{4.117}$$

where we indicated by the superscript 0,1 the order of the expansion in γ. We can now analyse the near–tip behavior of the ISD and IASD fluxes, namely find the leading terms in an expansion near $\tau = 0$. We are interested in the origin of the singular behavior of such modes.

For the imaginary part, we can see from the infrared expansions of the KS fields (2.37) that the only component that contributes to the singularity is F_3^{345}. Since $2P - F_0 \sim 2P - \tau^2/6$, from $\lambda(\tau)^{345}$ in (4.115) we recover the imaginary part of the $G_\pm$ fluxes (4.108), (4.109). For the real part, we find that the only relevant component is F_3^{125}. We have $(\star F_3^{125})^0 = e^{-2y_0}\, F_0\, g_3 \wedge g_4 \wedge g_6 \sim \frac{2}{3} P\, g_3 \wedge g_4 \wedge g_6$, while $(\star F_3^{125})^1 = e^{-2y_0}(\tilde{\phi}_7 - 2F_0\,\tilde{\phi}_2)\, g_3 \wedge g_4 \wedge g_6$. Since

$$e^{-2y_0}(\tilde{\phi}_7 - 2F_0\,\tilde{\phi}_2) = \frac{1}{\tau}\left(\frac{32}{3}\left(\frac{2}{3}\right)^{1/3} P h_0 X_1\right) + \mathcal{O}(\tau^0)\,, \tag{4.118}$$

we see that the two terms in the real part of G_+ (4.116) give the same singularity as in the real part of G_-, in agreement with (4.108), (4.109). Before discussing the interpretation of these results, let us show the expansions of the modes $\Phi_\pm$ at the first order in γ

$$\Phi_-^1 = -\frac{1}{\tau}\left(16\left(\frac{2}{3}\right)^{1/3} X_1\right) + \frac{32 j_0 X_1}{3} - \frac{32}{15}\left(\frac{2}{3}\right)^{1/3} X_1 \tau + \mathcal{O}(\tau^2)\,, \tag{4.119}$$

$$\Phi_+^1 = -\frac{32 j_0 X_1}{3} - \frac{4Y_4^{IR}}{h_0^2 P^4} + \mathcal{O}(\tau^2)\,. \tag{4.120}$$

The singularity in Φ_-^1 is expected since the Green's function $j(\tau)$ at the linearized level diverges at the tip. The regular behavior of Φ_+^1 is one of the infrared boundary conditions that we imposed in section 4.5.1 and ensures that no regular D3 branes are present at the tip.

Let us summarize our findings. The ISD and IASD three–form fluxes in the linearized anti–D3 solution have a singularity of order τ^{-1} in the infrared. Another mode in the solution has the same τ^{-1} singularity, namely the mode Φ_- which is coupled to the anti–D3 branes.

We can see that the singularity in G_-^1 compensates the singularity in Φ_- in the equation of motion (4.94) [67, 140]. Indeed, at $\tau \sim 0$ we find

$$\Phi_+^0 G_-^1 + \Phi_-^1 G_+^0 = \mathcal{O}(\tau)\,. \tag{4.121}$$

Based on this observation, it was argued in [67, 140] that at the non–linear level, since Φ_- will be finite at the tip, the G_- singularity will disappear in the full backreacted solution. We remark however that in the linearized solution also the G_+ mode (4.109) have a singular behavior near $\tau = 0$, as shown in figure 4.6.

4.7.3 Discussion

A similar situation to the one discussed above was found for the full backreaction of anti–D6 branes in [39]. While this latter setup differs in many aspects from the

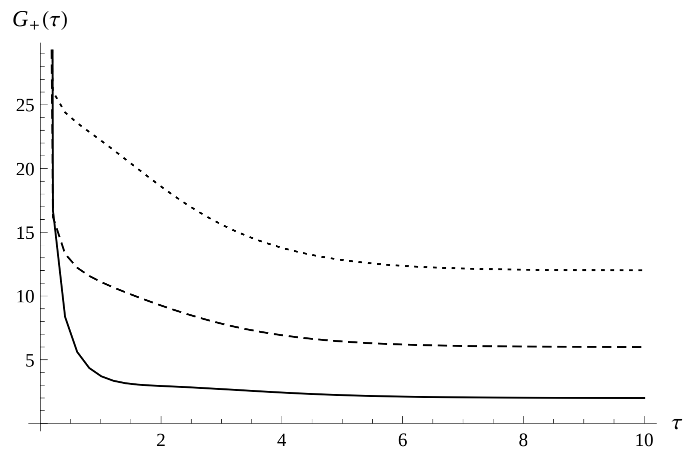


Figure 4.6: The ISD flux with legs on g_3, g_4 and g_5 for the Klebanov–Strassler geometry perturbed by anti–D3 branes. The plot is for $P = 1, 3, 6$ (solid, dashed and dotted lines) and $\bar{N} = 1$.

Klebanov–Strassler background, it displays the same kind of singular behavior of our linearized solution, as we will now explain. One can perform three T–dualities along the worldvolume of the anti–D6 branes and finds that this setup will describe anti–D3 branes on $\mathbb{R}^3 \times T^3$. If one regards the three–torus as a large radius limit of the finite S^3 at the tip of the Klebanov–Strassler throat, we expect that the anti–D6 solution will describe the behavior of the three–form flux F_3 with legs on the three–sphere. From the result of [39], we then expect that for this flux the *full* backreacted solution will be described by the relation

$$H = -\lambda(\tau)\, e^{\phi} \star F_3\,, \tag{4.122}$$

with a divergent $\lambda(\tau)$ in the near–brane region (with $\lambda(\tau) \to +\infty$). We can now compare this expectation to our result for the linearized anti–D3 solution (4.115). We see that the three–form flux with legs on the S^3 (i.e. the $g_3 \wedge g_4 \wedge g_5$ component) is precisely described by a relation of the form (4.122), with $\lambda(\tau) = \lambda(\tau)^{345}$. As we established in (4.116), (4.117), this analogy would point towards a divergency in the imaginary part of both the ISD and IASD fluxes at the full non–linear order. However, the leg structure of the three–form flux in our linearized anti–D3 solution is more complicated, and there is another component of the flux, F_3^{125}, which contributes to the singularity in $\lambda(\tau)$, making the anti–D6 analogy alone not fully conclusive.

A discussion on the interpretation of the behavior described by (4.122) can be found in [40], where it is argued that it describes an H–flux accumulation toward the anti-branes, which will eventually lead to a critical value for which the barrier against brane/flux annihilation is destroyed. This is a time-dependent resolution of the singularity. In the next section we will study in detail other possible ways to resolve it. In particular we will consider the possibility, as schematically depicted in [59, 67], that in the very near–tip region the solution might be altered by the polarization process in which the anti–branes form a fuzzy five–brane wrapping an $S^2 \subset S^3$. In particular, one may hope that the three–form flux singularity will be cured much as in the Polchinski–Strassler solution [146]. If the geometry is smoothed–out in this way, then the effects of the backreaction will alter only quantitatively the KPV model, by making the bound on $\bar{N}/M$ for the existence of a metastable state more strong. However, at least for smeared anti–D3 branes, the singular ISD flux (4.109) does not have the correct legs needed for brane polarization, as also mentioned in [25].

To conclude, in this last section we have computed the ISD and IASD fluxes for the linearized backreaction of $\bar{N}$ anti–D3 branes on the Klebanov–Strassler geometry. Both these modes have an infrared singularity. While it has been suggested [67, 140] that the IASD mode may be regular in the full non–linear solution, this argument does not apply to the ISD flux. In fact, by analogy with the anti–D6 backreaction [37, 38, 39], one can even argue that some components of the IASD flux could be singular at the non–linear level as well. It is clearly important to verify this by computing the full backreaction near the anti–D3 branes. If confirmed, one should address the question of whether the interpretation of this singularity is compatible or not with the existence of a metastable state. In the next chapter, we will address these questions and we will confirm the results conjectured above.

Chapter 5

Infrared singularities and brane polarization

As we discussed in great detail in the previous chapters, anti-D3 branes at the tip of the Klebanov-Strassler solution with D3-charge dissolved in fluxes give rise, in the probe approximation, to a metastable state. However, as we showed in section 4.7, the linearized back-reacted smeared solution has singular three-form fluxes in the IR. In this chapter we will prove that these singularities persist in the fully non-linear backreaction, by solving the second-order equations of motion directly. The presence of such singularities in the full solution suggests a stringy resolution by brane polarization à la Polchinski-Strassler. We thus investigate the polarization process and we show that there is no polarization into anti-D5-branes wrapping the S^2 of the conifold at a finite radius. We will then discuss the implications of this result for phenomenology, in particular for uplift AdS and obtain a very large landscape of de Sitter vacua in string theory. This Chapter is based on the paper [27].

5.1 Introduction

In this chapter we will focus on the singularity of the solution corresponding to smeared anti-D3 branes at the tip of the KS cone, constructed in the previous chapter. This singularity has been argued to go away when considering the full backreaction of the anti-D3 branes [67], but as we discussed before this argument does not fully work and we expect the singularity to be a true feature of the non-linear supergravity solution. We will rigorously prove this in the next sections, and we will then consider the problem of a string theory resolution of the singularity.

Singularities in string theory have been studied extensively over more than ten years, and there are two very important lessons that have come out of this study: the first is that if a solution has a singularity one cannot hope to obtain correct physics by doing calculations in some region far away from the singularity, where the curvature is low; the resolution of the singularity may involve low-mass modes that

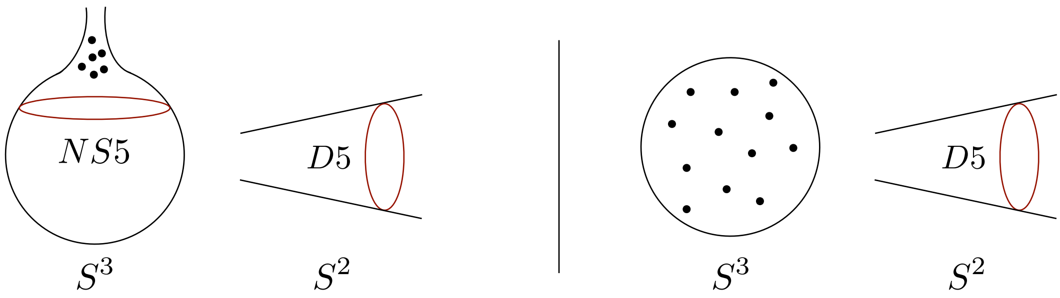


Figure 5.1: *Left*: localized anti-D3 branes at the north pole of the S^3 can polarize into a NS5 brane wrapping a two sphere $S^2 \subset S^3$ and into a D5 brane wrapping the shrinking S^2 of the conifold. *Right*: smearing the anti-D3 branes on the S^3 wipes out the KPV channel but the D5 channel still survives.

modify the spacetime at macroscopic distances away from the singularity, or may signal an instability of the whole spacetime. A second lesson, which is a corollary of the first, is that in the context of the AdS-CFT correspondence only singularity-free solutions are dual to vacua of the gauge theory, while singular solutions (such as the Polchinski-Strassler unpolarized solution [81, 146], the singular giant graviton [139, 125] or the Klebanov-Tseytlin solution [118]) are not dual to any vacuum of the gauge theory and have to be discarded as unphysical.

Thus, the healthy instinct when seeing a singular solution is to discard it, unless there is a good physical reason to accept it. For anti-D3 branes in Klebanov-Strassler one would expect that there is such a reason: brane polarization [142] à la Polchinski-Strassler [146]. Indeed, in the probe approximation, the probe anti-D3 branes were found to polarize into NS5 branes that wrap a contractible S^2 inside the large S^3 at the tip of the conifold [111], as drawn in Figure 5.1, and reviewed in section 2.4. One might thus expect that this polarization will continue to happen in the fully backreacted solution. However, to check this directly one would need to construct a solution for multiple anti-D3 branes localized at the north pole of the S^3. Unfortunately, constructing non-supersymmetric solutions that depend on two variables is beyond current technology[1], so the resolution of the singularity via polarization into NS5 branes cannot be directly checked.

However, one can use a less direct route to this result by remembering a very important feature of the Polchinski-Strassler construction: the D3 branes that polarize into NS5 branes wrapping an S^2 inside a three-plane can also polarize into D5 branes wrapping an S^2 inside an orthogonal plane, and more generally into a (p, q) five-brane wrapping an S^2 inside a diagonal three-plane. Hence, if the NS5 polarization channel is present and can cure the anti-brane singularity, so should be the other (p, q) five-brane channels, as well as the D5 channel. The latter channel

[1]Only the supersymmetric KS solution with localized D3 branes is known [120].

would correspond to a polarization of the anti-D3 branes localized at the north pole of a large S^3 at the tip of KS into a D5 brane wrapping the contractible S^2 of the deformed conifold at a finite distance away from the KS tip (see Figure 5.1). At fist glance this calculation looks as hopeless as the previous one, as it appears to also require the backreacted localized anti-D3 solution. However, things are much better: as shown in [146] and as we will review in Section 5.5.2, the polarization potential is independent of the location of the branes that polarize, and hence the potential for the D3's to polarize into D5 branes can be calculated from the smeared near-antibrane solution. The purpose of this chapter is to calculate this polarization potential.

We find that this potential has exactly the same type of terms as the polarization potential in Polchinski-Strassler, which confirms the expectation that brane polarization may be the mechanism of choice for resolving this singularity. However, the coefficients of the terms are not the same as in [146]; in particular, these coefficients depend nontrivially on two parameters that can only be fixed if one knows the full interpolating solution between the IR and the UV. Thus, in general, our potential could have had either SUSY minima (as in [146]), stable non-SUSY minima, metastable minima, or no polarization whatsoever. However, we find that, for any values of the unknown parameters, the terms are such that no polarization is possible.

Hence, our result shows that the singularity of the smeared anti-D3 infrared solution of [26] cannot be resolved by brane polarization, and by the arguments above, that also the localized anti-D3 brane solution will not be resolvable by polarization into D5 branes. Of course, our result does not directly rule out a resolution of the antibrane singularity in KS by polarization into NS5 branes that wrap the S^2 inside the S^3 at the tip à la KPV. However, the fact that D3 brane polarization always happens in multiple channels, and the fact that at least one of this channels is absent, suggests that the KPV polarization channel into NS5 branes might also be absent at full backreaction.

Our calculation pose the problem of whether anti D-branes in solutions with charge dissolved in fluxes can give rise to metastable vacua. If the the KPV channel is also absent in the full backreaction, two immediate corollaries follow: the first one is that the dual gauge theories, despite having a intricate structure of supersymmetric vacua [68], do not have metastable vacua. The other is that the mechanism for uplifting AdS vacua with stabilized moduli to dS vacua by adding anti-D3 branes in regions of high warp factor [110] will probably not work and will have to be replaced by another uplift mechanism. While there are several other uplift mechanisms in the market (F/D-term uplifting [153, 124] and Kahler-uplift [10, 152]), none is as generic as anti-D3 uplifting. Hence, it may be necessary to revisit the idea that string theory has a large landscape of dS vacua and to fall back to the old "non-anthropic" approach to understanding the physics of our universe.

5.2 The setup

We will use the PT Ansatz discussed in details in the previous Chapters and we will apply the technique introduced in 3.2 for the full non-linear case.

We recall that the potential in the one-dimensional Lagrangian (3.4) can be obtained from a superpotential W:

$$V = \frac{1}{8} G^{ab} \frac{\partial W}{\partial \phi^a} \frac{\partial W}{\partial \phi^b} \,. \tag{5.1}$$

In fact this equation has two different solutions, and therefore V has two possible superpotentials:

$$W^{\pm} = e^{4(p+A)} \left(\cosh y \pm e^{-6p-2x} \pm \frac{1}{2} e^{-2x} K \right) \,. \tag{5.2}$$

Using either of the two W, the supersymmetry conditions can be neatly written as a first-order flow equation

$$G_{ab} \dot{\phi}^b - \frac{1}{2} \frac{\partial W}{\partial \phi^a} = 0 \,. \tag{5.3}$$

The presence of the two superpotentials follows directly from the invariance of the type IIB action under the flip $(C_4, H_3) \to (-C_4, -H_3)$. In our notations this corresponds to the change of sign of f, k, Q and, as a result, of K. The first-order equations following from the two superpotentials impose either an imaginary self-duality (ISD) or imaginary anti-self-duality (IASD) condition on the complex 3-form, $G_3 \equiv F_3 + i e^{-\Phi} H_3$. As the subscript suggests, in our conventions, the supersymmetric solution derived from W^+ is the Klebanov-Strassler background with ISD 3-form, while W^- leads to the anti-Klebanov-Strassler solution with IASD fluxes. The two solutions preserve different supersymmetries. Consequently the supersymmetric KS solution can also include arbitrary number of the mobile D3-branes, $Q > 0$, but no anti-D3 branes, $Q < 0$; and vice versa for the supersymmetric anti-KS background.

5.2.1 The KS and anti-KS solutions

Before proceeding it is worth recalling how the eight integration constants of the eight first-order superpotential equations (5.3) (with $W = W^+$) are fixed in the KS solution with Q smeared mobile D3 branes ($Q > 0$):

- The zero-energy condition of the effective Lagrangian fixes the τ-redefinition gauge freedom and is automatically solved, but the constant shift $\tau \to \tau + \tau_0$ still remains unfixed, and so τ_0 appears as a “trivial" integration constant.
- The conifold deformation parameter ϵ and the constant dilaton e^{ϕ_0} give two other free parameters.

- An additional parameter renders the conifold metric singular in the IR [47] and has to be discarded.

- The three equations for the flux functions f, k and F appear to have three free parameters [122]. The first corresponds to an IR singular $(2,1)$ complex 3-form $G_3 \equiv F_3 + ie^{-\phi}H_3$, the second gives a $(0,3)$ form which is singular in the UV[2], and the third is related to the B-field gauge transformation $(f,k) \to (f+c, k+c)$, which is just a shift of the D3 brane charge and can be absorbed in the redefinition of Q.

- The warp function $h \sim e^{2x-4(p+A)}$ can only be determined up to a constant, which is fixed requiring that $h(\tau)$ vanishes at infinity:

$$h(\tau) = \int_{\tau}^{\infty} d\bar{\tau} \frac{\Big(4\pi Q + 32 g_s P^2 \; (\bar{\tau}\coth(\bar{\tau}) - 1)\,(\sinh(\bar{\tau}))^{-2}\left(\frac{1}{2}\sinh(2\bar{\tau}) - \bar{\tau}\right)\Big)}{\left(\frac{1}{2}\sinh(2\bar{\tau}) - \bar{\tau}\right)^{2/3}} \,. \tag{5.4}$$

  It is important to stress here that for the anti-KS solution with anti-D3's $(Q<0)$ one has to put $|Q|$ instead of Q, since otherwise $h(\tau)$ is negative for small τ. This is contrary to the flux $K(\tau)$ which flips sign once we go from the KS to the anti-KS solution.

As has been explained above, the anti-KS solution can be easily found by flipping the sign of the functions f and k. Notice that the $(2,1)$ and the $(0,3)$ 3-forms will be now $(1,2)$ and $(3,0)$. The remaining functions are exactly the same for the solutions derived from W^+ and W^-.

5.2.2 The first-order formalism

In order to solve for the anti-D3 backreaction we will need to solve the full set of second-order equations of motion, which we show in Appendix B. We now introduce a computational technique that will be extremely useful: the idea, explained in section 3.2, is to recast the eight second-order EOMs for the scalars ϕ^a as a set of sixteen coupled first-order equations by introducing conjugate momenta ξ_a, defined as

$$\xi_a = G_{ab}\dot{\phi}^b - \frac{1}{2}\frac{\partial W}{\partial \phi^a} \,. \tag{5.5}$$

Since we have two superpotentials that govern the system, W^+ and W^-, we can introduce *two* sets of conjugate modes, denoted by ξ_a^+ and ξ_a^- respectively. With this notation the supersymmetric KS first-order flow equations (with ISD fluxes) are simply $\xi_a^+ = 0$, while the first-order equations corresponding to supersymmetric anti-KS solutions (with IASD fluxes) are $\xi_a^- = 0$. It is easy to verify that solutions of

[2] Importantly the singular $(2,1)$ form is supersymmetric exactly as the 3-form of the KS solution, while the $(0,3)$ form breaks SUSY [90, 93].

these eight first-order equations solve also the full set of EOM. Indeed, by plugging the definition (5.5) into the second order EOMs we obtain:

$$\dot{\xi}_a = \frac{1}{2}\left[\frac{\partial G^{bc}}{\partial \phi^a}\xi_b\xi_c + \frac{\partial G^{bc}}{\partial \phi^a}\frac{\partial W}{\partial \phi^b}\xi_c - G^{bc}\frac{\partial^2 W}{\partial \phi^a \partial \phi^b}\xi_c\right], \tag{5.6}$$

which is equivalent to equation (3.13) and which is indeed trivially solved by putting all of ξ_a's to zero.

Replacing the second-order EOMs for the eight fields ϕ_a by sixteen first-order ones, equations (5.5) and (5.6), proves very efficient when studying supersymmetry breaking perturbatively as we showed in the previous Chapters, and turns out to be extremely useful for our purpose as well. As we will review in the next section, it was shown in [26] that without introducing singular fluxes it is not possible to interpolate between the ISD Klebanov-Strassler solution in the UV and the anti-D3 branes ($Q < 0$) boundary conditions in the IR. The regularity conditions on the fields near the anti-branes determine almost uniquely the leading-order behavior of the fields ξ_a's derived from W^-, which in turn appears to be incompatible with the equations (5.6). Moreover, we will provide a "topological" argument leading to the same conclusion but using instead the ξ_a functions derived from W^+. Since we will make an extensive use of both functions ξ, the following should be useful to keep track of the notation:

$$\begin{aligned}
W^+ &= W_{\text{KS}}\,, \qquad \text{BPS solution}\;:\; \xi^+ = 0\,, \quad G_3\,\text{ISD}\,, \quad F_{D3} = 0\\
W^- &= W_{\text{AKS}}\,, \qquad \text{BPS solution}\;:\; \xi^- = 0\,, \quad G_3\,\text{IASD}\,, \quad F_{\overline{D3}} = 0
\end{aligned}$$

where in the last equality we have added the force on probe D3 and anti-D3 branes.

The explicit form of (5.5) for the conjugate momenta is:

$$\begin{aligned}
\xi_1^{\pm} &= -e^{4(p+A)}\left(\dot{x} - 2\dot{p} - 2\dot{A} \mp \frac{1}{2}e^{-2x}K\right)\\
\xi_2^{\pm} &= -e^{4(p+A)}\left(\dot{x} + \dot{p} - 2\dot{A} + \cosh y - \frac{1}{2}e^{-6p-2x}\right)\\
\xi_3^{\pm} &= -6e^{4(p+A)}\left(\dot{p} + \dot{A} - \frac{1}{2}e^{-6p-2x}\right)
\end{aligned}$$

$$\begin{aligned}
\xi_y^{\pm} &= -\frac{1}{2}e^{4(p+A)}\left(\dot{y} + \sinh y\right)\\
\xi_\Phi^{\pm} &= -\frac{1}{4}e^{4(p+A)}\dot{\phi}\\
\xi_f^{\pm} &= -\frac{1}{4}e^{-2x+4(p+A)}\left(e^{-\Phi-2y}\dot{f} \pm (2P - F)\right)\\
\xi_k^{\pm} &= -\frac{1}{4}e^{-2x+4(p+A)}\left(e^{-\Phi+2y}\dot{k} \pm F\right)
\end{aligned}$$

$$\xi_F^\pm = -\frac{1}{2}e^{-2x+4(p+A)}\left(e^\Phi \dot{F} \pm \frac{1}{2}(k-f)\right) , \qquad (5.7)$$

where

$$\xi_1^\pm \equiv \xi_x^\pm - \frac{\xi_p^\pm}{3} + \frac{\xi_A^\pm}{3} , \, \xi_2^\pm \equiv \xi_x^\pm + \frac{\xi_p^\pm}{6} + \frac{\xi_A^\pm}{3} , \, \xi_3^\pm \equiv \xi_p^\pm - \xi_A^\pm . \qquad (5.8)$$

The ξ_1 redefinition[3] is will prove to be especially convenient, since we can show that this mode has a very clear physical meaning: it parameterizes the force felt by D3 branes probing a given solution. Indeed, the force on probe D3 and anti-D3 branes only depends on ξ_1 and no other ξ_a:

$$F_{D3} = -2e^{-2x}\xi_1^+ , \quad F_{\overline{D3}} = -2e^{-2x}\xi_1^- . \qquad (5.9)$$

As expected, adding a probe D3 brane to a solution derived from the superpotential W^+ (with ISD fluxes) does not break supersymmetry, and hence the force on probe D3 branes, F_{D3}, vanishes. Analogously, an anti-D3 brane in the anti-KS solution does not break supersymmetry and therefore feels no force. In a general non-supersymmetric solution, such as the singular anti-D3 in KS backreacted solution that we analyze in Section 5.4, both forces are nonzero.

5.3 A regular solution does not exist

In this section we will prove that there is no IR-regular solution with smeared anti-D3 branes ($Q < 0$) at the tip of the conifold and with KS asymptotics in the UV. Indeed, starting with a singularity-free anti-brane solution in the IR, one necessarily ends up with an anti-KS solution in the UV. Moreover, we will prove that the only regular solution with $|Q|$ anti-D3 branes is exactly the anti-KS version of the solution with Q mobile anti-branes we described in the previous subsection.

5.3.1 Regular boundary conditions for anti-D3 branes

In order to prove our statement, we need to understand first the IR boundary conditions corresponding to the presence of smeared anti-D3 branes at the tip of the KS geometry. We will also impose regularity of the 3-form fluxes. These conditions, which we will call IR regularity conditions , are the following:

- The dilaton is finite at $\tau = 0$.
- the $6d$ conifold metric has the tip structure of the KS solution: the 2-sphere (the $g_1^2 + g_2^2$ part of the $6d$ metric) shrinks smoothly at $\tau = 0$ and the 3-sphere (the $g_3^2 + g_4^2 + \frac{1}{2}g_5^2$ term) has finite size. The former condition is equivalent to

[3] In the notation of Chapter 4 we have $\xi_1^+ = \frac{1}{3}\tilde{\xi}_1$.

$2e^{-6p-x} \approx e^{x-y}$ near $\tau = 0$, and the latter requires $\tau^2 e^{-6p-x} \approx 2e^{x+y}$. All in all, we find that

$$e^{6p+2x} = \tau + \mathcal{O}(\tau^2)\,, \qquad e^y = \frac{\tau}{2} + \mathcal{O}(\tau^2)\,. \tag{5.10}$$

- The warp factor comes from $|Q|$ anti-D3 branes smeared on the 3-sphere, and hence goes like $h(\tau) \sim |Q|/\tau$. In our notation it amounts to demanding that both e^{12p+2x} and $e^{4(p+A)-2x}$ go like τ. The precise proportionality coefficients, though, cannot be fixed in this approach. Instead, one coefficient can be eliminated by a proper rescaling of the $4d$ space-time coordinates, while the other is a free parameter that measures the size of the non-shrinking 3-sphere (the conifold deformation parameter ϵ). We will use the $4d$ rescaling to match the expansion of e^{6p+x} to the supersymmetric solution (see (5.4) and (4.2)):

$$e^{12p+2x} = \frac{4}{\pi Q} \cdot \tau + \mathcal{O}(\tau^2)\,, \qquad e^{4(p+A)-2x} = c_0 \frac{4}{\pi Q} \cdot \tau + \mathcal{O}(\tau^2)\,. \tag{5.11}$$

  For the KS solution one finds $c_0 = 2^{-10/3} 3^{-2/3} \epsilon_0^{8/3}$.

- There is no singularity in the three-form fluxes; their energy densities, H_3^2 and F_3^2, do not diverge at $\tau = 0$. From (3.27) we obtain that

$$\begin{aligned} |F_3|^2 &= F_{\mu\nu\rho} F^{\mu\nu\rho} = 6 e^{6p-x} \left(e^{2y} (2P - F)^2 + e^{-2y} F^2 + 2\dot{F}^2 \right) \\ |H_3|^2 &= H_{\mu\nu\rho} H^{\mu\nu\rho} = 6 e^{6p-x} \left(e^{-2y} \dot{f}^2 + e^{2y} \dot{k}^2 + \frac{1}{2} (k - f)^2 \right)\,. \end{aligned} \tag{5.12}$$

  Hence, using (5.10) and (5.11) the Taylor expansions of the functions f, k and F start from τ^3, τ and τ^2 terms respectively, exactly like in the KS background (see (4.2)). To be more precise, in a solution with branes at the tip, the functions f, k and F can also start with non-integer powers ($\tau^{9/4}$, $\tau^{1/4}$ and $\tau^{5/4}$), but it is not hard to show that the logarithmic terms in x, p, A and y imply that the IR expansion of the solution proceeds only with integer powers of τ. In either situation the expansion of K starts with a constant Q term.

Let us summarize the leading IR terms in the expansion of the metric functions:

$$\begin{gathered} e^{\Phi} = e^{\Phi_0} + \mathcal{O}(\tau)\,, \qquad e^{2x} = \frac{\pi Q}{4} \cdot \tau + \mathcal{O}(\tau^2)\,, \qquad e^y = \frac{\tau}{2} + \mathcal{O}(\tau^2)\,,\,, \\ e^{6p} = \frac{4}{\pi Q} + \mathcal{O}(\tau)\,, \qquad e^{6A} = c_0^{\frac{3}{2}} \frac{\pi Q}{4} \cdot \tau^3 + \mathcal{O}(\tau^4)\,, \\ f = \mathcal{O}(\tau^3)\,, \qquad k = \mathcal{O}(\tau)\,, \qquad F = \mathcal{O}(\tau^2)\,, \qquad K = -\frac{\pi Q}{4} + \mathcal{O}(\tau^3)\,. \end{gathered} \tag{5.13}$$

Even if we arrived at the boundary conditions (5.13) by physical arguments, one my wonder whether these are the most general conditions we can impose. We checked that if we start by allowing a general Taylor expansion for the functions x, y, p and A, the equations of motion imply precisely the behavior summarized in (5.13).

For our proof that this IR behavior does not glue to a solution with ISD fluxes in the UV, it will be essential to determine the leading-order behavior of the conjugate modes ξ_a^+'s and ξ_a^-'s (defined in (5.7)) in the IR. Let us denote by n_a^+ and n_a^- the lowest possible leading orders of these two functions respectively. We find that the boundary conditions (5.13) imply:

$$\begin{aligned} \left(n_1^+, n_2^+, n_3^+, n_y^+, n_f^+, n_k^+, n_F^+, n_\Phi^+\right) &= (1,2,2,2,1,3,2,2) \\ \left(n_1^-, n_2^-, n_3^-, n_y^-, n_f^-, n_k^-, n_F^-, n_\Phi^-\right) &= (2,2,2,2,1,3,2,2)\ . \end{aligned} \tag{5.14}$$

The only difference between the two sets is in n_1^+ and n_1^-. Indeed, from (5.13) one sees that the leading (logarithmic) terms cancel out in the parenthesis of ξ_1^-, eq. (5.7), and sum up for ξ_1^+. Similar cancelations happen also for ξ_2^+, ξ_3^+, ξ_y^+ and their ξ_a^- counterparts. However, for the 3-form ξ_a's we cannot argue for such a cancelation neither for ξ_f^+, ξ_k^+ and ξ_F^+ nor for ξ_f^-, ξ_k^- and ξ_F^-. This is since we have no control over the coefficients of the leading terms in the expansions of f, k and F.

It is important to stress again that in arriving at (5.14) we have not imposed neither the ISD nor the IASD flux condition. Instead, we insisted on having a regular 3-form flux in the IR, with all other components of the solution being that of a smeared D3-brane solution: $1/\tau$ behavior of the warp function, constant 5-form flux proportional to Q, plus a constant dilaton.

Finally, we should also briefly mention the UV boundary conditions, although their details will not be used in our discussion. In general we must insist on KS (and not anti-KS) asymptotic with some normalizable UV modes turned on. Having only normalizable modes in the UV should be essential for the construction, since the new solution must describe a new vacuum in the same theory. Since in the UV region the non-supersymmetric solution should be just a small perturbation of the KS solution, one can use the linearized version of the equations of motion. A careful analysis reveals (see Chapter 4) that $\xi_f^+(\tau)$ and $\xi_k^+(\tau)$ approach the same non-zero constant value at large τ, while all the other functions $\xi_a^+(\tau)$ vanish.

We would like to demonstrate now that one cannot meet both the IR and the UV boundary conditions advocated above. We will do it in two different ways. We will find that the only possible solution is $\xi_a^- = 0$ for all a's, meaning that one has anti KS solution not only in the IR, but also all the way to the UV. Hence, any solutions with anti-D3 branes in the infrared must necessarily have singular three-form fluxes. This result is in agreement with the linearized analysis of [25, 21], where the equations of motion (5.5)-(5.6) were solved perturbatively in the number of antibranes. Similar results were obtained for other types of anti-branes in background with opposite charge dissolved in fluxes [24, 135, 78, 37, 38, 39].

We will provide two proofs of this claim. First, we will argue that the IR conditions (5.14) are in odds with the ξ^-'s equations of motion. This analysis has been carried out originally in [26], where it was referred to as the "IR obstruction". Second, we will present a "global" argument which is also based on the ξ^-'s equations of motion, but does not employ the Taylor expansion of these functions.

5.3.2 The first proof

Our immediate goal is to show that when solving the equations (5.6) for ξ_1^-, ξ_f^-, ξ_k^- and ξ_F^- in the IR (small τ) and imposing the IR regularity conditions, one finds only trivial solutions for these functions. This essentially means that the IASD conditions $\xi_f^-, \xi_k^-, \xi_F^- = 0$ will be satisfied all the way to the UV and not only at $\tau = 0$.

The equations we need are:

$$\dot{\xi}_1^- + Ke^{-2x}\xi_1^- = 4e^{2x-4(p+A)}\left[e^{\Phi+2y}(\xi_f^-)^2 + e^{\Phi-2y}(\xi_k^-)^2 + \frac{1}{2}e^{-\Phi}(\xi_F^-)^2\right] \tag{5.15}$$

and

$$\begin{aligned}
\dot{\xi}_f^- &= \frac{1}{2}e^{-2x}(2P-F)\xi_1^- + \frac{1}{2}e^{-\Phi}\xi_F^- \\
\dot{\xi}_k^- &= \frac{1}{2}e^{-2x}F\xi_1^- - \frac{1}{2}e^{-\Phi}\xi_F^- \\
\dot{\xi}_F^- &= \frac{1}{2}e^{-2x}(k-f)\xi_1^- + e^{\Phi}\left(e^{2y}\xi_f^- - e^{-2y}\xi_k^-\right) .
\end{aligned} \tag{5.16}$$

The equations of motion for the remaining four $\dot{\xi}_a^-$ modes are:

$$\begin{aligned}
\dot{\xi}_2^- &= -Ke^{-2x}\xi_1^- + 3e^{-6p-2x}\xi_2^- \\
&\quad -e^{-4(p+A)}\left((\xi_1^-)^2 + \frac{2}{3}\xi_2^-\xi_3^- - \frac{1}{18}(\xi_3^-)^2 + 2(\xi_y^-)^2 + 4(\xi_\Phi^-)^2\right) \\
\dot{\xi}_3^- &= 6e^{-6p-2x}\xi_2^- \\
\dot{\xi}_y^- &= \cosh y \cdot \xi_y^- + \frac{1}{3}\sinh y \cdot \xi_3^- - 2e^{\Phi}\left(e^{2y}(2P-F)\xi_f^- - e^{-2y}F\xi_k^-\right) \\
&\quad +4e^{\Phi+2x-4(p+A)}\left(e^{2y}(\xi_f^-)^2 - e^{-2y}(\xi_k^-)^2\right) \\
\dot{\xi}_\Phi^- &= -e^{\Phi}\left(e^{2y}(2P-F)\xi_f^- + e^{-2y}F\xi_k^-\right) + \frac{1}{2}e^{-\Phi}(k-f)\xi_F^- \\
&\quad +2e^{2x-4(p+A)}\left(e^{\Phi}\left(e^{2y}(\xi_f^-)^2 + e^{-2y}(\xi_k^-)^2\right) - \frac{1}{2}e^{-\Phi}(\xi_F^-)^2\right) .
\end{aligned} \tag{5.17}$$

Remember that if the fluxes are regular, the IR expansions of $f(\tau)$, $k(\tau)$ and $F(\tau)$ can only start from τ^3, τ and τ^2 respectively (see the discussion around (5.13)). As we have pointed out earlier, lower but non-integer powers are not ruled out. One can easily check, though, that our proof still goes through even in this situation.

Let us denote by n the lowest power in the Taylor expansion of ξ_1^-, i.e. $\xi_1^- = a_1\tau^n + \dots$, We already know from (5.14) that $n \geqslant 2$. Together with (5.13), equation

(5.15) implies that the leading terms in the expansions of ξ_f^-, ξ_k^- and ξ_F^- are

$$\xi_f^- = a_f\,\tau^{(n-2)/2} + \ldots\,, \quad \xi_k^- = a_k\,\tau^{(n+2)/2} + \ldots\,, \quad \xi_F^- = a_F\,\tau^{n/2} + \ldots\,, \tag{5.18}$$

Note that an additional comparison with (5.14) shows that actually for a regular solution $n \geqslant 4$. Moreover, since all the terms on the right hand side of (5.15) are non-negative and $Ke^{-2x} = \tau^{-1} + \ldots$, at least one of the constants a_f, a_k and a_F has to be non-zero. Next, plugging these expansions into the last two equations of (5.16) we see that for $n \geqslant 4$, the terms involving ξ_1^- and ξ_f^- disappear from the leading-order expansions of all these equations. A simple calculation then reveals that (5.16) has only two possible solutions, $\xi_F^- \sim \tau$ or $\xi_F^- \sim \tau^{-2}$, and both yield a singular 3-form flux.[4] Thus we have to put $a_k, a_F = 0$, in which case the first equation in (5.16) gives $n = -2$, and so we arrive at a contradiction.

We observe, therefore, that with regular boundary conditions at $\tau = 0$, the equations (5.15) and (5.16) have only the trivial solution $\xi_1^- = \xi_f^- = \xi_k^- = \xi_F^- = 0$. This means that we obtain an IASD solution all the way from the IR to the UV. In other words, one cannot "glue" the solution near the smeared anti D3-branes to the KS solution, since the latter has an ISD 3-form.

Importantly, with a bit of an effort we can demonstrate that the anti-KS geometry with mobile anti-D3's at the tip is the only regular solution of the remaining equations of motion. In other words, there is no non-singular solution with anti-D3 branes in the IR and anti-KS asymptotics in the UV. To do this we have to prove that all the remaining ξ^- functions identically vanish, exactly as ξ_1^-, ξ_f^-, ξ_k^- and ξ_F^-.

Plugging $\xi_{1,f,k,F}^- = 0$ into (5.17) we find that $\xi_\Phi^- = 0$ (otherwise the dilaton diverges), while the remaining functions satisfy:

$$\begin{aligned}
\dot{\xi}_2^- &= 3e^{-6p-2x}\xi_2^- - e^{-4(p+A)}\left(\frac{2}{3}\xi_2^-\xi_3^- - \frac{1}{18}(\xi_3^-)^2 + 2(\xi_y^-)^2\right) \\
\dot{\xi}_3^- &= 6e^{-6p-2x}\xi_2^- \\
\dot{\xi}_y^- &= \cosh y\,\xi_y^- + \frac{1}{3}\sinh y\,\xi_3^-\,.
\end{aligned} \tag{5.19}$$

In the (anti) KS solution $e^{-4(p+A)}$ goes to zero as $e^{-4\tau/3}$ for large τ, while e^{-6p-2x} asymptotes to $2/3$. From the first two equations we find that $\dot{\xi}_2^- \approx 2\xi_2^-$. The functions ξ_2^-, ξ_3^- and ξ_y^-, exactly like the functions ξ_2^+, ξ_3^+ and ξ_y^+, have to vanish at infinity both for KS and anti KS solutions. So we have to put $\xi_2^- = 0$. This in turn implies that $\xi_3^- = \xi_y^- = 0$.

We have shown that a regular solution with anti-D3 branes in the IR remains anti-KS all the way to the UV using the conjugate variables ξ. But actually, the most straightforward way to see it is to solve the second-order ϕ_a equations (shown in Appendix B) directly in powers of τ subject to the regularity conditions (5.13). We

[4] We will come back to the $\xi_F^- \sim \tau$ singular solution in the next section.

found that to order τ^{15} the space of solutions is parameterized by three independent parameters, none of which breaks the IASD condition confirming that ξ_f^-, ξ_k^- and ξ_F^- are indeed zero for the IR regular solution. Furthermore, one parameter leads to

$$\xi_2^- = 3c\tau^3 + \ldots, \qquad \xi_3^- = 6c\tau^3 + \ldots, \qquad \xi_y^- = -c\tau^3 + \ldots, . \tag{5.20}$$

This is consistent with (5.19) and, as we already know, produces a UV divergent solution. The remaining two parameters correspond to two UV-singular solutions of the supersymmetric $\xi_a^- = 0$ equations that we have already mentioned in the previous section. The first introduces the $(0,3)$ complex 3-form and the second shifts the warp function.

To sum up, IR regularity of the 3-form fluxes implies that all of the ξ_a^-'s identically vanish. The integration constants emerging from the $\xi_a^- = 0$ equations are then fixed by the UV regularity and we end up with the anti KS background with $|Q|$ mobile anti D3 branes.

5.3.3 The second proof

We can also present a “global" argument why the functions ξ_1^-, ξ_f^-, ξ_k^- and ξ_F^- have to vanish in a regular solution, without focusing on their Taylor expansions. The proof for the remaining four functions proceeds precisely as above.

Our key observation is that the flux functions $f(\tau)$, $k(\tau)$ and $F(\tau)$ appear only in equations (5.15) and (5.16). None of the remaining $\dot{\xi}^-{}_a$ equations has any flux function in it. Next, the equations in (5.16) might be derived from the following reduced Lagrangian:

$$\mathcal{L}_{\text{fluxes}} = 4e^{2x-4(p+A)} \left[e^{\Phi+2y}(\xi_f^-)^2 + e^{\Phi-2y}(\xi_k^-)^2 + \frac{1}{2}e^{-\Phi}(\xi_F^-)^2 \right] + e^{-4(p+A)}(\xi_1^-)^2 . \tag{5.21}$$

Recall that the ξ^-'s are first order in the derivatives of ϕ's and so the Lagrangian is of second order in τ-derivatives, as it should be. It differs from the flux part of the one-dimensional Lagrangian (3.3) for the KS field only by total derivative terms. Written this way, however, $\mathcal{L}_{\text{fluxes}}$ has a remarkable property: it is strictly non-negative and vanishes only if all the functions ξ_1^-, ξ_f^-, ξ_k^- and ξ_F^- are zero.

Again, we treat $\mathcal{L}_{\text{fluxes}}$ as the effective Lagrangian only for the fields $f(\tau)$, $k(\tau)$ and $F(\tau)$. In particular, it means that the first three terms in (5.21) are kinetic terms, while the last one is a potential term. We assume now that one first solves (5.16) for these three fields and for arbitrary x, y, p, A, Φ (but with the proper boundary conditions ensuring regularity of the metric), and then substitutes the result into the remaining five EOM.

Since $\mathcal{L}_{\text{fluxes}}$ is bounded from below, in other words has a global minimum for

$$\xi_f^-(\tau), \quad \xi_k^-(\tau), \quad \xi_F^-(\tau), \quad \xi_1^-(\tau) \quad = \quad 0\,, \tag{5.22}$$

one may wonder whether this trivial IASD solution is, in fact, the unique solution of the EOM (5.16). The answer depends on the boundary conditions for $f(\tau)$, $k(\tau)$ and $F(\tau)$. If these are incompatible with (5.22), the final solution will be more complicated. If on the other hand, the regular boundary conditions we imposed on the 3-form flux are compatible with the trivial IASD solution, then the latter will also be the only possible solution.

For our Lagrangian (5.21) the fields ξ_f^-, ξ_k^- and ξ_F^- are the conjugate momenta of the fields f, k and F respectively. In general, one may impose boundary conditions either on these fields or on their conjugate momentum, in the IR or/and in the UV.

The regularity requirement we considered in the previous sections, however, constrains all the three flux functions and their conjugate momenta in the IR. Indeed, we saw that both (f, k, F) and $(\xi_f^-, \xi_k^-, \xi_F^-)$ have to vanish at $\tau = 0$ for a regular solution. Furthermore, $\xi_1^- = 0$ in the IR, therefore the IR boundary conditions following from the regularity are consistent with the trivial solution (5.16). Thus we see that requiring regularity in the IR forces upon us the anti-KS solution.

This proof, though, has to be taken with a grain of salt, since the EOM for the flux fields are strictly speaking singular at $\tau = 0$, and so we cannot rule out completely the possibility that there are two different solutions of (5.16) subject to the same boundary conditions. One can promptly make our proof more rigid by listing Taylor τ^n-expansions of all six possible solutions in the IR and verifying that only one of them, the IASD, is not at odds with (5.22). However, the main goal of this subsection is to prepare the ground for the localized case discussion, where the power counting method of the first proof will be most likely unavailable making a “topological" argument we presented here a more efficient tool.

5.4 The singular anti-D3 solution

In the previous section we proved that by imposing the regular IR boundary conditions summarized in (5.13), it is not possible to find a supersymmetry-breaking solution (except the one that we have mentioned before, corresponding to ISD fluxes with a $(0, 3)$ component, which diverges in the UV). Thus, the regular IR boundary conditions are incompatible with the presence of anti-D3 branes in the infrared. One can try to construct a singular solution describing the backreaction of these anti-D3 branes by relaxing the assumptions we made in the previous section, and considering a more general expansion for the fields. In this section we therefore analyze the equations of motion dropping the assumption of regularity in the three-form fluxes discussed in Section 5.3.1.

Let us start by noticing that even in a solution with singular 3-forms, all ξ_a^-'s, but ξ_f^-, ξ_k^- and ξ_F^-, have the same leading term powers at small τ as for any regular solution, see (5.14). In particular, we still have $\xi_1^- \sim \tau^2$, since otherwise the solution will not describe anti D3's at the tip of the conifold. At the same time, ξ_f^-, ξ_k^- and ξ_F^- will now start with lower powers of τ. Remarkably, equation (5.15) suffices to determine this behavior. Indeed, since the left hand side is still of order τ exactly like in the regular case, and the right hand side is a sum of positive terms, we see that:

$$\xi_f^- = b_f + \mathcal{O}(\tau)\,, \quad \xi_k^- = b_k\,\tau^2 + \mathcal{O}(\tau^3)\,, \quad \xi_F^- = b_F\,\tau + \mathcal{O}(\tau^2)\,. \tag{5.23}$$

In deriving this result we used the first two lines of (5.13). The expansions of the original flux functions are:

$$f = -\frac{\pi Q}{8c_0} e^{\Phi_0} b_f\,\tau^2 + \mathcal{O}(\tau^3)\,, \quad k = \frac{\pi Q}{c_0}\left(b_F + 2e^{\Phi_0} b_k\right) + \mathcal{O}(\tau)\,, \quad F = \frac{\pi Q}{c_0} b_k\,\tau + \mathcal{O}(\tau^2)\,, \tag{5.24}$$

where in going from (5.23) to $f(\tau)$, $k(\tau)$ and $F(\tau)$ we have eliminated two additional solutions (see the end of the previous section): the first one is the "very" singular $(1,2)$ solution with $k \sim \tau^{-2}$ and $F \sim \tau^{-1}$ which we will not consider, and the second corresponds to the gauge transformation $(f,k) \to (f+c, k+c)$ we mentioned earlier. We fix the gauge freedom by requiring that $f(\tau)$ vanishes at $\tau = 0$.

It seems that, all in all, we have a singular solution in the UV parameterized by three independent parameters b_f, b_k and b_F. However, only two parameters are independent, since both the $\dot{\xi}_k^-$ and the $\dot{\xi}_F^-$ equations in (5.16) imply that

$$b_F = -4e^{\Phi_0} b_k\,. \tag{5.25}$$

Thus we have (at least) a two-dimensional space of singular solutions in the IR. At the same time, by gluing the solution to the UV we expect to arrive at a unique solution for the entire range of τ that depends on two parameters Q and P. The UV regularity will then impose an additional constraint on b_f and b_k (as well as on all the other "free" IR parameters like the dilaton), so that one will have to switch on both these modes in order to avoid a divergent UV solution. In fact, the perturbative solution constructed in Chapter 4 at linear order in Q/P has exactly this singularity structure (5.24) with $b_f = -12 b_k \sim 0.02\,\epsilon_0^{8/3}\,P^{-2}$. However, for the full solution this result is expected to change.

We see now that the singular solution will necessarily have a non-zero ξ_f^- at $\tau = 0$. In this case the arguments from the end of the previous section do not apply and, as a result, there is no "global minimum" obstruction for the singular solution.

As a consistency check we may show that the net force on a probe D3 brane in this background will be pointed towards the tip, as expected for a solution with smeared anti-D3 branes. This force is given by (5.9) and in our conventions it means that ξ_1^+ should be non-negative. Let us demonstrate it with the help of the ξ_1^+ equation of motion:

$$\dot{\xi}_1^+ - Ke^{-2x}\xi_1^+ = 4e^{2x-4(p+A)}\left[e^{\Phi+2y}(\xi_f^+)^2 + e^{\Phi-2y}(\xi_k^+)^2 + \frac{1}{2}e^{-\Phi}(\xi_F^+)^2\right]\,. \tag{5.26}$$

We know from (5.13) that $\xi_1^+ = \frac{1}{2}c_0^2\tau + \ldots$ near $\tau = 0$. Thus, for function $\xi_1^+(\tau)$ to vanish at some $\tau = \tau_\star$ and to become negative for $\tau > \tau_\star$, we must have $\dot{\xi}_1^+(\tau_\star) < 0$.

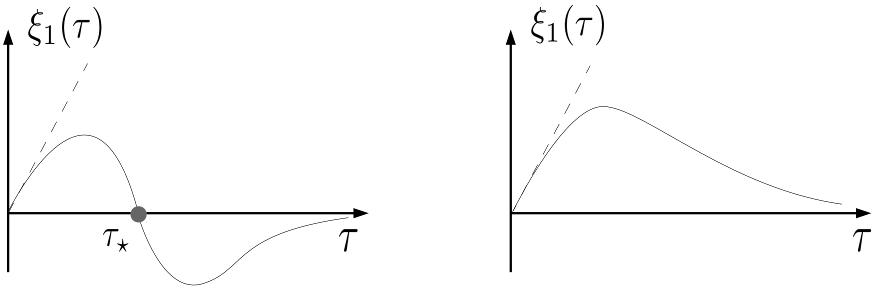


Figure 5.2: The function $\xi_1(\tau)$ is positive for small τ but cannot have a zero (*left*) at finite $\tau = \tau_\star$, since $\dot{\xi}_1(\tau) < 0$ is not allowed. As a consequence, it will be everywhere positive (*right*). Notice that it goes to zero at infinity, otherwise we do not get asymptotic KS solution.

This, however, is at odds with the equation (5.26), since its right hand side is non-negative. We conclude that $\xi_1^+(\tau) > 0$ for $\tau \in (0, \infty)$, see Figure 5.2.

Let us now come back to the ξ^- equations of motion. We may further use (5.15) in order to extract a relation between b_f, b_k, b_F and the constant b_1 defined by

$$\xi_1^- = b_1 \tau^2 + \mathcal{O}(\tau^3)\,. \tag{5.27}$$

Plugging (5.23) into (5.15) we get:

$$b_1 = \frac{\pi Q}{3c_0}\left(e^{\Phi_0}\left(\frac{b_f^2}{4} + 4b_k^2\right) + e^{-\Phi_0}\frac{b_F^2}{2}\right) = \frac{\pi Q}{12 c_0} e^{\Phi_0}\left(b_f^2 + 48 b_k^2\right) \geqslant \frac{\pi Q}{12 c_0} e^{\Phi_0} b_f^2\,. \tag{5.28}$$

This last inequality will play a crucial rôle in the next section when we will determine the form of the polarization potential. For this we will also need the explicit expressions for the RR 4 and 6-form gauge fields:[5]

$$\begin{aligned} C_4 &= \left(-2\chi_1 + e^{-2x+4(p+A)}\right) dx_0 \wedge \ldots \wedge dx_3 \\ C_6 &= \chi_f \cdot g_1 \wedge g_2 + \chi_k \cdot g_3 \wedge g_4\,, \end{aligned} \tag{5.29}$$

where[6]

$$\dot{\chi}_1 = e^{-2x}\xi_1^-\,, \qquad \dot{\chi}_f = 4e^{2y+\Phi}\xi_f^- + 2\dot{f}\chi_1\,, \qquad \dot{\chi}_k = 4e^{-2y+\Phi}\xi_k^- + 2\dot{k}\chi_1\,. \tag{5.30}$$

[5]In our conventions $dC_6 = e^{\Phi} \star_{10} F_3 - H_3 \wedge C_4$.

[6]Notice that C_6 depends only on ξ^-'s and vanishes for the anti KS solution. It is also zero for the KS background as one can show using (5.7).

The integration constants of χ_1, χ_f and χ_k can be eliminated by gauge transformations of C_4 and C_6. We will fix the freedom by requiring that all these functions vanish at $\tau = 0$. Using (5.23) and (5.13) we can find the leading order behavior of χ_1 and χ_f:

$$\chi_1 = \frac{2}{\pi Q} b_1 \cdot \tau^2 + \dots \ , \qquad \chi_f = \frac{1}{3} e^{\Phi_0} b_f \cdot \tau^3 + \dots \ . \tag{5.31}$$

We end this section by making explicit the singular character of our solution, and explaining the various terms that contribute to the singularity. First, it is easy to verify by plugging the IR behavior into (5.12) that the 3-form flux densities diverge, namely

$$|H_3|^2 \sim \frac{(b_f)^2 + 8(b_k)^2}{\sqrt{\tau}} + \mathcal{O}(\tau^0) \qquad |F_3|^2 \sim \frac{(b_k)^2}{\sqrt{\tau}} + \mathcal{O}(\tau^0) \ . \tag{5.32}$$

It is useful to characterize the singularity of our solution in terms of the ISD and IASD components of the three form flux. Let us recall the notation of Section 4.7, where we defined three scalar functions of the radial variable λ^A, by

$$e^{-\Phi} H_3 = -\lambda(\tau)_A * F_3^A \tag{5.33}$$

where F_3^A denotes each of the three components of F_3, namely along g_{125}, g_{345} and $d\tau\,(g_{13} + g_{24})$. These definitions ensure that for ISD (IASD) fluxes, $\lambda^A = 1$. We find that the component with legs g_{345} is singular:

$$\lambda_{345}(\tau) = -\frac{\pi Q}{2 c_0 P} b_f \tau^{-1} + \mathcal{O}(\tau^0) \ , \tag{5.34}$$

while the other two are regular. We should note that in the linearized anti-D3 solution of [25, 20, 21], there was actually an additional singular λ, namely λ_{125}. We thus see that at the full non-linear level one singularity gets resolved, but the singularity in λ_{345} (which corresponds to three-form field strengths that have the legs on the S^3) persists, precisely confirming our conjecture in Section 4.7.

We end this section by comparing our results to the ones in the solution considered in [40], corresponding to anti-D6-branes wrapping a T^3. As explained in [28], that solution can be T-dualized three times, and will yield a KS-like solution where the warped deformed conifold is replaced by $T^3 \times \mathbb{R}^3$. As argued in Section 4.7 this solution can be regarded as a toy model for the KS infrared region. Indeed, there is a flux singularity very much like the one found here, but in a sense simpler: the fluxes can be parameterized by a single function λ, defined by $H_3 = \lambda(\tau) *_3 F_0$, where F_0 is the mass parameter in massive type IIA, which is the toy-model version of the dual 3-form F_3 on the S^3. The fully backreacted anti-D6 solution has [39] $\lambda(\tau) = \lambda_0\, \tau^{-1} + \mathcal{O}(\tau^0)$, and the whole (IR singular) solution can be parameterized by λ_0.

5.5 D5 polarization

In this section we would like to address the main question of this paper: can the 3-form flux singularity of the anti-D3 brane putative solution be cured by the polarization of the anti-D3 branes into D5 branes? The singularity occurs at $\tau = 0$ and if we find that there is a stable configuration with a polarized D5 brane wrapping the 2-sphere at a finite distance away from the tip, it will imply that the singularity is still physically meaningful. In the first subsection we will compute the potential of a probe D5 brane with anti-D3 charge n in the singular solution sourced by Q anti-D3 branes smeared on the KS tip. We will then argue in the second subsection that this potential also governs the polarization of all Q anti-D3 branes into D5 branes.

5.5.1 The D5 potential

In order to see if the anti-branes polarize or not into D5-branes we need to compute the potential of a probe D5-brane that wraps the S^2 of the deformed conifold and has n anti-D3 branes dissolved in it. The D5 brane action (in string frame) is

$$S_{D5} = S_{DBI} + S_{WZ} \tag{5.35}$$

with

$$S_{DBI} = -\mu_5 \int d^6\xi e^{-\phi}\sqrt{-\det\left(g + 2\pi\mathcal{F}_2\right)}, \quad S_{WZ} = \mu_5 \int \left(C_6 + 2\pi\mathcal{F}_2 \wedge C_4\right), \tag{5.36}$$

where $2\pi\mathcal{F}_2 \equiv 2\pi\mathfrak{f}_2 - B_2$ and $\mathfrak{f}_2$ is the D5 worldvolume gauge field strength that gives the number of anti-D3 branes dissolved in the D5:

$$\mathfrak{f}_2 = \frac{n}{2}\omega_{S^2}, \tag{5.37}$$

where ω_{S^2} is proportional to $g_1 \wedge g_2$. The larger n the easier to polarize it is, and in that limit one can expand the DBI action in a $1/n$ series. The leading term cancels the leading term in the WZ action, and the polarization potential has in general the following form:

$$V(\tau) \sim 2\pi n \cdot c_2\tau^2 - c_3\tau^3 + \frac{1}{2\pi n}c_4\tau^4, \tag{5.38}$$

where the quadratic term comes from the imperfect WZ-DBI cancelation (and is equal to the force on a probe anti-D3 brane), the cubic term[7] comes from the C_6 term in the WZ action and the quartic terms is the subleading term in the $1/n$ expansion of the DBI action.

It is easy to show that if the following relation is satisfied

$$(c_3)^2 < \frac{32}{9}c_2c_4, \tag{5.39}$$

[7] In our conventions c_3 is positive.

then the potential (5.38) has no minima for any τ away from zero, and thus there is no polarization. In our singular solution we obtain

$$c_2 = \lim_{\tau\to 0}\left(\frac{\chi_1}{\tau^2}\right), \qquad c_3 = \lim_{\tau\to 0}\left(\frac{\chi_f}{\tau^3}\right), \qquad c_4 = \lim_{\tau\to 0}\left(\frac{e^{4(p+A)+2y+\Phi}}{\tau^4}\right), \tag{5.40}$$

where χ_1 and χ_f are defined in eqs. (5.29), (5.30), and their IR behavior is given in (5.31). Using this and (5.13), we arrive at the following result:[8]

$$c_2 = \frac{2}{\pi Q}b_1\,, \qquad c_3 = \frac{1}{3}e^{\Phi_0}b_f\,, \qquad c_4 = \frac{1}{4}c_0 e^{\Phi_0}\,. \tag{5.41}$$

We can now rewrite the inequality in (5.28) in terms of c_2, c_3 and c_4 and find that in all anti-D3 singular solutions:

$$(c_3)^2 \leqslant \frac{8}{3}c_2 c_4\,. \tag{5.42}$$

From this result we see that the condition (5.39) is always satisfied. This is the main result of this Chapter. It proves that the potential (5.38) has no minimum, not even a metastable one. Thus no polarization into D5 branes occurs and the 3-form flux singularity appears to be genuine. Even more importantly, we were able to prove this statement without extending the solution from the IR all the way to the UV.

In fact, the story here is strikingly similar to the D6 toy model of [28] that we briefly mentioned in the previous section. Remarkably, in this model there is also no need to determine the full backreacted solution in order to see that the polarization potential has no minimum away from zero. Moreover, the inequality (5.42) was exactly saturated. Our potential is more complicated, and reduces to the one of [28] if one sets $b_k = b_F = 0$. However, turning this parameter back on makes polarization even more difficult, and hence does not modify the physics that the toy model predicted.

5.5.2 The mean field argument

To understand the relation between the potential for probe anti-D3 branes that we calculated in the previous section, the potential that governs the polarization of all the smeared D3 branes into smeared D5 branes, and the potential for the polarization of *localized* D3 branes into D5 branes it is important to recapitulate several very important features of the Polchinski-Strassler construction [146].

Despite the absence of a fully-backreacted solution, Polchinski and Strassler compute in [146] the potential for all the D3 branes that source the $AdS_5 \times S^5$ geometry to polarize into D5, NS5 or (p,q)-5 branes. This computation has three ingredients. One starts from a singular solution sourced by N D3 branes, and calculates the potential of a probe D5 brane that wraps a topologically-trivial S^2 and has n units

[8] Notice that neither b_k nor b_F appear in the potential.

of D3 brane charge inside, where $n \ll N$. This potential has three terms, that go respectively like r^4, r^3 and r^2. One then finds that in the r^4 term the various factors of the warp function of the backreacted D3 branes cancel out, and this term is therefore independent of the location of the backreacted branes (all the information about the angular location of the D branes is stored in the warp factor). Furthermore, the r^3 term is proportional to the IASD three-form, which is closed and co-closed, and hence depends only on the asymptotic boundary conditions; hence, this term is also independent of the location of the backreacted D3 branes.

The r^2 term in [146] is much more complicated, as it comes from the the backreaction of the fluxes on the metric, dilaton and five-form field strength. When supersymmetry is present, one can find this term by completing the squares in the supersymmetric polarization potential [146]. However, computing this term directly is much more painful, and has been done in [73, 158]. Not surprisingly, the two calculations agree, and the r^2 term also turns out to be independent of the warp factor sourced by the backreacted D3 branes, although this is much more difficult to see from the supergravity calculation. When supersymmetry is broken by the introduction of a fourth fermion mass, one can still compute the r^2 term by using various supersymmetric limits as well as the fact that this term comes from interacting three-form field strengths (see for example section IV of [146]) and one still finds that this term is independent of the warp factor, and therefore of the position of the backreacting D3 branes. Hence, both in supersymmetric and in non-supersymmetric Polchinski-Strassler backgrounds the polarization potential for a probe D5 brane with D3 charge n is independent of the position of the N D3 branes that source the solution.

Armed with this fact, one can consider then the much more general problem of a large number of D5 branes that have charges n_i, such that $\sum_i n_i = N$ and $n_i \ll N$. Each of these D5 branes can now be treated as a probe in the supergravity solution created by the other branes, and because the polarization potential is independent of the position of the D3 branes that source the background, the potential felt by each D5 brane in this configuration is the same as the potential of this D5 brane in the singular solution above. Hence, one can construct self-consistently the full solution by requiring that each probe is at a minimum in the background sourced by the other probes. This "mean-field" construction can then be generalized straightforwardly to D3 branes polarizing into multiple shells that can also have NS5 or more general (p, q)-5-brane dipole charge. More generally, this construction can also be used to study all the other types of brane polarization that occur in the region where the branes that polarize dominate the geometry. The correctness of this "mean-field" Polchinski-Strassler construction of vacua with polarized branes has been confirmed in the few examples where the fully-backreacted brane polarization supergravity solution exists, such as the mass-deformed M2 brane theory [31, 125], or the supergravity dual of the mass-deformed 5D Super Yang-Mills theory [17]. Hence the probe calculation that we presented in the previous section gives the full potential for the smeared anti-D3 branes to polarize into D5 branes at a finite distance away from the tip.

However, one can do much more: one can use this independence of the Polchinski-Strassler polarization potential on the location of the polarizing branes to compute the potential for N D3 branes that are *localized* near the north pole of the large S^3 at the bottom of the KS solution to polarize into a D5 brane wrapping the conifold S^2 at a finite distance from the tip. By the arguments above, this potential is the same as the potential for several probe D3 branes to polarize on this S^2 in the singular geometry sourced by a large number of D3 branes that are localized on the KS three-sphere, as long as the polarization occurs in the region where these D3 branes dominate the geometry. In turn, this potential is independent of the location of the D3 branes that dominate the geometry, and hence is the same as the potential for several probe D3's to polarize into a D5 brane in the geometry where these D3 branes are *smeared*, which we calculated in the previous subsection.

Hence, our calculation indicates that neither smeared nor localized anti-D3 branes do not polarize into D5 branes, and therefore that brane polarization à la Polchinski-Strassler does not appear to cure the singularity of antibranes in KS.

5.5.3 Validity of approximations

We now discuss the range of validity of our calculation. We see from the probe D5 potential (5.38) and the expressions (5.41) that the radius τ_* at which the D5 would sit is of the order

$$\tau_* \sim n\frac{c_3}{c_4} \sim n\, b_f\,. \tag{5.43}$$

Here we immediately face a problem. Since we do not know the full solution we cannot fix the dependence of b_f and all other coefficients on P (the 5-form flux) and Q (the number of the anti D3's). We can still *estimate* however this dependence using the method utilized in [28]. The full solution is expected to be unique, namely having no parameters other than P and Q. We, therefore, anticipate that for a fixed order in the τ expansion the contributions coming from various terms in the EOM will be of the same order in terms of P and Q. In other words, there should be a detailed balance between different terms.

Let us introduce the following notation:

$$\xi_f^- = b_f^{(0)} + b_f^{(1)}\tau + b_f^{(2)}\tau^2 + \ldots \tag{5.44}$$

and similarly for the other ξ^-'s. The additional index stands for the power of τ and in terms of the notation introduced in the previous section we have $b_f^{(0)} = b_f$, $b_k^{(2)} = b_k$, *etc.*

We can start our analysis, for instance, from the τ^2 contribution to the following term in the ξ_1^- equation (5.15)

$$e^{2y}\left(\xi_f^-\right)^2 + e^{-2y}\left(\xi_k^-\right)^2\,. \tag{5.45}$$

We see that the detailed balance implies $b_f^{(0)} \sim b_k^{(2)}$. Next, the $e^{2y}\xi_f^- - e^{-2y}\xi_k^-$ term in the $\dot{\xi}_F^-$ equation gives $b_f^{(0)} \sim b_k^{(4)}$. We conclude that $b_k^{(2)} \sim b_k^{(4)}$. With a bit of effort, one can further show that in fact all $b_f^{(i)}$'s and $b_k^{(i)}$'s are of the same order of magnitude. Moreover, $b_F^{(i)} \sim e^{\Phi_0} b_k^{(j)}$ for all i and j.

Let us now consider the $b_1^{(i)}$ coefficients. From (5.15) and the $\dot{\xi}_f^-$ equation in (5.16) we learn that $b_1^{(2)} \sim Q e^{\Phi_0} \left(b_f^{(0)}\right)^2$ and $b_f^{(2)} \sim PQ^{-1} b_1^{(2)}$ respectively. Combining the two we see that $b_f^{(i)} \sim e^{-\Phi_0} P^{-1}$ and $b_F^{(i)} \sim P^{-1}$.

Finally, we have to compare the ξ_f^- and ξ_f^{-2} terms on the right hand side of the $\dot{\xi}_\Phi^-$ equation in (5.17). We find $b_f^{(1)} \sim P/Q$ and comparing this with the observations of the previous two paragraphs we see eventually that $e^{\Phi_0} \sim Q/P^2$.

To summarize, we find that:

$$b_f^{(i)} \sim b_k^{(i)} \sim \frac{P}{Q}\,, \qquad b_F^{(i)} \sim \frac{1}{P}\,, \qquad \text{and} \qquad e^{\Phi_0} \sim \frac{Q}{P^2}\,. \tag{5.46}$$

The remaining coefficients are irrelevant for our analysis.

We are now in a position to check the validity region of our polarization calculation. In order to trust our computation we need to assume the following conditions:

- The anti-D3 charge of the probe D5 should be much smaller than the anti-D3 charge of the background
$$n \ll Q\,. \tag{5.47}$$
We recall that in our conventions Q is the number of anti-D3 branes.

- In order to trust the IR expansions we should demand that τ_* is small compared to the ratio between the leading and next-to-leading terms in the series. Since, for instance, all of the $b_f^{(i)}$ are of the same order, we must require $\tau_* \ll 1$. This in turn amounts to
$$n \ll \frac{Q}{P}\,. \tag{5.48}$$

- Since we expanded the square root in the DBI action we should demand that $\det(2\pi\mathcal{F}_2) \gg \det(g_\perp)$. Recalling that in our Ansatz $\det(g_\perp) = e^{2x+2y} \sim Q\tau^3 + \ldots$ we obtain $n^2 \gg \tau_*^3 Q$ or
$$n \ll \frac{Q^2}{P^3}\,. \tag{5.49}$$

- The radius of the S^2 at which the D5 brane would polarize should be large in string units. Since the radius is given by $(\det(g_\perp))^{1/4}$ this amounts to demanding $\tau_*^3 Q \gg 1$ or
$$n \gg \frac{Q^{2/3}}{P}\,. \tag{5.50}$$

- The string coupling should be small at τ_*. This means that

$$Q \ll P^2 \,. \tag{5.51}$$

To conclude, we have the following criteria

$$\frac{Q^{2/3}}{P} \ll n \ll \frac{Q^2}{P^3} \ll \frac{Q}{P} \ll Q \,. \tag{5.52}$$

This can be easily achieved. For example, we can set $n \sim \sigma$, $Q \sim \sigma^7$ and $P \sim \sigma^4$ for large σ.

5.6 Discussion

We reviewed in detail the non-linear solution corresponding to Q anti-D3-branes smeared on the S^3 at the tip of the deformed conifold, focusing on the fact that such solution has singular three-form fluxes in the infrared. These singularities could have suggested a stringy resolution by polarization à la Polchinski-Strassler. However, we showed that the anti-D3-branes do not polarize into anti-D5-branes wrapping the S^2 at a finite radius, and therefore such mechanism of resolution of singularities is not in place here.

In order to show that, we computed the polarization potential, which has quadratic, cubic and quartic terms in the radial variable, but with coefficients such that there is no minimum, regardless of any UV data. All information needed to reach that conclusion are the IR boundary conditions reviewed in detail in the text. This result is quite strong, as on one hand we had shown that any solution with anti-D3-brane boundary conditions leads to either an anti-KS solution or to a singular solution, and on the other hand we are showing that this singularity is not resolved by polarization into anti-D5-branes, no matter what irrelevant or relevant operators one adds in the UV.

It is worth mentioning again the striking similarities between our results and those on anti-D6-branes in backgrounds with D6-charge dissolved in fluxes, which serves indeed as a toy model for the IR of KS. They both have the same type of singularities, and in neither case these can be resolved by polarizing into anti-branes of two dimensions higher. Furthermore, the potential for polarization in the case of anti-D3 branes reduces exactly to the one for anti-D6 if one integration constant is set to zero. The second integration constant, which should be related to the first one by UV boundary conditions, only makes things worse in terms of getting a minimum.

Our result also suggests that in the fully back-reacted solution there may be no polarization into NS5-branes, opposite to what happens in the probe calculation. In order to pin down this question one would need the localized solution, though, as smearing the charge on the S^3 wipes out this polarization channel. However, one might hope that, as was the case here, only very few details of the solution are needed to get an answer, and such details might be within reach.

Chapter 6

Metastable states in M-theory

In this chapter we will focus on non-supersymmetric eleven dimensional supergravity solutions. By using the same method followed for the deformed conifold in six dimensions, we construct an M–theory background dual to a metastable state in a (2+1)-dimensional field theory, which corresponds to placing a stack of anti–M2 branes at the tip of a warped Stenzel space, which is a higher dimensional generalization of the deformed conifold. With this purpose we analytically solve for the linearized non–supersymmetric deformations around the warped Stenzel space, preserving the $SO(5)$ symmetries of the supersymmetric background, and which interpolate between the IR and UV region. We identify the supergravity solution which corresponds to a stack of $\bar{N}$ backreacting anti–M2 branes by fixing all the 12 integration constants in terms of $\bar{N}$. In the UV this solution has the desired features to describe the conjectured metastable state of the dual (2+1)-dimensional theory, while in the IR it suffers from a singularity in the four–form flux, which we describe in some details and which is similar in nature to the singularity of the anti-D3 brane backreaction studied in the previous sections. This Chapter is based on the paper [135].

6.1 Introduction and motivation

In the previous Chapters we discussed in great details the construction of non-supersymmetric cone-like four-dimensional compactifications of type IIB string theory. The reason to focus on these constructions is the immediate phenomenological interest in constructing de Sitter vacua and cosmological models in the four-dimensional space time. However, a large part of the motivation also comes from the gauge/gravity correspondance, since these solutions are conjectured to be dual to non-supersymmetric states in the field theory, and to deformations by a full set of non-normalizable modes. Attention to mechanisms of metastable supersymmetry breaking in quantum field theories was drawn by the work of Intriligator, Seiberg and Shih [106]. Since the constructions of such states involve strongly coupled regimes, it is natural to address the study of this phenomenon in stringy realizations of the supersymmetric theories.

The approach we followed in this thesis is to consider brane realisations which extend the AdS/CFT correspondance to non–conformal or less–supersymmetric theories and try to construct metastable states in this context. One way to achieve this is to start by a configuration of branes placed at some Calabi-Yau singularity and consider the supergravity solution obtained after smoothing the singular point. While in the previous sections we explored this possibility for the six-dimensional Klebanov-Strassler cone, it is of obvious interest to address the same question in different contexts where metastable states are conjectured to exist, by string theory arguments, in supergravity backgrounds dual to strongly coupled field theories.

Here we focus on the case of an AdS_4/CFT_3 correspondence, which involves an $\mathcal{N}=2$ supersymmetric (2+1)–dimensional theory, whose supergravity dual is $AdS_4\times V_{5,2}$, where $V_{5,2}$ is the 7–dimensional Sasaki–Einstein space $V_{5,2}=SO(5)/SO(3)$. A gravity dual for a long-lived metastable state has been proposed in [115] based on the probe analysis, by placing a stack of anti–M2 branes at the tip of the warped M–theory background with transverse Stenzel space [156]. Here, the analogue of the KS solution is the supersymmetric solution of Cvetič, Gibbons, Lü and Pope (CGLP) [54]; indeed, the 8–dimensional Stenzel space is a part of a family of Ricci–flat solutions parametrized by the dimension n, which include the deformed conifold for $n=6$. The mechanism for which the false vacuum decays is similar to the KPV process [111] (see Section 2.4): the anti–M2 branes fall in the warped throat and at the tip they polarize into M5–branes wrapping an $S^3\subset S^4$. The probe analysis of [115] shows that this state is metastable if $p/\tilde{M}\leq 0.054$, where $\tilde{M}$ is the number of units of the 4–form flux of the CGLP background.

The effects of the backreaction of the anti–M2 branes on the transverse geometry have been partially studied in [24], where the linearized equations that govern the first–order backreacted solution have been solved implicitly in terms of integrals by using the first–order formalism introduced by Borokhov and Gubser [41] and discussed in section 3.2; the full solution was presented separately in the small and large radius limit. The main purpose of that work was to study the IR behavior of the perturbed solution, and the conclusion of this analysis was similar to the anti–D3 case, namely that the conjectured solution dual to the metastable state exhibits certain singularities which in the anti–M2 case lead to a divergent action in the IR[1].

As we discussed for the deformed conifold solution, it is important to check whether the solution, beside the singularity, has the correct features to correspond to the metastable state in the field theory. For this, one needs to connect the IR and the UV region and to see if the ultraviolet region has the desired properties. In this perspective, it is clearly interesting to perform such an analysis in the anti–M2 brane configuration, which in the IR can be thought as the M-theory generalization of the Type IIB KS solution, but has a rather different behavior in the UV. For ex-

[1]See [78] for a similar analysis in a Type IIA context.

ample, in the M-theory background there is not a logarithmic running of the charge, which is an important feature of the KS background, and was crucial in the analysis of the backreaction, as we discussed in Chapter 4. In this Chapter we perform the analysis outlined above and by extending the results of [24] we present the full analytic solution of the linearized supergravity equations which describe the most general non–supersymmetric deformation of the warped Stenzel space compatible with the symmetries of the CGLP background. With this result we are able to study the effects of IR boundary conditions on the ultraviolet behavior of the supergravity modes and we identify the unique solution which has the desired features to describe anti–M2 branes in the CGLP background (leaving open the issue of the singularity discussed above).

This Chapter is organized as follows. In Section 6.2 we review the computational formalism and we solve analytically the system of first–order differential equations governing perturbations around the CGLP supersymmetric solution. Our full solution, which is shown in Appendix E, contains few single integrals that cannot be explicitly performed, but they can easily be handled with numerical integration. In Section 6.3 we show the expansions of our solution in the IR and in the UV region in terms of a set of twelve integration constants denoted (X_a, Y_a). In Section 6.4 we discuss the various charges in the Stenzel background and we identify the perturbation due to the presence of M2 branes. In Section 6.5 we impose the boundary conditions that arise from placing a stack of anti–M2 branes at the tip ($r = 0$) of the geometry and we discuss the problems associated to an infrared singularity in the fluxes. We then summarize the asymptotic behavior of the anti–M2 solution, which is expressed in terms of the number of anti–M2 branes. As a check of our boundary conditions, we compute the force exerted on a probe M2 brane and we show that it agrees with the one derived from the brane/antibrane potential (which we review in Appendix F). We end with a discussion in Section 6.6.

6.2 Linearized equations and their solutions

The linearized equations governing the deformations around the warped Stenzel space have been derived in [24] by using the Borokhov–Gubser [41] first–order formalism. We use the ansatz for the $SO(5)$–invariant supergravity solution of [115]:

$$ds_{11}^2 = e^{-2z(r)} dx_\mu dx^\mu + e^{z(r)} \left[e^{2\gamma(r)} dr^2 + e^{2\alpha(r)} \sigma_i^2 + e^{2\beta(r)} \tilde{\sigma}_i^2 + e^{2\gamma(r)} \nu^2 \right] , \qquad (6.1)$$

where σ_i, $\tilde{\sigma}_i$ ($i = 1, 2, 3$) and ν are the 1–forms in the coset $SO(5)/SO(3)$ and $\mu = 0, 1, 2$. The four–form field strength G_4 is given by

$$G_4 = dK(r) \wedge dx^0 \wedge dx^1 \wedge dx^2 + m\, F_4 \,, \qquad (6.2)$$

where the internal flux F_4 is parametrized by

$$F_4 = f'(r) dr \wedge \tilde{\sigma}_1 \wedge \tilde{\sigma}_2 \wedge \tilde{\sigma}_3 + h'(r)\, \epsilon^{ijk} dr \wedge \sigma_i \wedge \sigma_j \wedge \tilde{\sigma}_k \qquad (6.3)$$

$$+\frac{1}{2}(4h(r)-f(r))\epsilon^{ijk}\nu\wedge\sigma_i\wedge\tilde{\sigma}_j\wedge\tilde{\sigma}_k-6\,h(r)\,\nu\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3\,,$$

and the function $K(r)$ is fixed in terms of the other functions by the equation of motion

$$d\star G_4=\frac{1}{2}G_4\wedge G_4\,. \tag{6.4}$$

We recall that the method introduced in [41] relies on the existence of a superpotential W defined such that its square gives the potential, namely

$$V(\phi)=\frac{1}{8}\,G^{ab}\,\frac{\partial W}{\partial\phi^a}\,\frac{\partial W}{\partial\phi^b}\,. \tag{6.5}$$

We consider an expansion for the fields ϕ^a $(a=1,...,n)$ around the supersymmetric background

$$\phi^a=\phi_0^a+\phi_1^a(X)+\mathcal{O}(X^2)\,, \tag{6.6}$$

where X represents the set of perturbation parameters, ϕ_1^a is linear in them, and ϕ_0^a are the functions in the CGLP solution, written explicitly in (6.15). We will denote the set of functions ϕ^a, $a=1,...,6$ in the following order

$$\phi^a=(\alpha,\beta,\gamma,z,f,h)\,. \tag{6.7}$$

The first order formalism gives a set of $2n$ linear first–order differential equations for the perturbations ϕ_1^a and their conjugates ξ^a (see section 3.2):

$$\frac{d\xi_a}{d\tau}+\xi_b\,M^b{}_a(\phi_0)=0\,, \tag{6.8}$$

$$\frac{d\phi_1^a}{d\tau}-M^a{}_b(\phi_0)\,\phi_1^b=G^{ab}\,\xi_b\,, \tag{6.9}$$

where

$$\xi_a\equiv G_{ab}(\phi_0)\left(\frac{d\phi_1^b}{d\tau}-M^b{}_d(\phi_0)\,\phi_1^d\right)\,,\qquad M^b{}_d\equiv\frac{1}{2}\frac{\partial}{\partial\phi^d}\left(G^{bc}\frac{\partial W}{\partial\phi^c}\right)\,. \tag{6.10}$$

The equations (6.9) are the definitions of the ξ_a,while the n equations (6.8) form a closed set and imply the equations of motion [41]. The functions ξ_a should additionally satisfy the zero–energy condition

$$\xi_a\,\frac{d\phi_0^a}{d\tau}=0\,. \tag{6.11}$$

The field–space metric in (6.10) is

$$G_{ab}\,\phi'^a\,\phi'^b=\frac{1}{2}e^{-\alpha-3(\beta+z)}\Big[3e^{4\alpha+6\beta+3z}\big(3z^2-4\alpha^2-12\alpha\beta-4\beta^2-4\alpha\gamma-4\beta\gamma\big)$$
$$+e^{4\alpha}m^2f^2+12e^{4\beta}m^2h^2\Big]\,, \tag{6.12}$$

and the superpotential is given by [24]

$$W(\phi) = -3e^{2\alpha+2\beta}\big(e^{2\alpha} + e^{2\beta} + e^{2\gamma}\big) - 6m^2 e^{-3z}\Big[h(f-2h) - \frac{1}{54}\Big]. \tag{6.13}$$

The background fields satisfy the flow equation[2]

$$\frac{d\phi_0^a}{d\tau} = \frac{1}{2}G^{ab}\frac{\partial W}{\partial \phi_0^b} \tag{6.14}$$

and they are given by the CGLP solution [54]

$$\begin{aligned}
e^{2\alpha_0} &= \frac{1}{3}(2+\cosh 2r)^{1/4}\cosh r\,, \qquad (6.15)\\
e^{2\beta_0} &= \frac{1}{3}(2+\cosh 2r)^{1/4}\sinh r \tanh r\,,\\
e^{2\gamma_0} &= (2+\cosh 2r)^{-3/4}\cosh^3 r\,,\\
f_0 &= \frac{1-3\cosh^2 r}{3^{3/2}\ \cosh^3 r}\,,\\
h_0 &= -\frac{1}{2\,3^{3/2}\ \cosh r}\,,\\
z_0 &= \frac{1}{3}\log(m^2 H(r))\,,
\end{aligned}$$

where the warp factor H is defined by the following integral:

$$H(r) = \int_r^\infty \frac{3\,\mathrm{sech}^3 u\ \tanh u}{(2+\cosh 2u)^{3/4}}\,du = \sqrt{2}\,\frac{y\,(7-5y^4)}{(y^4-1)^{3/2}} + 5\sqrt{2}\,F\Big(\ \arcsin(y^{-1})| - 1\Big), \tag{6.16}$$

where

$$y = (2+\cosh(2r))^{1/4} \tag{6.17}$$

and F is the incomplete elliptic integral of the first kind

$$F(\phi|q) = \int_0^\phi (1-q\sin^2(\theta))^{-1/2}d\theta\,. \tag{6.18}$$

As shown in [24], it is useful to solve for the following linear combinations of the fields ξ_a and ϕ_a

$$\tilde{\xi}_a = (\xi_1+\xi_2+\xi_3, \xi_1-\xi_2+3\,\xi_3, \xi_1+\xi_2-3\xi_3, \xi_4, \xi_5, \xi_6)\,, \tag{6.19}$$

$$\tilde{\phi}_a = (\phi_1-\phi_2, \phi_1+\phi_2-2\,\phi_3, \phi_3, \phi_4, \phi_5, \phi_6)\,. \tag{6.20}$$

[2] Note that the equations for $a = 1,2,3$ are equivalent to the the Ricci–flat Kähler condition for the metric [133], while the ones for $a = 5,6$ give the self–duality of the internal form F_4.

The first–order systems of coupled differential equations for the fields $\tilde{\xi}_a$ and $\tilde{\phi}_a$ are:

$$\tilde{\xi}_4' = \frac{1}{9} e^{-3(z_0+\alpha_0+\beta_0)} m^2 \big(54\, h_0(f_0 - 2h_0) - 1\big)\, \tilde{\xi}_4\,, \tag{6.21}$$

$$\tilde{\xi}_1' = \frac{2}{9} e^{-3(z_0+\alpha_0+\beta_0)} m^2 \big(54\, h_0(f_0 - 2h_0) - 1\big)\, \tilde{\xi}_4\,, \tag{6.22}$$

$$\tilde{\xi}_5' = \frac{1}{2} e^{\alpha_0-\beta_0} \tilde{\xi}_6 - 2m^2 h_0 e^{-3(z_0+\alpha_0+\beta_0)} \tilde{\xi}_4\,, \tag{6.23}$$

$$\tilde{\xi}_6' = 6e^{-3\alpha_0+3\beta_0} \tilde{\xi}_5 - 2e^{\alpha_0-\beta_0} \tilde{\xi}_6 - 2m^2(f_0 - 4h_0) e^{-3(z_0+\alpha_0+\beta_0)} \tilde{\xi}_4\,, \tag{6.24}$$

$$\tilde{\xi}_3' = 4e^{-\alpha_0-\beta_0+2\gamma_0} \tilde{\xi}_3 + \frac{2}{9} e^{-3(z_0+\alpha_0+\beta_0)} m^2 \big(54(f_0 - 2h_0)h_0 - 1\big) \tilde{\xi}_4\,, \tag{6.25}$$

$$\begin{aligned}\tilde{\xi}_2' = {} & 2\cosh(\alpha_0 - \beta_0)\tilde{\xi}_2 - \frac{3}{2} e^{\alpha_0-\beta_0} \tilde{\xi}_1 + \frac{3}{2} e^{-\alpha_0-\beta_0} (e^{2\alpha_0} - 2e^{2\gamma_0}) \tilde{\xi}_3 \\ & - 36 h_0 e^{-3\alpha_0+3\beta_0} \tilde{\xi}_5 + e^{\alpha_0-\beta_0} (f_0 - 4h_0) \tilde{\xi}_6\,,\end{aligned} \tag{6.26}$$

and

$$\tilde{\phi}_1' = -2\cosh(\alpha_0 - \beta_0)\tilde{\phi}_1 + \frac{1}{12} e^{-3(\alpha_0+\beta_0)} (-3\tilde{\xi}_1 + 4\tilde{\xi}_2 + 3\tilde{\xi}_3)\,, \tag{6.27}$$

$$\tilde{\phi}_2' = -4e^{-\alpha_0-\beta_0+2\gamma_0} \tilde{\phi}_2 - 6\sinh(\alpha_0 - \beta_0)\tilde{\phi}_1 + \frac{1}{12} e^{-3(\alpha_0+\beta_0)} (-3\tilde{\xi}_1 + 7\tilde{\xi}_3)\,, \tag{6.28}$$

$$\tilde{\phi}_3' = 3\sinh(\alpha_0 - \beta_0)\tilde{\phi}_1 + \frac{3}{2} e^{-\alpha_0-\beta_0+2\gamma_0} \tilde{\phi}_2 + \frac{1}{12} e^{-3(\alpha_0+\beta_0)} (\tilde{\xi}_1 - 3\tilde{\xi}_3)\,, \tag{6.29}$$

$$\tilde{\phi}_5' = \frac{2}{m^2} e^{-3\alpha_0+3\beta_0} (-3m^2\tilde{\phi}_6 + 9m^2 h_0 \tilde{\phi}_1 + e^{3z_0} \tilde{\xi}_5)\,, \tag{6.30}$$

$$\tilde{\phi}_6' = \frac{1}{6m^2} e^{\alpha_0-\beta_0} (-3m^2\tilde{\phi}_5 + 12m^2\tilde{\phi}_6 - 3m^2(f_0 - 4h_0)\tilde{\phi}_1 + e^{3z_0}\tilde{\xi}_6)\,, \tag{6.31}$$

$$\begin{aligned}\tilde{\phi}_4' = {} & \frac{1}{9} e^{-3(z_0+\alpha_0+\beta_0)} \Big(2e^{3z_0}\tilde{\xi}_4 + m^2((1 + 54h_0(2h_0 - f_0))\tilde{\phi}_2 + 2\tilde{\phi}_3 + \tilde{\phi}_4 \\ & + 18(h_0(-3(f_0 - 2h_0)(2\tilde{\phi}_3 + \tilde{\phi}_4) + \tilde{\phi}_5) + (f_0 - 4h_0)\tilde{\phi}_6))\Big)\,.\end{aligned} \tag{6.32}$$

6.2.1 Solutions for $\tilde{\xi}_a$

We first note that the equation for $\tilde{\xi}_4$ can be easily integrated by using the flow equations (6.14); for $a = 4$ this reads:

$$z_0'(r) = 2e^{-3(z_0+\alpha_0+\beta_0)} m^2 \Big(h_0(f_0 - 2h_0) - \frac{1}{54}\Big). \tag{6.33}$$

This shows that $\tilde{\xi}_4$ is proportional to the warp factor:

$$\tilde{\xi}_4 = m^2 H(r) X_4. \tag{6.34}$$

In terms of the radial variable r the solutions for the remaining modes are

$$\tilde{\xi}_4 = m^2 X_4 H(r)\,, \tag{6.35}$$

$$\tilde{\xi}_1 = 2\,m^2 X_4 H(r) + X_1\,,$$

$$\tilde{\xi}_5 = -\mathrm{csch}^2 r\,\mathrm{sech} r\,\Big(\frac{3\sqrt{3}}{2} m^2 X_4 \int^r \frac{\mathrm{csch}^3 u}{(\cosh 2u+2)^{3/4}} du + X_5\Big)$$
$$- \cosh r\,\coth^2 r\,\Big(- \frac{3\sqrt{3}}{2} m^2 X_4 \int^r \frac{\cosh 2u\,\mathrm{csch}^3 u\,\mathrm{sech}^4 u}{(\cosh 2u+2)^{3/4}} du + X_6\Big)\,,$$

$$\tilde{\xi}_6 = (3\cosh 2r + 1)\,\mathrm{csch}^2 r\,\mathrm{sech}^3 r\,\Big(\frac{3\sqrt{3}}{2} m^2 X_4 \int^r \frac{\mathrm{csch}^3 u}{(\cosh 2u+2)^{3/4}} du + X_5\Big)$$
$$+ (4\coth r\,\mathrm{csch} r - 2\cosh r)\,\Big(- \frac{3\sqrt{3}}{2} m^2 X_4 \int^r \frac{\cosh 2u\,\mathrm{csch}^3 u\,\mathrm{sech}^4 u}{(\cosh 2u+2)^{3/4}} du + X_6\Big)\,,$$

$$\tilde{\xi}_3 = -6\sinh^4 r\,(\cosh 2r + 2)\Big(m^2 X_4 \int^r \frac{\mathrm{csch}^3 u\,\mathrm{sech}^4 u}{(\cosh 2u+2)^{7/4}} du + X_3\Big)\,,$$

$$\tilde{\xi}_2 = \sinh r\cosh r\Big[X_2 + \frac{3}{2} X_1 \coth r + 9 X_3\,\sinh^3 r\,\cosh r + \frac{4}{3\sqrt{3}} X_5\,\mathrm{csch} r\,\mathrm{sech}^5 r$$
$$+ \frac{4}{3\sqrt{3}} X_6\,(\coth r - 3\tanh r) + m^2 X_4\,\Big(3H(r)\coth r - \frac{2\tanh r\,\mathrm{sech}^3 r}{(\cosh 2r+2)^{3/4}}$$
$$+ 2\,\mathrm{csch} r\,\mathrm{sech}^5 r \int^r \frac{\mathrm{csch}^3 u}{(\cosh 2u+2)^{3/4}} du + 9\sinh^3 r\cosh r \int^r \frac{\mathrm{csch}^3 u\,\mathrm{sech}^4 u}{(\cosh 2u+2)^{7/4}} du$$
$$+ 4(\cosh 2r - 2)\,\mathrm{csch} 2r \int^r \frac{\cosh 2u\,\mathrm{csch}^3 u\,\mathrm{sech}^4 u}{(\cosh 2u+2)^{3/4}} du\Big)\Big]\,.$$

The zero energy condition (6.11) reads

$$X_2 = 0. \tag{6.36}$$

By using the change of variables (6.17) it is easy to show (see Appendix E) that the solution can be expressed in terms of only two integrals, the warp factor $H(r)$ and the Green's function [19]

$$G(r) = \int^r \frac{3\sqrt{3}\,\mathrm{csch}^3 u}{2\,(\cosh 2u+2)^{3/4}} du = \frac{\sqrt{3}}{2}\Bigg[\frac{3y(y^4-1)^{1/2}}{\sqrt{2}\,(9-3y^4)} - 3\,2^{-1/2}\,F\Big(\arcsin(y^{-1})|1\Big)$$
$$- \sqrt{2}\,\Pi\Big(-\sqrt{3}; -\arcsin(y^{-1})|1\Big) - \sqrt{2}\,\Pi\Big(\sqrt{3}; -\arcsin(y^{-1})|1\Big)\Bigg]\,, \tag{6.37}$$

where we use the standard definition for the incomplete elliptic integral of the third kind

$$\Pi(n;\phi|m) = \int_0^\phi (1 - n\sin^2(\theta))^{-1}(1 - m\sin^2(\theta))^{-1/2} d\theta\,. \tag{6.38}$$

6.2.2 Solutions for $\tilde{\phi}_a$

We now present the solution for the $\tilde{\phi}_a$ modes. Here we show the result in a compact form in terms of the variable r and we relegate to Appendix E the involved analytic expressions which are obtained by explicitly performing the integrations. The first–order perturbations to the metric modes and fluxes are

$$
\begin{aligned}
\tilde\phi_1 &= -\frac{1}{\sinh 2r}\int^r \frac{9\,\coth u\,\mathrm{csch} u}{2\,(2+\cosh 2u)^{3/4}}\,\tilde\xi_{123}\,du + \frac{Y_1}{\sinh 2r}\,, \qquad (6.39)\\
\tilde\phi_2 &= -\frac{9\,\mathrm{csch}^4 r}{4\,(2+\cosh 2r)}\int^r \frac{\sinh u}{(2+\cosh 2u)^{3/4}}\Big[(15+3\cosh 2u)\,\tilde\xi_1 - 12\,\tilde\xi_2 - (23+7\cosh 2u)\,\tilde\xi_3\Big]du\\
&\quad - \frac{3}{2+\cosh 2r}\tilde\phi_1 + \frac{\mathrm{csch}^4 r}{2+\cosh 2r}\,Y_2\,,\\
\tilde\phi_3 &= Y_3 - \frac{9}{32}\int^r \frac{\mathrm{csch}^3 u}{(2+\cosh 2u)^{3/4}}\Big[\tilde\xi_1 + 3\cosh 2u\,\tilde\xi_{123} + 3\,\tilde\xi_3\Big]du - \frac{3}{8}\cosh 2r\,\tilde\phi_1 - \frac{3}{8}\,\tilde\phi_2\,,\\
\tilde\phi_5 &= \sinh^3 r\,\tanh^3 r\,\Lambda_5 + \frac{1}{2}\cosh r\,(5-\cosh 2r)\,\Lambda_6\,,\\
\tilde\phi_6 &= -\frac{1}{4}(3+\cosh 2r)\sinh r\,\tanh r\,\Lambda_5 + \frac{1}{2}\cosh^3 r\,\Lambda_6\,,
\end{aligned}
$$

where we defined

$$
\begin{aligned}
\Lambda_5 &= Y_5 + \frac{1}{24}\int^r \Big[12\sinh u\,\tilde\xi_5 - (5-\cosh 2u)\coth^2 u\,\mathrm{csch} u\,\tilde\xi_6\Big]\,H(u)\,du \qquad (6.40)\\
&\quad + \frac{\sqrt{3}}{8}\int^r \frac{(2-\cosh 2u)\mathrm{csch}^3 u}{(2+\cosh 2u)^{3/4}}\,\tilde\xi_{123}\,du + \frac{2-\cosh 2r}{6\sqrt{3}}\,\tilde\phi_1\,,\\
\Lambda_6 &= Y_6 + \frac{1}{12}\int^r \mathrm{sech} u\,\tanh^3 u\Big[3\,(3+\cosh 2u)\,\tilde\xi_5 + \cosh^2 u\,\tilde\xi_6\Big]\,H(u)\,du\\
&\quad - \frac{\sqrt{3}}{8}\int^r (2+\cosh 2u)^{1/4}\mathrm{sech}^3 u\,\tanh u\,\tilde\xi_{123}\,du - \frac{(2+\cosh 2r)\tanh^4 r}{6\sqrt{3}}\,\tilde\phi_1\,,
\end{aligned}
$$

and we dubbed $\tilde\xi_{123}$ the following combination of $\tilde\xi_a$

$$
\tilde\xi_{123} = 3\,\tilde\xi_1 - 4\,\tilde\xi_2 - 3\,\tilde\xi_3\,.
$$

The last mode we solve for is the perturbation to the warp factor $\tilde\phi_4$. Its integral expression is

$$
\begin{aligned}
\tilde\phi_4 &= \frac{1}{m^2 H(r)}\int^r \frac{6m^2\,\mathrm{csch}^3 u\,H(u)}{(2+\cosh 2u)^{3/4}}\,\tilde\xi_4\,du + \frac{1}{m^2 H(r)}\int^r \frac{3m^2\,\mathrm{sech}^3 u\,\tanh u\,(\tilde\phi_2 + 2\tilde\phi_3)}{(2+\cosh 2u)^{3/4}}\,du\\
&\quad - \frac{1}{m^2 H(r)}\int^r \frac{3\sqrt{3}\,m^2\,\mathrm{csch} u\,\mathrm{sech} u\,(\mathrm{csch}^2 u\,\tilde\phi_5 + 2\,\mathrm{sech}^2 u\,\tilde\phi_6)}{(2+\cosh 2u)^{3/4}}\,du + \frac{Y_4}{m^2 H(r)}\,.
\end{aligned}
\qquad (6.41)
$$

We now briefly explain the procedure we followed in order to obtain this solution. We firstly solve the system (6.27)–(6.32) using the Lagrange method of variation of parameters. While in principle this produce a solution with an increasing number of nested integrations, we found that successive integrations by parts reduce the outcome of this method to the compact form (6.39). We note that since the solution for the $\tilde{\xi}_a$ modes is analytic in the variable y, the aforementioned solution for the $\tilde{\phi}_a$ modes contain at most single integrals of the from

$$\int^y f(u)\mathbf{L}(u)\,du, \tag{6.42}$$

where $\mathbf{L}(y)$ is a combination of incomplete elliptic integrals and $f(y)$ is a polynomial function of the variable y. In this form the expressions for the modes $\tilde{\phi}_a$ can be easily evaluated numerically, and thus provide a full interpolating solution which connects the IR and the UV region.

The space of solutions we solved for is parametrized by twelve integration constants X_a, Y_a, of which only ten are physical since X_2 can be eliminated through the zero energy condition (6.36) and Y_3 corresponds to a rescaling of the three–dimensional coordinates. In Appendix E we show the full solution obtained after replacing the analytic expressions for the modes $\tilde{\xi}_a$ and by recasting some of the integrations in terms of incomplete elliptic integrals. We were not able to further simplify the resulting solution, but we stress that the crucial improvement that permits to easily handle numerical evaluation is the absence of nested integration (as opposed for example to what happens for the anti–D3 case).

6.3 Asymptotic behavior

In order to impose the desired boundary conditions we need to calculate the behavior of the solution presented in the previous section in the small and large r limits. For that we need the expansions of the elliptic integrals that enter in the expressions for the $\tilde{\phi}_a$ modes. In the IR the first terms of the relevant functions are

$$F\Big(\arcsin(y^{-1})|-1\Big) = F_0 - \frac{r^2}{2\sqrt{2}\,3^{3/4}} + \frac{r^4}{12\sqrt{2}\,3^{3/4}} + \mathcal{O}(r^6)\,, \tag{6.43}$$

$$\Pi\Big(-\sqrt{3};-\arcsin(y^{-1})|-1\Big) = K_1 + \frac{r^2}{4\sqrt{2}\,3^{3/4}} - \frac{r^4}{48\sqrt{2}\,3^{3/4}} + \mathcal{O}(r^6)\,, \tag{6.44}$$

$$\Pi\Big(\sqrt{3};-\arcsin(y^{-1})|-1\Big) = K_2 + \frac{3^{1/4}\,\log(r)}{\sqrt{2}} - \frac{r^2}{4\sqrt{2}\,3^{3/4}} + \frac{r^4}{40\sqrt{2}\,3^{3/4}} + \mathcal{O}(r^6)\,, \tag{6.45}$$

where in order to keep notation intelligible, we used the following abbreviations:

$$F_0 = F\Big(\arcsin\Big(\frac{1}{3^{1/4}}\Big)|-1\Big) \approx 0.7896\,, \quad K_1 = \Pi\Big(-\sqrt{3};-\arcsin\Big(\frac{1}{3^{1/4}}\Big)|-1\Big) \approx -0.6142$$

and $K_2 \sim -0.9102$. We also encounter the constant

$$K(-1) = 2\sqrt{\frac{2}{\pi}}\,\Gamma\left(\frac{5}{4}\right)^2 \approx 1.3110\,,$$

where K is the complete elliptic integral of the first kind[3]. Finally, we need the expansion of the warp factor (6.16) which is

$$H(r) = H_0 - \frac{1}{2}\, 3^{1/4} r^2 + \frac{7\, r^4}{4\, 3^{3/4}} + \mathcal{O}(r^6)\,, \qquad H_0 = -4\, 3^{1/4} + 5\sqrt{2} F_0 \approx 0.3187\,. \tag{6.46}$$

With these expansions we can easily find the IR behavior of the solution. In order to match with the UV behavior we only need to perform a numerical integration to find the expansions of the integrals that appear as the coefficients of X_4 in the solution shown in Appendix E.

6.3.1 Numerical matching

We now briefly describe the numerical method used to relate the UV and IR expansions of the integrals that appear in the solution for the $\tilde{\phi}_a$ modes. They are of the form (6.42), thus by using (6.43)–(6.45) we easily get the IR expansions for the integrands. By performing an indefinite integration we therefore get the desired expansions up to an integration constant which is generically different in the IR and in the UV. Since these integrals are divergent in the small r limit but vanish at $r = \infty$, we chose to do the definite integration in the range $[r, \infty]$; in this way the UV integration constant is zero and we only need to match in the IR. This can be done up to an arbitrary precision p by fixing an r_0 smaller than the radius of convergence of the IR series, evaluating the IR expansion $\mathcal{S}$ of the indefinite integral at r_0 up to the appropriate order n and then fixing a constant k such that

$$\left| \mathcal{S}_n(r_0) + k - \int_{r_0}^{\infty} f(u)\, \mathbf{L}(u)\, du \right| < 10^{-p}. \tag{6.47}$$

We kept a precision of $p \approx 10$, which we found enough for our purposes. As a check, we can verify that the expansions obtained in the aforementioned way approximates well the numerical solution for small and large r, as shown in Figure 6.1 for one of the perturbation modes.

6.3.2 Infrared expansions

We now show the IR expansions of the modes $\tilde{\phi}_a$, focusing on the singular behavior which is needed in order to impose boundary conditions in Section 6.5. These expansions appeared already in [24] and apart from making sure our results are fine here we relate Y^{IR} to Y^{UV}, a crucial step in order to try and write the backreacted state at linearized order for all radii. Here the integration constants X_a and Y_a are those appearing in the analytic solution shown in section 6.2.1 and 6.2.2 and we defined the $\tilde{Y}_a$ as

$$\tilde{Y}_a = Y_a + m^2\, X_4\, k_{\tilde{\phi}_a}\,, \tag{6.48}$$

[3]Defined as $K(q) = F(\frac{\pi}{2}|q)$.

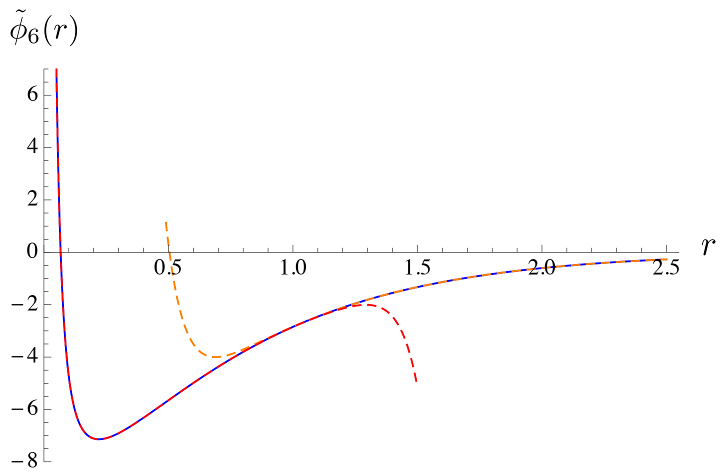


Figure 6.1: The solution for the mode $\tilde{\phi}_6$, for $X_2 = 0$, $X_1 = X_3 = X_5 = X_6 = 1$, $X_4 = 10$, $Y_a = 1$, $m = 1$ (underlying blue solid line). The red and orange dashed curves correspond to the IR and UV expansions (respectively up to 20 and 15 terms).

$$k_{\tilde{\phi}_1} = 7.45479\,, \quad k_{\tilde{\phi}_2} = 0.301287\,, \quad k_{\tilde{\phi}_3} = 0.112188\,, \quad k_{\tilde{\phi}_5} = 0.576358\,, \quad k_{\tilde{\phi}_6} = -0.00504419\,.$$

The constants $k_{\tilde{\phi}_a}$ are obtained with the numerical procedure outlined in the previous subsection. We also impose the zero energy condition (6.36) and so in what follows we set $X_2 = 0$. With these remarks and notations in mind, we now provide the IR expansions for the first–order perturbation modes

$$\begin{aligned}
\tilde{\phi}_1 = {}& \frac{1}{4\,3^{3/4} r^2}\Big[-27\,X_1 - 16\sqrt{3}\,(X_5 + X_6) - 30\,m^2\,H_0\,X_4\Big] \qquad (6.49)\\
&+ \frac{1}{12r}\Big[6\tilde{Y}_1 + \sqrt{2}\,3^{1/4}(-45\sqrt{3}\,X_1 - 162\sqrt{3}\,X_3 + 40\,X_5 + 112\,X_6)K(-1)\Big]\\
&+ \frac{1}{12\cdot 3^{3/4}}\Big(189\,X_1 - 80\sqrt{3}\,(X_5 + X_6) + 6\,m^2\,X_4(83\,H_0 - 3^{1/4}\,33)\Big) + \mathcal{O}(r)\,,\\
\tilde{\phi}_2 = {}& \frac{1}{r^4}\bigg[\frac{\tilde{Y}_2}{3} + \frac{4}{105}\,H_0\,(63\,X_1 + 432\,X_3 - 7\sqrt{3}\,(5\,X_5 + 11\,X_6)) + \frac{63\,3^{1/4}}{5}\,X_1 + \frac{2808}{35}\,3^{1/4}\,X_3\\
&- \frac{72}{5}\,3^{3/4}\,X_6\bigg] + \frac{1}{r^2}\bigg[-\frac{291}{20}\,3^{1/4}\,X_1 - \frac{3744}{35}\,3^{1/4}\,X_3 + 2\,3^{3/4}\,X_5 + \frac{106}{5}\,3^{3/4}\,X_6 \qquad (6.50)\\
&+ H_0\Big(-\frac{16}{5}\,X_1 - \frac{768\,X_3}{35} + \frac{3\,3^{1/4}\,m^2\,X_4}{2} + \frac{16\,(5\,X_5 + 11\,X_6)}{15\sqrt{3}}\Big) - \frac{4\,\tilde{Y}_2}{9}\bigg]\\
&- \frac{1}{12r}\Big[6\,\tilde{Y}_1 - \sqrt{2}\,3^{1/4}(45\sqrt{3}\,X_1 + 162\sqrt{3}\,X_3 - 40\,X_5 - 112\,X_6)K(-1)\Big]
\end{aligned}$$

$$+\frac{137\,3^{1/4}\,X_1}{25}+\frac{12792\,3^{1/4}\,X_3}{175}+6\sqrt{3}\,m^2\,X_4-\frac{1}{675}(225\,3^{3/4}\,X_5+9081\,3^{3/4}\,X_6-205\tilde{Y}_2)$$
$$+H_0\Big(\frac{164}{75}\,X_1+\frac{2624}{175}\,X_3-\frac{23\,3^{1/4}\,m^2\,X_4}{2}-\frac{164\,(5\,X_5+11\,X_6)}{225\sqrt{3}}\Big)+\mathcal{O}(r)\,,$$

$$\tilde{\phi}_3=\frac{1}{r^4}\bigg[\frac{H_0}{70}\Big(-63\,X_1-432\,X_3+7\sqrt{3}(5\,X_5+11\,X_6)\Big)-\frac{189\,3^{1/4}\,X_1}{40}-\frac{1053}{35}\,3^{1/4}\,X_3 \tag{6.51}$$
$$+\frac{27\,3^{3/4}\,X_6}{5}-\frac{\tilde{Y}_2}{8}\bigg]+\frac{1}{r^2}\bigg[\frac{2H_0}{105}\Big(63\,X_1+432\,X_3-7\sqrt{3}(5\,X_5+11\,X_6)\Big)$$
$$+\frac{237\,3^{1/4}\,X_1}{40}+\frac{1404\,3^{1/4}\,X_3}{35}-\frac{3^{3/4}}{10}(5\,X_5+77\,X_6)+\frac{\tilde{Y}_2}{6}\bigg]$$
$$+\frac{61}{160}3^{1/4}\,X_1-\frac{1167}{100}3^{1/4}\,X_3-\frac{67}{16}\sqrt{3}m^2\,X_4-\frac{671\,X_5-1033\,X_6}{120\,3^{1/4}}-\frac{41}{360}\,\tilde{Y}_2+\tilde{Y}_3$$
$$+\frac{F_0}{630\sqrt{2}}\Big(-10836\,X_1-69444\,X_3+6300\,3^{1/4}\,m^2\,X_4+7\sqrt{3}\,(725\,X_5+1847\,X_6)\Big)$$
$$+\frac{K_1+K_2}{4\sqrt{2}}\Big(21\,X_1-4\sqrt{3}(X_5+X_6)\Big)+30\sqrt{2}\,3^{1/4}\,m^2\,X_4\,F_0\log r$$
$$+\frac{1}{8}\Big(21\,3^{1/4}\,X_1-210\,\sqrt{3}\,m^2\,X_4-4\,3^{3/4}\,(X_5+X_6)\Big)\log r+\mathcal{O}(r)\,,$$

$$\tilde{\phi}_5=\frac{1}{5}\Big(-12\,3^{3/4}\,X_1-81\,3^{3/4}\,X_3+18\,3^{1/4}\,X_6+10\,\tilde{Y}_6\Big) \tag{6.52}$$
$$-\frac{H_0}{60}\Big(21\sqrt{3}\,X_1+4\,(27\sqrt{3}\,X_3+10\,X_5+4\,X_6)\Big)+\mathcal{O}(r^3)\,,$$

$$\tilde{\phi}_6=\frac{H_0}{12\,r^2}\Big[\sqrt{3}\,H_0\,m^2\,X_4-2\,(X_5+X_6)\Big]-H_0\,\Big(\frac{7\sqrt{3}\,X_1}{80}+\frac{9\sqrt{3}\,X_3}{20}+\frac{7\,m^2\,X_4}{24\,3^{1/4}}+\frac{10\,X_5+X_6}{90}\Big)$$
$$-\frac{H_0^2\,m^2\,X_4}{12\sqrt{3}}-\frac{43\,3^{3/4}\,X_1}{80}-\frac{81\,3^{3/4}\,X_3}{20}+\frac{25\,X_5+106\,X_6}{30\,3^{3/4}}+\frac{\tilde{Y}_6}{2}+\mathcal{O}(r)\,, \tag{6.53}$$

$$\tilde{\phi}_4=\frac{1}{r^2}\Big[-\frac{3}{4}\;3^{1/4}\,H_0^2\,m^2\,X_4-\frac{1}{H_0}\Big(\frac{207\sqrt{3}\,X_1}{40}+\frac{2403\sqrt{3}\,X_3}{70}-\frac{54}{5}\,X_6+\frac{\tilde{Y}_2}{8\,3^{3/4}}-3^{3/4}\,\tilde{Y}_6\Big)$$
$$-\frac{33}{40}\,3^{1/4}\,X_4-\frac{333}{70}\,3^{1/4}\,X_3-\frac{5\,X_5-X_6}{5\,3^{1/4}}\Big]+\mathcal{O}(r^0)\,. \tag{6.54}$$

6.3.3 Ultraviolet expansions

Here we show the leading terms in the UV expansions of the perturbation modes $\tilde{\phi}_a$

$$\tilde{\phi}_1=-\frac{27\,X_3}{2^{1/4}}e^{-r/2}+2e^{-2r}\,Y_1-e^{-5r/2}\,2^{7/4}\Big(27\,X_1+81\,X_3-16\sqrt{3}\,X_6\Big) \tag{6.55}$$
$$+\frac{1}{20\,2^{1/4}}e^{-9r/2}\Big(3267\,X_3-1024\sqrt{3}\,X_6\Big)+\mathcal{O}(e^{-6r})\,,$$

$$\tilde{\phi}_2 = -\frac{63\,X_3}{10\,2^{1/4}}e^{3r/2} - \frac{52569\,X_3}{280\,2^{1/4}}e^{-5r/2} - 12e^{-4r}\,Y_1 + \mathcal{O}(e^{-9r/2})\,, \tag{6.56}$$

$$\tilde{\phi}_3 = Y_3 - \frac{3Y_1}{8} + \frac{81\,X_3}{20\,2^{1/4}}e^{3r/2} - \frac{29079\,X_3}{560\,2^{1/4}}e^{-5r/2} + \frac{15}{4}e^{-4r}\,Y_1 + \mathcal{O}(e^{-9r/2})\,, \tag{6.57}$$

$$\begin{aligned}\tilde{\phi}_5 = {} & \frac{Y_5 - Y_6}{8}e^{3r} - \frac{9\,(Y_5 - Y_6)}{8}e^{r} + \frac{1}{24}e^{-r}\Big(- 8\sqrt{3}\,Y_1 + 117\,Y_5 + 27\,Y_6\Big) \\ & - 38\,2^{3/4}\sqrt{3}e^{-3r/2}\,X_3 + \frac{1}{72}e^{-3r}\Big(168\sqrt{3}Y_1 - 9\,(111\,Y_5 + Y_6)\Big) \\ & + \frac{2}{195}2^{3/4}e^{-7r/2}(-3348\sqrt{3}X_1 + 1323\sqrt{3}X_3 + 6160\,X_6) + \mathcal{O}(e^{-5r})\,,\end{aligned} \tag{6.58}$$

$$\begin{aligned}\tilde{\phi}_6 = {} & -\frac{Y_5 - Y_6}{16}e^{3r} - \frac{3\,(Y_5 - Y_6)}{16}e^{r} + \frac{1}{144}e^{-r}\Big(- 8\sqrt{3}\,Y_1 + 117\,Y_5 + 27\,Y_6\Big) \\ & - 5\,2^{3/4}\sqrt{3}\,e^{-3r/2}\,X_3 + \frac{1}{48}e^{-3r}\Big(8\sqrt{3}Y_1 - 51\,Y_5 + 3\,Y_6)\Big) \\ & + \frac{1}{585}2^{3/4}e^{-7r/2}\Big(- 1188\sqrt{3}X_1 + 243\sqrt{3}X_3 + 2320\,X_6) + \mathcal{O}(e^{-5r})\,,\end{aligned} \tag{6.59}$$

$$\begin{aligned}\tilde{\phi}_4 = {} & \frac{3\,Y_4^{UV}}{16\,2^{3/4}m^2}\,e^{9r/2} + \frac{27\,Y_4^{UV}}{26\,2^{3/4}\,m^2}\,e^{5r/2} - \frac{27\,X_3}{10\,2^{1/4}}\,e^{3r/2} + \frac{350271\,Y_4^{UV}}{182872\,2^{3/4}\,m^2}\,e^{r/2} \\ & + \frac{3\,Y_1}{2} - 2\,Y_3 - 2\sqrt{3}\,Y_5 + 2\sqrt{Y_6} - \frac{324}{325}\,2^{3/4}\,X_3\,e^{-r/2} + \frac{484605\,Y_4^{UV}}{298792\,2^{3/4}\,m^2}\,e^{-3r/2} \\ & - \frac{24}{13}(Y_1 - 6\sqrt{3}\,Y_6)\,e^{-2r} - \frac{11957859009\,X_3}{28155400\,2^{1/4}}\,e^{-5r/2} + \frac{7978373883\,Y_4^{UV}}{21130570240\,2^{3/4}\,m^2}\,e^{-7r/2} + \mathcal{O}(e^{-4r})\,.\end{aligned} \tag{6.60}$$

From these expansions we can extract the UV behavior of the fields $\tilde{\phi}_a$, which is important to understand the holographic physics. For this purpose we have to relate our radial variable r to the standard AdS coordinate ρ_{AdS} as

$$\rho_{AdS} \sim e^{3r/2}\,. \tag{6.61}$$

A discussion of the holographic behavior can be found in [24], where it was shown that the integration constants X_a and Y_a are paired into normalizable and non-normalizable mode. In order to be self–contained we tabulate in Table 6.1 (which is adapted from [24]), the leading terms coming from each modes. Note that since we obtain the asymptotic behavior from an analytic solution, we can relate the integration constants of [24] to the IR singular behavior of the same modes. In particular, one can explicitly check if an IR regularity condition on one integration constant is compatible with the absence of the respective non–normalizable mode in the UV. We will come back on this point in the next section. In the following table Δ is the dimension of the local operator $\mathcal{O}$ holographically associated to the two supergravity modes whose asymptotic is $\rho_{AdS}^{-\Delta}$ (dual to the vacuum expectation value of $\mathcal{O}$) and $\rho_{AdS}^{\Delta-3}$ (dual to a deformation of the action $\delta S = \int d^3x\,\mathcal{O}$). Also, the combination which appears at dimension $\Delta = 7/3$ is the linear combination of Y_1, Y_5 and Y_6 which appears in the corresponding terms in (D.16) and (D.17).

dim Δ	non-norm/norm	integration constants
6	$\rho_{AdS}^{3}/\rho_{AdS}^{-6}$	Y_4/X_4
5	$\rho_{AdS}^{2}/\rho_{AdS}^{-5}$	$Y_5 - Y_6/X_5 - X_6$
4	$\rho_{AdS}/\rho_{AdS}^{-4}$	X_3/Y_2
3	$\rho_{AdS}^{0}/\rho_{AdS}^{-3}$	$Y_1 + Y_3/X_2$
$\frac{7}{3}$	$\rho_{AdS}^{-2/3}/\rho_{AdS}^{-7/3}$	$Y_5 + Y_6 + Y_1/X_5 + X_6$
$\frac{5}{3}$	$\rho_{AdS}^{-4/3}/\rho_{AdS}^{-5/3}$	Y_1/X_1

Table 6.1: The UV behavior of all fourteen modes for the $SO(5)$-symmetric deformation around the CGLP solution, extracted from the asymptotic of our analytic solution.

6.4 Charges and M2–branes

The space of solutions we solved for in the previous sections should contain the linearized perturbation of the warped Stenzel space due to the presence of a stack of smeared anti–M2 branes placed at the tip of the geometry. This configuration was studied in the probe approximation in [115] and corresponds in the dual gauge theory to a metastable supersymmetry breaking state. In order to identify the back-reacted solution, we need to impose the correct boundary conditions associated to the presence of the anti–branes at the tip. In this section we start by discussing the standard notions of charge in the Stenzel background (see for example [96, 3, 99]) and as a warmup we identify the BPS perturbation of the CGLP solution ascribed to the presence of M2 branes. The anti–M2 brane perturbation will be discussed in the next section.

In the Stenzel background we can define a "running" M2 charge by integrating $\star_{11}G_4$ on a 7–dimensional section $\mathcal{M}_r = V_{5,2}$ of the transverse cone at a fixed r

$$Q_{M2}(r) = \frac{1}{(2\pi l_p)^6}\int_{\mathcal{M}_r} \star_{11} G_4\,, \tag{6.62}$$

where l_p is the Planck length in eleven dimensions. We can also integrate G_4 over the 4–sphere which has a finite size at the tip and define the quantity

$$q(r) = \frac{1}{(2\pi l_p)^3}\int_{S_4} G_4\,. \tag{6.63}$$

For the parametrization (6.1)-(6.2) and for the CGLP background we find from (6.33) (see also Appendix F)

$$Q^0_{M2}(r) = -\frac{6\,2^{11}\,m^2\,\mathrm{Vol}_{V_{5,2}}}{3^4\,(2\pi l_p)^6}\Big(h_0(f_0 - 2h_0) - \frac{1}{54}\Big)\,, \tag{6.64}$$

$$q^0(r) = -\frac{16\pi^2 m}{(2\pi l_p)^3} h_0(r)\,, \tag{6.65}$$

where $\mathrm{Vol}_{V_{5,2}} = 27\pi^4/128$ [33]. Substituting the zeroth–order solution (6.15) we find

$$Q^0_{M2}(r) = \frac{m^2\, \tanh^4 r}{108\,\pi^2\, l_p^6}\,, \quad q^0(r) = \frac{m\,\mathrm{sech} r}{3\sqrt{3}\,\pi\, l_p^3}\,, \tag{6.66}$$

which is the known result for the CGLP solution [115].

We now want to calculate the first–order corrections to these charges from the first–order perturbation of the Stenzel geometry. The simpler case is the BPS one, where a stack of M2–branes smeared over the S^4 is placed at the origin $r = 0$. The perturbation on the geometry should still preserve supersymmetry, so we are forced to set $X_a = 0$, $a = 1, \ldots, 6$ since the "conjugate–momenta" $\tilde{\xi}_a$ are the modes that parametrize the supersymmetry breaking. Note that in this case the solutions for the modes $\tilde{\phi}_a$ are given by the homogenous solutions of the coupled system of ODE's (6.27)–(6.32) and they are easily found by setting $\tilde{\xi}_a = 0$ in (6.39). The perturbation due to the presence of M2 branes at the tip is found by imposing the following conditions on the Y_a integration constants: $Y_1 = Y_2 = 0$ to cancel IR divergencies in $\tilde{\phi}_1$ and $\tilde{\phi}_2$, $Y_4 = Y_5 - Y_6 = 0$ to cancel the divergent terms in the UV expansions of $\tilde{\phi}_4$ and $\tilde{\phi}_5$, and finally $Y_3 = 0$ to fix the freedom of rescaling the three–dimensional coordinates. The first–order perturbation to (6.64) is proportional to $h_0(\tilde{\phi}_5 - 4\tilde{\phi}_6) + f_0\tilde{\phi}_6$ and at the linearized level the running M2 charge is given by

$$Q_{M2} = Q^0_{M2} + Q^1_{M2} = \frac{m^2\, l_p^{-6}}{108\,\pi^2}\left[\tanh^4 r + 6\sqrt{3}\, Y_6\Big(1 - \tanh^4 r\Big)\right]. \tag{6.67}$$

The profile of the charge is shown in Figure 6.2 for different values of the constant Y_6. The asymptotic behavior is the following

$$Q^{IR}_{M2} = \frac{m^2\, l_p^{-6}}{6\sqrt{3}\,\pi^2}\, Y_6 + \mathcal{O}(r^4)\,, \qquad Q^{UV}_{M2} = \frac{m^2\, l_p^{-6}}{108\,\pi^2} + \mathcal{O}(e^{-2r})\,. \tag{6.68}$$

The integral of G_4 over the four–sphere is given by the behavior of the mode $h \sim \tilde{\phi}_6$ and thus

$$q = q^0 + q^1 = \frac{m\, l_p^{-3}}{3\sqrt{3}\pi}\Big(1 - 3\sqrt{3}\, Y_6\Big)\,\mathrm{sech} r\,. \tag{6.69}$$

We see that q vanishes at infinity while in the IR it approaches a constant value

$$q^{IR} = \frac{m\, l_p^{-3}}{3\sqrt{3}\pi}\Big(1 - 3\sqrt{3}\, Y_6\Big) + \mathcal{O}(r^2)\,, \qquad q^{UV} = \mathcal{O}(e^{-r})\,. \tag{6.70}$$

We will denote $\tilde{M} = q^0(0)$ the number of G_4 flux units through the non–vanishing S^4 at the tip. Note that for the zeroth-order solution we have

$$\frac{(\tilde{M})^2}{4} = Q^{UV}_{M2}\,. \tag{6.71}$$

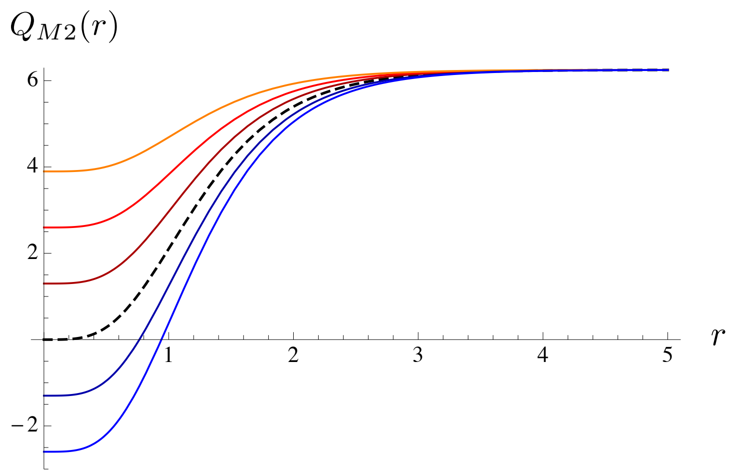


Figure 6.2: The profile of the M2 charge Q_{M2} for the BPS perturbation, for different values of the parameter Y_6. The black dashed line is the zeroth–order solution ($Y_6 = 0$). Note that at the linearized level the perturbations vanish in the UV.

At first–order, we expect a term related to the explicit brane charge in the IR; in fact, we easily see from (6.70) that our solution satisfies

$$\frac{(q^{IR})^2}{4} = Q_{M2}^{UV} - Q_{M2}^{IR}\,, \tag{6.72}$$

which indeed reduces to (6.71) when $Y_6 = 0$, which corresponds to having no regular M2–branes[4]. Allowing a nonzero Y_6 introduces a singularity in the warp factor $\tilde{\phi}_4$

$$\tilde{\phi}_4 = \frac{3^{3/4}\,Y_6}{H_0\,r^2} + \mathcal{O}(r^0)\,, \tag{6.73}$$

which is the expected divergency due to smeared M2 branes on the S^4 at the tip.

We could have derived these results without relying on the actual solution for the $\tilde{\phi}_a$ modes. In fact, the linearized BPS perturbation can be obtained by simply shifting the fluxes as follows[5]

$$\begin{aligned} \tilde{\phi}_5 &= 2c\,, \\ \tilde{\phi}_6 &= \frac{c}{2}\,, \end{aligned} \tag{6.74}$$

[4]Note that equation (6.72) is just the standard relation introduced in [96]. The quantities of equation (2.16) of that reference are $\Phi = Q_{M2}^{UV}$, $N = Q_{M2}^{IR}$ and $\int G \wedge G = \frac{(q^{IR})^2}{4}$ (see also [98]).

[5]By shifting the fluxes $f \to f + 2\,c$, $h \to h + \frac{c}{2}$ we can obtain the full nonlinear solution, but this introduce terms proportional to c^2 which are not seen in our linearized deformation space.

where c is the number of M2 branes. The M2 charge thus changes in the following way

$$Q^{(1)}_{M2} = -\frac{m^2\, l_p^{-6}}{2\,\pi^2}\Big(h_0(\tilde\phi_5 - 4\tilde\phi_6) + f_0\tilde\phi_6\Big) = -\frac{c\,m^2\, l_p^{-6}}{4\,\pi^2}\, f_0 = \frac{c\,m^2\, l_p^{-6}}{6\sqrt{3}\,\pi^2} + \mathcal{O}(r^4)\,, \tag{6.75}$$

while for the warp factor we have, from (6.32)

$$\Delta\tilde\phi_4' = c\, m^2 f_0(r)\, e^{-3(z_0+\alpha_0+\beta_0)} \overset{r\to 0}{\sim} -\frac{2\,3^{3/4}\,c}{H_0\, r^3}\,, \tag{6.76}$$

from which we get

$$\tilde\phi_4 = \frac{3^{3/4}\,c}{H_0\, r^2} + \mathcal{O}(r^0)\,, \tag{6.77}$$

which agrees with (6.73) with the identification $Y_6 = c$. From this result we can also extract the correct mass/charge normalization between the warp factor divergency and the charge sourced by the branes, which will be useful in the next section

$$m^2\, H_0\, r^2\, \tilde\phi_4 = 18\cdot 3^{1/4}\,\pi^2\, l_p^6\, |Q^{IR}_{M2}|\,. \tag{6.78}$$

In the next section, we will turn to the case of interest in which we add a stack of anti–M2 branes at the tip of the transverse Stenzel space. In this case the expressions (6.64), (6.65) evaluated at first–order in perturbation theory will depend on all of the X_a, Y_a integration constants. However, we have to impose appropriate regularity conditions for the IR and UV behavior of the modes $\tilde\phi_a$, and we will see that this fixes all the integration constants in terms of X_4, which is the one responsible for the force on a probe M2 brane in this background (see section 6.5.3), and Y_6. We thus expect the expressions for the charges in the BPS case to be modified by some pieces proportional to X_4. By requiring the variation in the M2 charge $Q^{(1)}_{M2}$ to be commensurate to the singularity introduced in the warp factor, equation (6.78), we will derive a relation which fixes Y_6 in terms of X_4 and so we will fix all the integration constants in terms of the number of anti–M2 branes.

6.5 The anti–M2 brane perturbation

In this section we consider the perturbed solution corresponding to a stack of $\bar N$ anti–M2 branes at the tip of the transverse geometry. It was shown in [115] that in the probe approximation, for $\bar N/\tilde M \lesssim 0.054$ this configuration is metastable and will eventually decay into a supersymmetric configuration in which $\tilde M - 1 - \bar N$ M2 branes are present at the tip [6]. In order to find the supergravity dual to the metastable state,

[6] The units of G_4 flux for the susy state are then $\tilde M - 2$. A way to understand these values is to look at (6.72). We then see that these are the correct values so that the charge at infinity is conserved: $Q^{UV}_{\rm susy} = \frac{(\tilde M-2)^2}{4} + \tilde M - 1 - \bar N - = \frac{q^2}{4} - \bar N = Q^{UV}_{\rm ms}$, where $Q_{\rm susy}$ and $Q_{\rm ms}$ are the charges for the susy and metastable states.

we will adopt the following strategy. Firstly, we consider the IR behavior and we allow only for divergencies that are directly sourced by the anti-branes. Secondly, we demand that the UV non-normalizable modes described in section 6.3.3 are absent, so that the UV asymptotic is the same as for the original CGLP background. As we will show, these requirements (together with the mass/charge normalization discussed in the previous section) provide enough independent contraints on the deformation space to fix every integration constants in terms of a single physical quantity, namely the number of anti–M2 branes present at the tip. We then compute the relevant charges for the perturbed solution, as well as the explicit expression for the force felt by probe M2 branes in the backreacted anti–M2 background.

6.5.1 IR and UV boundary conditions

We now proceed to impose regularity conditions on the IR behavior of the modes $\tilde{\phi}_a$. We demand that divergencies are zero except for the singularity in the warp factor $\tilde{\phi}_4$ which is directly sourced by the anti–M2 branes. We first impose the zero energy condition, which amounts to setting

$$X_2 = 0\,. \tag{6.79}$$

From regularity of $\tilde{\phi}_1$ we derive

$$\begin{aligned} X_1 &= -\frac{2}{27}\left[8\sqrt{3}(X_5+X_6)+15\,m^2\,X_4\,H_0\right], \qquad (6.80)\\ X_5 &= \frac{27\sqrt{3}\,X_3}{20}-\frac{8\,X_6}{5}-\frac{Y_1}{20\sqrt{2}\,3^{1/4}K(-1)}-\frac{m^2\,X_4\left(100\,H_0\,K(-1)+\sqrt{2}\,3^{1/4}k_{\tilde{\phi}_1}\right)}{80\sqrt{3}\,K(-1)}\,, \end{aligned}$$

while from the singular terms in $\tilde{\phi}_2$ we derive X_6 and Y_2 in terms of X_3, X_4 and Y_1

$$\begin{aligned} X_6 &= \frac{9\sqrt{3}\,X_3}{4}-\frac{Y_1}{12\sqrt{2}\,3^{1/4}K(-1)}-\frac{m^2\,X_4\left(220\,H_0\,K(-1)+\sqrt{2}\,3^{1/4}k_{\tilde{\phi}_1}\right)}{48\sqrt{3}\,K(-1)}\,, \qquad (6.81)\\ Y_2 &= \frac{594}{35}\,(3\,3^{1/4}-H_0)\,X_3-\frac{\sqrt{2}3^{1/4}(9\,3^{1/4}+H_0)\,Y_1}{5\,K(-1)}-\frac{m^2\,X_4}{10\,K(-1)}\Big[\sqrt{2}3^{1/4}(9\,3^{1/4}+H_0)k_{\tilde{\phi}_1}\\ &\quad +30K(-1)k_{\tilde{\phi}_2}+8H_0(153\,3^{1/4}+2H_0)K(-1)\Big]\,. \end{aligned}$$

We can check that with these conditions the other IR divergencies of the modes $\tilde{\phi}_a$ are automatically canceled, except for a $\log r$ mode in the IR behavior of the field $\tilde{\phi}_3$, which is a perturbation of the metric, which is proportional to X_4. It is not clear why one should not be able to kill such divergent behavior. However, after imposing the previous boundary conditions, the solution presents an even worse singularity appearing in the field strength F_4^2 [24]

$$F_4^2 \sim \frac{X_4^2}{r^4}\,, \tag{6.82}$$

which is quite analogous to the divergence found in the anti–D3 solution, with the difference that now the action is divergent. This behavior is sub–leading with respect to the energy density associated to the divergency in the warp factor, which is of order r^{-6}. Note that this is an IR phenomenon insensible to UV boundary conditions; in fact, the integration constant X_4 cannot be set to zero for the very simple reason that it parametrizes the force felt by a probe M2 brane [24] and thus is indicative of the presence of anti–M2 branes at the tip. Despite arguments in the literature, there is not a rigorous proof that shows if this singularity is acceptable or not. Given the difficulties in proving this, we will assume that the singularity is harmless and we will try to see if the anti–M2 solution develops problems in the UV; if this is not the case, the solution we find describes the holographic dual of the conjectured metastable state in the field theory, but clearly a more detailed study of the IR singularity, along the lines of Chapter 5, is required to decide whether this supergravity solution can be trusted or not.

We now proceed by imposing boundary conditions in the UV, where we demand that non–normalizable modes in the UV expansions for the modes $\tilde{\phi}_a$ are absent. The first condition is from the $e^{3r/2}$ term in $\tilde{\phi}_2$, from which we get

$$X_3 = 0\,. \tag{6.83}$$

From the divergent term in $\tilde{\phi}_5$ we get

$$Y_5 = Y_6\,, \tag{6.84}$$

and finally from the term e^{-2r} in $\tilde{\phi}_1$ we get

$$Y_1 = 0\,. \tag{6.85}$$

Note that we should allow an e^{-r} term in the fluxes, which is dual to the dimension $\Delta = 7/3$ operator, since it is the charge mode sourced by the branes. We thus see that we fixed the ten physically relevant integration constants in terms of X_4 and Y_6, which are related respectively to the force on a probe M2 brane and to the number $\bar{N}$ of anti–M2 branes placed at the tip [24].

6.5.2 Charges and anti–M2 branes

In order to relate X_4 and Y_6 we look at the M2 charge (6.62). Once all the boundary conditions are imposed, we get that

$$Q^{IR}_{M2} = \frac{m^2\,\tilde{\phi}_5(0)}{12\sqrt{3}\,\pi^2\,l_p^6} = \frac{m^2}{6\sqrt{3}\,\pi^2\,l_p^6}\,(Y_6 - \alpha\, m^2\, X_4)\,, \tag{6.86}$$

where the coefficient α is the following combination of the numerical constants which enters in the expansions for the modes $\tilde{\phi}_a$

$$\alpha = \frac{H_0(63\,3^{1/4} + 22H_0)}{60\sqrt{3}} - k_{\tilde{\phi}_6} + \frac{(27 + 3^{3/4}\,H_0)\,k_{\tilde{\phi}_1}}{360\,\sqrt{2}K(-1)} \approx 0.900178\,. \tag{6.87}$$

We impose that this variation gives the correct singularity in the infrared expansion for the warp factor, which is found to be

$$\tilde{\phi}_4 = \frac{3^{3/4}\,(Y_6 - \beta\, m^2\, X_4)}{H_0\, r^2} + \mathcal{O}(r^0)\,, \tag{6.88}$$

with

$$\beta = \alpha + \frac{H_0^2}{\sqrt{3}} \approx 0.958828\,. \tag{6.89}$$

From the mass/charge normalization (6.78) we thus get the following condition

$$-Y_6 + \alpha\, m^2\, X_4 = Y_6 - \beta\, m^2\, X_4\,, \tag{6.90}$$

which results in

$$Y_6 = \frac{1}{2}(\alpha+\beta)\, m^2\, X_4 = \Big(\alpha + \frac{H_0^2}{2\sqrt{3}}\Big)\, m^2\, X_4\,. \tag{6.91}$$

If we now plug this relation back into the expression for the charge (6.86), we find the following relation

$$Q^{IR}_{M2} = -\bar{N} = \frac{H_0^2\, m^4\, X_4}{36\,\pi^2\, l_p^6}\,. \tag{6.92}$$

We note that this result does not depend on the UV boundary conditions. Indeed, although it is not clear from our derivation, it is easy to show that if we only impose IR boundary conditions the terms proportional to X_3, Y_5 and Y_1 that appear in (6.86) and (6.88) cancel in (6.91).

Since Q^{IR}_{M2} is related to the number $\bar{N}$ of anti–M2 branes placed at the tip, from (6.92) we determine X_4 as a function of $\bar{N}$ and thus we fix all the integration constants in terms of this parameter.

With these results, we can explicitly compute the charges associated to the anti–M2 brane perturbation (in Figure 6.3 we show the profile of the first–order perturbation to the Maxwell charge $Q^{(1)}_{M2}$). In particular, the M2 charge Q_{M2} evaluated at a holographic screen at infinity should be the same for the metastable and the supersymmetric state. This condition ensures that the metastable state is a state in the same theory which is dual to the supersymmetric vacuum. Unfortunately, we see from (6.68) that the perturbation to the M2 charge vanishes at infinity at the linearized level, and so in our backreacted solution the value of Q^{UV}_{M2} is fixed. We expect shifts of this quantity to appear only at second–order in perturbation theory. While we cannot directly check whether the value of the charge at infinity is conserved, we can look in the IR and check whether the charges are perturbed in a consistent way. From the probe computation (see footnote 6), we expect relation (6.72) to be satisfied. For a supersymmetric domain wall, this easily follows from the equation of motion (6.4) and the self–duality of the flux G_4, and indeed we found that the

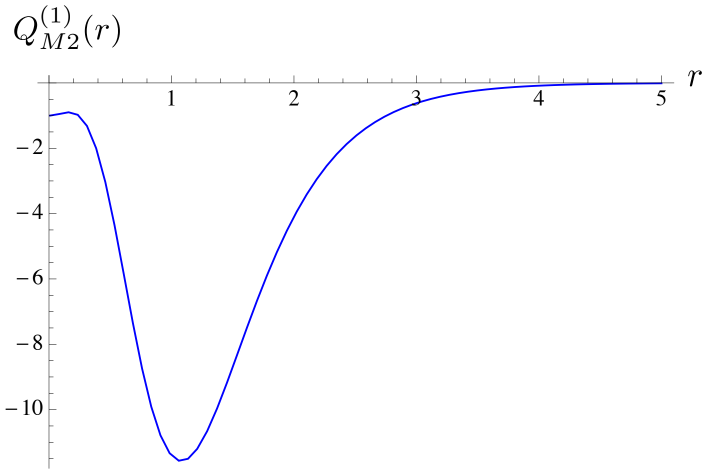


Figure 6.3: The profile of the first–order M2 charge $Q^{(1)}_{M2}$ for the anti–M2 solution, setting $\bar{N} = 1$.

BPS perturbation considered in section 6.4 is consistent with this constraint at the linearized level. For the non–supersymmetric case, one should be more careful. It is useful to write the first–order perturbation to the Maxwell charge in the IR in the following way

$$Q^{IR}_{M2} = \frac{m^2\, l_p^{-6}}{2\pi^2}\left[\frac{1}{6\sqrt{3}}\left(\tilde{\phi}_5(0) - 4\,\tilde{\phi}_6(0)\right) + \frac{2}{3\sqrt{3}}\tilde{\phi}_6(0)\right], \tag{6.93}$$

from which we derive, at the linearized level

$$\frac{(q_{IR})^2}{4} = \frac{m^2\, l_p^{-6}}{\pi^2}\left[-\frac{1}{6\sqrt{3}}+\tilde{\phi}_6(0)\right]^2 = Q^{UV}_{M2}-Q^{IR}_{M2}+\frac{m^2\, l_p^{-6}}{12\sqrt{3}\pi^2}\left[\tilde{\phi}_5(0)-4\,\tilde{\phi}_6(0)\right]+\mathcal{O}(X^2)\,. \tag{6.94}$$

After imposing the anti–M2 IR boundary conditions, we find that the term in the brakets in the right hand side of the last equation is not zero

$$\tilde{\phi}_5(0) - 4\,\tilde{\phi}_6(0) = H_0\, 3^{-1/4}\, m^2\, X_4\,. \tag{6.95}$$

Indeed, this is the term which gives rise to the singularity in the field strength F_4^2 that we discussed in section 6.5.1. This result is consistent with the fact that at the linearized level the self–duality of the four–form flux is spoiled, and we do not expect relation (6.72) to be satisfied for the anti–M2 solution. As we discussed in the previous subsection, it is possible that this result is an artifact of the perturbation theory. While we cannot address this issue within our first–order technology, we believe that further investigation is needed on this problem.

6.5.3 The force on a probe brane

With the results obtained in the previous subsections, we are able to compute explicitly the coefficient of the force exerted on a probe M2–brane in the anti–M2 backreacted background, whose functional form has been derived in [24, 19]:

$$F_{M2} = -\frac{18\,X_4\,\mathrm{csch}^3 r}{(2+\cosh 2r)^{3/4}}\,. \tag{6.96}$$

Inserting the expression for X_4 that we derive from (6.92) we obtain

$$F_{M2} = \frac{648\,\pi^2\,l_p^6\,\bar{N}\,\mathrm{csch}^3 r}{m^4\,H_0^2\,(2+\cosh 2r)^{3/4}}\,. \tag{6.97}$$

This result has to be compared to the one given by the probe anti–brane potential à la KKLMMT [109], which is given in [19] and reviewed in Appendix F. The result of this computation is given in (F.10). Once we substitute d_2 we see that the two expressions exactly agree. This is a nontrivial check that our IR boundary conditions are the correct ones to describe anti–M2 branes in the Stenzel geometry.

6.5.4 Asymptotic of the anti–M2 solution

We now collect the results we obtained for the twelve (X_a, Y_a) integration constants and which determine the anti–M2 solution in terms of the constant X_4, which is fixed in terms of $\bar{N}$ by (6.92)

$$X_4 = -\frac{36\,\pi^2\,l_p^6}{m^4\,H_0^2}\,\bar{N}\,. \tag{6.98}$$

For the X_a integration constants we have

$$\begin{aligned} X_1 &= -2\,H_0\,m^2\,X_4\,, \quad X_2 = 0\,, \quad X_3 = 0\,, \\ X_5 &= \left[73\,H_0 + 2\,3^{1/4}\sqrt{\pi}\,k_{\tilde{\phi}_1}\,\Gamma\left(\frac{1}{4}\right)^{-2}\right]\frac{m^2\,X_4}{12\sqrt{3}}\,, \\ X_6 &= \left[-\,55\,H_0 - 2\,3^{1/4}\sqrt{\pi}\,k_{\tilde{\phi}_1}\,\Gamma\left(\frac{1}{4}\right)^{-2}\right]\frac{m^2\,X_4}{12\sqrt{3}}\,, \end{aligned} \tag{6.99}$$

For the Y_a integration constants we have

$$\begin{aligned} Y_1 &= 0\,, \\ Y_2 &= \left[-\,\frac{4}{5}\,H_0(153\,3^{1/4} + 2\,H_0) - 3\,k_{\tilde{\phi}_2} - \frac{4}{5}\,3^{1/4}(9\,3^{1/4} + H_0)\sqrt{\pi}\,\Gamma\left(\frac{1}{4}\right)^{-2}\right] m^2\,X_4\,, \\ Y_3 &= 0\,, \\ Y_5 = Y_6 &= \left[\sqrt{3}\,H_0(63\,3^{1/4} + 52\,H_0) - 180\,k_{\tilde{\phi}_6} + 2\,(27 + 3^{3/4}\,H_0)\sqrt{\pi}\,k_{\tilde{\phi}_1}\,\Gamma\left(\frac{1}{4}\right)^{-2}\right]\frac{m^2\,X_4}{180}\,. \end{aligned} \tag{6.100}$$

The IR and UV behavior of the backreacted anti–M2 solution, up to the desired order, can be read off from the analytic solution presented in Appendix E after imposing the boundary conditions (6.99), (6.100). For the reader's convenience, we show here the first few terms of the ultraviolet behavior of the solution.

$$\begin{aligned}\tilde{\phi}_1 =& \frac{4\,m^2\,X_4}{3K(-1)}\left[29\,2^{3/4}\,H_0\,K(-1)+6^{1/4}\,k_{\tilde{\phi}_1}\right]e^{-5r/2}\\&+\frac{16\,m^2\,X_4}{15K(-1)}\left[110\,2^{3/4}\,H_0\,K(-1)+6^{1/4}\,k_{\tilde{\phi}_1}\right]e^{-9r/2}\\&+\frac{7\,m^2\,X_4}{3K(-1)}\left[29\,2^{3/4}\,H_0\,K(-1)+6^{1/4}\,k_{\tilde{\phi}_1}\right]e^{-13r/2}+\mathcal{O}(e^{-17r/2})\,,\end{aligned}\tag{6.101}$$

$$\begin{aligned}\tilde{\phi}_2 =& \frac{8\,m^2\,X_4}{3K(-1)}\left[143\,2^{3/4}\,H_0\,K(-1)+4\,6^{1/4}\,k_{\tilde{\phi}_1}\right]e^{-9r/2}\\&-\frac{16\,m^2\,X_4}{5K(-1)}\left[8\,H_0\,(153\,3^{1/4}+2\,H_0)\,K(-1)+\sqrt{2}\,3^{1/4}(9\,3^{1/4}+H_0)\,k_{\tilde{\phi}_1}+30\,K(-1)\,k_{\tilde{\phi}_2}\right]e^{-6r}\\&+m^2\,X_4\left[2816\,2^{3/4}\,H_0+\frac{128\,6^{1/4}\,k_{\tilde{\phi}_1}}{5K(-1)}\right]e^{-13r/2}+\mathcal{O}(e^{-17r/2})\,,\end{aligned}\tag{6.102}$$

$$\begin{aligned}\tilde{\phi}_3 =& -\frac{4\,m^2\,X_4}{9K(-1)}\left[295\,2^{3/4}\,H_0\,K(-1)+8\,6^{1/4}\,k_{\tilde{\phi}_1}\right]e^{-9r/2}\\&+\frac{6\,m^2\,X_4}{5K(-1)}\left[8\,H_0\,(153\,3^{1/4}+2\,H_0)\,K(-1)+\sqrt{2}\,3^{1/4}(9\,3^{1/4}+H_0)\,k_{\tilde{\phi}_1}+30\,K(-1)\,k_{\tilde{\phi}_2}\right]e^{-6r}\\&-m^2\,X_4\left[\frac{14080}{13}\,2^{3/4}\,H_0+\frac{128\,6^{1/4}\,k_{\tilde{\phi}_1}}{13K(-1)}\right]e^{-13r/2}+\mathcal{O}(e^{-17r/2})\,,\end{aligned}\tag{6.103}$$

$$\begin{aligned}\tilde{\phi}_5 =& \frac{m^2\,X_4}{30}\left[\sqrt{3}\,H_0(63\,3^{1/4}+52\,H_0)-180\,k_{\tilde{\phi}_6}+2\,(27+3^{3/4}\,H_0)\sqrt{\pi}\,k_{\tilde{\phi}_1}\,\Gamma\left(\frac{1}{4}\right)^{-2}\right]e^{-r}\\&-\frac{7\,m^2\,X_4}{90}\left[\sqrt{3}\,H_0(63\,3^{1/4}+52\,H_0)-180\,k_{\tilde{\phi}_6}+2\,(27+3^{3/4}\,H_0)\sqrt{\pi}\,k_{\tilde{\phi}_1}\,\Gamma\left(\frac{1}{4}\right)^{-2}\right]e^{-3r}\\&-m^2\,X_4\left[\frac{48872\,2^{3/4}\,H_0}{585\sqrt{3}}+\frac{308\,2^{1/4}\,k_{\tilde{\phi}_1}}{117\,3^{1/4}\,K(-1)}\right]e^{-7r/2}\\&+\frac{m^2\,X_4}{6}\left[\sqrt{3}\,H_0(63\,3^{1/4}+52\,H_0)-180\,k_{\tilde{\phi}_6}+2\,(27+3^{3/4}\,H_0)\sqrt{\pi}\,k_{\tilde{\phi}_1}\,\Gamma\left(\frac{1}{4}\right)^{-2}\right]e^{-5r}\\&+m^2X4\left[\frac{301448\,2^{3/4}\,H_0}{585\sqrt{3}}+\frac{131012\,2^{1/4}\,k_{\tilde{\phi}_1}}{9945\,3^{1/4}\,K(-1)}\right]e^{-11r/2}+\mathcal{O}(e^{-7r})\,,\end{aligned}\tag{6.104}$$

$$\tilde{\phi}_6 = \frac{m^2\,X_4}{180}\left[\sqrt{3}\,H_0(63\,3^{1/4}+52\,H_0)-180\,k_{\tilde{\phi}_6}+2\,(27+3^{3/4}\,H_0)\sqrt{\pi}\,k_{\tilde{\phi}_1}\,\Gamma\left(\frac{1}{4}\right)^{-2}\right]e^{-r}\tag{6.105}$$

$$
\begin{aligned}
&- \frac{m^2\, X_4}{180}\left[\sqrt{3}\, H_0(63\, 3^{1/4} + 52\, H_0) - 180\, k_{\tilde{\phi}_6} + 2\,(27 + 3^{3/4}\, H_0)\sqrt{\pi}\, k_{\tilde{\phi}_1}\, \Gamma\left(\frac{1}{4}\right)^{-2}\right] e^{-3r} \\
&- m^2\, X_4 \left[\frac{10516\, 2^{3/4}\, H_0}{1755\sqrt{3}} + \frac{58\, 2^{1/4}\, k_{\tilde{\phi}_1}}{351\, 3^{1/4}\, K(-1)}\right] e^{-7r/2} \\
&+ \frac{m^2\, X_4}{180}\left[\sqrt{3}\, H_0(63\, 3^{1/4} + 52\, H_0) - 180\, k_{\tilde{\phi}_6} + 2\,(27 + 3^{3/4}\, H_0)\sqrt{\pi}\, k_{\tilde{\phi}_1}\, \Gamma\left(\frac{1}{4}\right)^{-2}\right] e^{-5r} \\
&+ m^2 X4\left[\frac{4244\, 2^{3/4}\, H_0}{1755\sqrt{3}} + \frac{1466\, 2^{1/4}\, k_{\tilde{\phi}_1}}{3315\, 3^{1/4}\, K(-1)}\right] e^{-11r/2} + \mathcal{O}(e^{-7r})\,. \\
\tilde{\phi}_4 = &\ \frac{4\sqrt{3}\, m^2\, X_4}{65}\left[\sqrt{3}\, H_0(63\, 3^{1/4} + 52\, H_0) - 180\, k_{\tilde{\phi}_6} + 2\,(27 + 3^{3/4}\, H_0)\sqrt{\pi}\, k_{\tilde{\phi}_1}\, \Gamma\left(\frac{1}{4}\right)^{-2}\right] e^{-2r} \\
&+ \frac{2192\, \sqrt{3} m^2\, X_4}{14365}\left[\sqrt{3}\, H_0(63\, 3^{1/4} + 52\, H_0) - 180\, k_{\tilde{\phi}_6} + 2\,(27 + 3^{3/4}\, H_0)\sqrt{\pi}\, k_{\tilde{\phi}_1}\, \Gamma\left(\frac{1}{4}\right)^{-2}\right] e^{-4r} \\
&- \frac{8\, m^2\, X_4}{135 K(-1)}\left[2617\, 2^{3/4}\, H_0\, K(-1) + 80\, 6^{1/4}\, k_{\tilde{\phi}_1}\right] e^{-9r/2} + \mathcal{O}(e^{-6r})\,. \qquad (6.106)
\end{aligned}
$$

6.6 Discussion

In this Chapter we constructed the analytic solution for the twelve–dimensional space of linearized non-supersymmetric deformations of the warped Stenzel space, consistent with the $SO(5)$ symmetries of the supersymmetric background. Our solution provides an interpolation between the IR and UV behaviors previously constructed in [24] and it should contain interesting informations about the dual (2+1)-dimensional gauge theory. In particular, we were interested in finding the supergravity solution dual to metastable states, which were conjectured in [115] to be described by a stack of anti–M2 branes placed at the tip of the transverse geometry. We were able to identify this solution by imposing suitable boundary conditions on the set of twelve integration constants (X_a, Y_a) that parametrize the full deformation space, and indeed we showed that this solution is unique and it depends only on the number $\bar{N}$ of anti–M2 branes placed at the tip. We then used this solution to compute the force exerted on a probe M2 brane placed in the anti–M2 backreacted supergravity background and we showed that it exactly agrees with the calculation à la KKLMMT [109] in which one considers the anti–M2 brane as probing the backreacted geometry of M2 branes on the Stenzel background

The linearized supergravity solution displays however an IR singularity in the four-form flux, which leads to a divergent action, whose nature is still poorly understood. Our analysis shows that this is the only drawback of the supergravity solution, which otherwise has the desired features to describe the metastable state in the dual gauge theory. It is thus of great importance to establish the nature of this singularity. It would be of interest to perform in the M-theory context the same analysis we described for the anti-D3 branes in Chapter 5.

Chapter 7

The geometry of non-supersymmetric conifolds

In this Chapter we present some results about the geometry of the non-supersymmetric conifold solutions derived analytically in Chapter 3. We also briefly mention how the first-order formalism described in that Section for one-dimensional cone-like solutions can be extended to a general flux compactification, by using the language of generalized complex geometry. This Chapter is based on unpublished results in collaboration with Mariana Graña.

7.1 Introduction

As we discussed in section 2.1, the study of supersymmetric flux compactifications with the use of generalized geometry [84, 87, 88] has received much attentions in the past years, in part because of the flexibility and the vast range of applications of differential geometry techniques. In this context, at least for type II theories compactified on a six–dimensional manifold $\mathcal{M}_6$, the equations that govern the metric and the fluxes are under very good control and they can be conveniently expressed in terms of pure spinors of $O(6,6)$. Schematically we have

$$d_H \left[e^{3A-\phi} \Phi_- \right] = 0 \,, \qquad d_H \left[e^{3A-\phi} \Phi_+ \right] + \text{R-R fluxes} = 0 \,. \tag{7.1}$$

The main advantage of this approach is that equations involving spinor quantities, that appear from the supersymmetry variations of the fermionc fields, are translated in equations involving differential forms of different degree (called polyforms), namely sections of the bundle $\Lambda^\bullet(T^\star \mathcal{M}_6)$. While traditionally one requires $\mathcal{M}_6$ to be compact, these techniques apply to non–compact manifolds as well, and in this context can be used to study solutions which are of interest for the AdS/CFT correspondance. In section 2.2 and 2.3 we provided some examples of this, by recovering the Klebanov-Strassler [116] and BGMPZ [45] solutions from pure spinor equations.

In those cases one usually deals with internal spaces which have a cone-like structure and demands the scalars that parametrize the solution to depend only on the radial variable. The presence of a particular radial direction is important for the holographic applications, in which radial evolution is associated to a RG flow [80, 74].

Given the success of generalized geometry in the study of supersymmetric compactifications, it is natural to ask whether a similar approach can be useful to study supersymmetry breaking. Although some progress in this direction has been made by restricting the analysis to particular classes of supersymmetry–breaking backgrounds (see for example [46, 127, 5, 100]), a general description of non–supersymmetric vacua remains a challenge. On the other hand, when restricted to cone-like solutions, superpotential methods [57, 155, 55, 41] have proven extremely useful in constructing non–supersymmetric deformations of the supersymmetric solutions, as we described in details in the previous chapters.

In this context it is possible to analyse very general kind of supersymmetry breaking, for which a six–dimensional geometric description is not known. Examples include softly broken supersymmetry induced by gaugino masses [122] and the backreaction of explicit susy–breaking sources such as anti–branes [25, 21, 24, 135] that we constructed in chapter 4. Let us note that even for cone-like compactifications, a superpotential approach is not always possible. There are cases in which such a function is very difficult to find. For example much effort [23, 126, 97] has been recently devoted to construct a superpotential for the Papadopouloss and Tseytlin Ansatz [144], which contains the supersymmetric solution describing the baryonic branch of the Klebanov–Strassler field theory [116]. However, it seems that such a superpotential would fail to reproduce the whole baryonic branch [77]. This essentially follows from the fact that the first-order equations imposed by supersymmetry, that we derived in Section 2.3, are not in the form of a simple system of ODEs, but they contain some algebraic constraints. One can indeed derive a superpotential which reproduces the equations only after these constraints are imposed [49].

Beyond these simple cone-like situations, supersymmetry breaking is more difficult to study and very few results are known. For this reason it would be extremely helpful to understand general properties of non-supersymmetric backgrounds and study their geometry. In this Chapter we will study a geometric approach to non-supersymmetric compactifications in a concrete example, where we have controls from other method. We then use this as a guide in trying to infer the general equations. The example we use is the analytic family of non-supersymmetric solutions constructed explicitly in Section 3. We recall that the solutions are of the form

$$ds^2_{10} = e^{2\tilde{A}} ds^2_{1,3} + ds^2_6 \,, \tag{7.2}$$

where the shape and the fluxes of the internal manifold with metric ds^2_6 are parametrized by scalars $\phi^a(\tau)$ which depend only on the radial direction. In this situation all the solutions to the type IIB equations of motions can be thought as paths on a moduli space parametrized by the scalars ϕ^a: the effective action which describes these paths

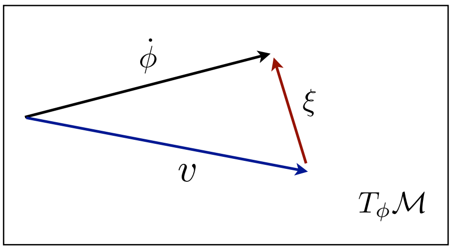


Figure 7.1: The tangent space at a point $\phi = (\phi^1, \ldots, \phi^N)$ on the moduli space $\mathcal{M}$. $\dot{\phi} = \dot{\phi}^A \partial_A$ is the vector tangent to the non–supersymmetric solution. The arrow v represents the velocity vector field that describes the supersymmetric first–order flow and we define a quantity ξ that parametrizes the supersymmetry breaking. At the linearized level in a perturbative expansion in terms of a susy breaking parameter γ these two fields satisfy decoupled systems of first–order ODEs.

can be obtained by dimensional reduction of the type IIB Lagrangian down to one radial dimension. The Hamilton-Jacobi equation for this one-dimensional system can be integrated and gives a superpotential W. This is the starting point of the first-order formalism described in section 3. Starting from a particular solution (i.e. a path on the moduli space), we can construct linearized perturbations around it that break supersymmetry, by solving two systems of *decoupled* ODEs. Let us recall the reason beyond this simplification. We call v the velocity vector field tangent to the given supersymmetric path (in our case, the Klebanov-Strassler solution), and we solve for a general solution $\phi(\tau)$ with velocity $\dot{\phi}$. We define a quantity

$$\xi = \dot{\phi} - v\,, \tag{7.3}$$

which parametrizes the supersymmetry breaking (see Figure 7.1). The type IIB supergravity equations of motion for $\phi(\tau)$ can then be recasted as equations for ξ:

$$\nabla_v \xi = -\nabla_\xi (v + \xi)\,. \tag{7.4}$$

At the linear level, namely if ξ is a first-order quantity in some supersymmetry breaking parameter, this equation becomes a decoupled system of first-order ODEs for the vector field ξ. It is clear that this is a very general feature and it is ultimately due to the integrability properties of the supersymmetric solution, namely that the first-order system obtained from supersymmetry requirement implies the supergravity equations of motion.

Guided by this example, one can try to apply the same idea to the general case. The pure spinors equations now provide the obvious definition of the supersymmetry breaking modes:

$$d_H \left[e^{3A-\phi} \Phi_- \right] = \Upsilon\,, \qquad d_H \left[e^{3A-\phi} \Phi_+ \right] + \text{R-R fluxes} = \Xi\,, \tag{7.5}$$

where now $\Upsilon, \Xi \in \Lambda^\bullet(T^\star\mathcal{M}_6)$. In the case of a cone-like solution, this should be equivalent to the definition (7.3), and the modes ξ^a are just the components of the polyforms Υ, Ξ. In this Chapter, we will explicitly construct these two polyforms for the example of the linearized perturbations around the deformed conifold. We will also discuss in detail the geometry of these solutions by computing some $SU(3)$ structure data such as the intrinsic torsion, that measure deviations from having $SU(3)$ holonomy.

The last step would be to "uplift" the equations of motion for the modes ξ^a to the bundle of polyforms, thus finding the equations for Υ, Ξ in a geometrical and general form. For this, we encounter the problem of recasting the equations of motion of type IIB supergravity compactified on a six-dimensional manifold in terms of pure spinors. In the last section we will discuss a possible approach to this problem.

7.2 Supersymmetry breaking polyforms

In section 2.2 we computed the pure spinor equations for the $\mathbb{Z}_2$ symmetric PT Ansatz [144]. Here we will explicitly determine how these equations fail to be satisfied for the family of non-supersymmetric solutions derived in chapter 3. We recall that we use an $SU(3)$ structure Ansatz, so the two pure spinors $\Phi_\pm$ are given by

$$\Phi_- = -i\Omega\,, \qquad \Phi_+ = e^{-iJ}\,. \tag{7.6}$$

The pure spinors quantities that we need are given in equations (2.35), (2.36), which we reproduce here for the reader's convenience:

$$\begin{aligned}
&e^{-3\tilde{A}+\phi} d_H\left[e^{3\tilde{A}-\phi}\Phi_-\right] = \\
&= \frac{1}{2}e^{-3p+\frac{x}{2}}\Big[2i d\tau\wedge g_1\wedge g_2\wedge g_3\wedge g_4\wedge g_5\left(f-k+e^{-y}f'-e^{y}k'\right) \\
&- d\tau\wedge(\wedge g_1\wedge g_3\wedge g_5+g_2\wedge g_4\wedge g_5)\left(2\cosh y+6p'-x'-6\tilde{A}'+2\phi'\right) \\
&- ie^{-y}d\tau\wedge g_3\wedge g_4\wedge g_5\left(2e^{y}+6p'-x'+2y'-6\tilde{A}'+2\phi'\right) \\
&- ie^{y}d\tau\wedge g_1\wedge g_2\wedge g_5\left(-2e^{-y}-6p'+x'+2y'+6\tilde{A}'-2\phi'\right)\Big] = 0\,,
\end{aligned} \tag{7.7}$$

$$\begin{aligned}
&e^{-3\tilde{A}+\phi} d_H\left[e^{3\tilde{A}-\phi}\Phi_+\right] + d\tilde{A}\wedge\bar{\Phi}_+ + e^{\phi}\star\lambda F = \\
&= e^{-6p}d\tau\wedge g_1\wedge g_2\wedge g_3\wedge g_4\left[-2+e^{6p+2x}(4\tilde{A}'+2x'-\phi')\right] \\
&+ e^{-2x}d\tau\left[e^{\phi}P\left(f(2P-F)+Fk\right)+e^{2x}\left(4\tilde{A}'-\phi'\right)\right] \\
&+ \frac{1}{2}\left(g_1\wedge g_3+g_2\wedge g_4\right)\wedge g_5\left[f-k-2e^{\phi}F'\right] \\
&+ d\tau\wedge g_1\wedge g_2\left[e^{2y+\phi}(F-2P)-f'\right]
\end{aligned} \tag{7.8}$$

$$+\,d\tau\wedge g_3\wedge g_4\left[-e^{-2y+\phi}F+e^{-\phi}k'\right]$$
$$+\,ie^{-6p-x}d\tau\wedge(g_1\wedge g_4-g_2\wedge g_3)\left[-1+e^{6p+2x}(2\tilde{A}'+x'-\phi')\right]=0\,.$$

The family of solutions we found is parametrized by 16 integrations constants, out of which 8 parametrize solutions of the first-order system

$$\phi'^a=v^a(\phi)=\frac{1}{2}G^{ab}\frac{\partial W_{KS}(\phi)}{\partial\phi^b}\,, \tag{7.9}$$

while the others 8 correspond to deviations from this system and are parametrized by supersymmetry breaking modes ξ^a. As we discussed in section 2.2 the flow equations (7.9) are not equivalent to the pure spinor equations above. In fact, the top form in (7.7) is proportional to the $(0,3)$-component of the three-form flux and it is not set to zero by (7.9). Apart from this component, we expect that for our family of solutions the right hand side of the pure spinor equations are given just in terms of the modes ξ^a. This is a big simplification since the equations for these modes are much simpler then the equations for the modes ϕ^a. It is easy to compute the various components by using the definitions (3.19). The result is the following: we define

$$e^{-3A+\phi}d_H\left[e^{3A-\phi}\Phi_-\right]=\Upsilon\,, \tag{7.10}$$

$$e^{-3A+\phi}d_H\left[e^{3A-\phi}\Phi_+\right]+dA\wedge\bar{\Phi}_++e^{\phi}\star\lambda F=\Xi\,, \tag{7.11}$$

and we find for the polyforms Υ and Ξ:

$$\begin{aligned}\Upsilon=&\frac{1}{2}e^{-4(A_0+p_0)-3p_0+\frac{x_0}{2}}\Big[12ie^{-3p_0+\frac{x_0}{2}}d\tau\wedge g_1\wedge g_2\wedge g_3\wedge g_4\wedge g_5\,G^{(0,3)} \qquad (7.12)\\&-d\tau\wedge(\wedge g_1\wedge g_3\wedge g_5+g_2\wedge g_4\wedge g_5)\left(-\tilde{\xi}_1-\frac{2}{3}\tilde{\xi}_3+\frac{1}{3}\tilde{\xi}_4+4\tilde{\xi}_8\right)\\&-ie^{-y_0}d\tau\wedge g_3\wedge g_4\wedge g_5\left(-\tilde{\xi}_1-\frac{2}{3}\tilde{\xi}_3+\frac{1}{3}\tilde{\xi}_4-4\tilde{\xi}_2+4\tilde{\xi}_8\right)\\&-ie^{y_0}d\tau\wedge g_1\wedge g_2\wedge g_5\left(\tilde{\xi}_1+\frac{2}{3}\tilde{\xi}_3-\frac{1}{3}\tilde{\xi}_4-4\tilde{\xi}_2-4\tilde{\xi}_8\right)\Big]\,,\\ \Xi=&e^{-4(A_0+p_0)+2x_0}\Big[-\frac{2}{3}d\tau\wedge g_1\wedge g_2\wedge g_3\wedge g_4\left(2\tilde{\xi}_1+\tilde{\xi}_3+\tilde{\xi}_4+6\tilde{\xi}_8\right) \qquad (7.13)\\&+\frac{2}{3}e^{-2x_0}d\tau\,\tilde{\xi}_1+2\,(g_1\wedge g_3+g_2\wedge g_4)\wedge g_5\,\tilde{\xi}_7\\&+2d\tau\wedge g_1\wedge g_2e^{2y_0}\left(\tilde{\xi}_5+\tilde{\xi}_6\right)+2d\tau\wedge g_3\wedge g_4e^{-2y_0}\left(\tilde{\xi}_5-\tilde{\xi}_6\right)\\&-\frac{i}{3}e^{-x_0}d\tau\wedge(g_1\wedge g_4-g_2\wedge g_3)\left(2\tilde{\xi}_1+\tilde{\xi}_3+\tilde{\xi}_4\right)\Big]\,,\end{aligned}$$

where $G^{(0,3)}$ cannot be expressed nicely in terms of $\tilde{\xi}^a$ and hence we omit it for simplicity. The functions $\xi^a(\tau)$ are parametrized, as expected, by 8 integrations constants X_a and their analytic solution is given in equations (3.55)–(3.61).

We now discuss some general features of this geometrical rewriting. First of all, we want to make contact with previous attempts to use generalized geometry to study non–supersymmetric backgrounds.

7.2.1 Domain wall supersymmetry breaking

The authors of [127] identified a particular class of $SU(3) \times SU(3)$ structure backgrounds that generalize the solution of Graña and Polchinski [90] and they used equations similar to (7.10), (7.11) to parametrize these solutions in terms of generalized geometric quantities. When restricted to the case of $SU(3)$ structures their solution indeed corresponds to SUSY–breaking by the $G^{(0,3)}$ flux component [90]. Their general class of vacua are described by the pure spinors equations (7.10), (7.11) with the condition

$$\Xi = 0\,, \tag{7.14}$$

which is motivated by some calibration condition. If we impose this condition in our explicit example (7.13), we see from the solution given in (3.55)-(3.61) that we are forced to set all the X_a integration constants to zero: $X_a = 0$, $a = 0, \ldots, 8$. This naively seems to imply that we are forced to consider only supersymmetric perturbations of the deformed conifold. However, as we discussed in the previous section, the ξ^a modes do not parametrize the full space of non–supersymmetric solutions. Since W_{KS} is a fake superpotential, there exist solutions with $\xi^a = 0$ which still break supersymmetry. In fact, by imposing that all the ξ^a modes are zero, the polyform Υ becomes

$$\Upsilon \sim -\frac{P^{3/2}\, Y_5 h(\tau)^{3/4} \sinh^2\tau}{\sqrt{6}}\, d\tau \wedge \cdots \wedge g_5\,. \tag{7.15}$$

This term is precisely the $G^{(0,3)}$ part of the fluxes which breaks supersymmetry. This is consistent with the analysis in [127], where condition (7.15) is the specialization to the $SU(3)$ structure case of what they call "domain wall (non)BPSness". We note that the class of vacua analysed in [127] captures only a one dimensional family of susy–breaking solutions, and it misses the remaining seven–dimensional space parametrized by the ξ^a modes (there are eight of such modes but one is fixed by the zero energy condition). While our analysis is carried out in a very specific example and only at first–order in perturbation theory, this situation clearly calls for further investigation of generic $\mathcal{N} = 0$ vacua. Our analysis suggests that both the polyforms Υ and Ξ should be non vanishing in such an attempt. We can name at least two interesting physical solutions which are captured by a non–zero polyform Ξ. One is the gravity dual of softly broken SUSY, first studied in [122]. This corresponds to a two–dimensional family of solution which is obtained from (3.55)-(3.61) by setting $X_2 \neq X_7 \neq 0$ and all others Xs to zero. Another example is the solution which corresponds to the backreaction of a stack of anti–D3 branes placed at the tip of the internal conifold which we discussed in detail in section 4.

7.3 Intrinsic torsion for non–susy conifolds

In this Section we derive the expression of other useful geometrical objects for our family of non-supersymmetric solutions. We recall that we are using an $SU(3)$ Ansatz. A useful characterization of a manifold with $SU(3)$ structure is in terms of $SU(3)$ representations of its geometrical objects and its fluxes. In particular, one can classify the geometry in terms of the intrinsic torsion, which is an obstruction to have $SU(3)$ holonomy. The component of the torsion classes in terms of $SU(3)$ representations are defined in (2.11). These forms provide very useful information about the geometry of the manifold. For example, in Table 7.1 we show the conditions on $\mathcal{W}_i$ that corresponds to well known classes of manifolds. For more details we refer to [84].

The expression for the torsion classes for the PT Ansatz can be computed by inverting the relations (2.11), for example by computing:

$$\mathcal{W}_1 = -\frac{i}{6}(dJ)^{ijk}\Omega_{ijk} \qquad (\mathcal{W}_4)_i = \frac{1}{2}(dJ)_{imn}J^{mn} \qquad (\mathcal{W}_5)_k = \frac{1}{8}(d\Omega)_k^{ijk}\Omega_{ijk}\,. \tag{7.16}$$

The result is:

$$\mathcal{W}_1 = 0 \tag{7.17}$$
$$\mathcal{W}_2 = e^{3p+\frac{3x}{2}-y}\left(e^{2y}g_1\wedge g_2 + g_3\wedge g_4\right)\left(\sinh y + y'\right) \tag{7.18}$$
$$\mathcal{W}_3 = 0 \tag{7.19}$$
$$\mathcal{W}_4 = dt\left(x' - e^{-6p-2x}\right) \tag{7.20}$$
$$\mathcal{W}_5 = -\frac{1}{4}(dt - ig_5)\left(2\cosh y + 6p' - x'\right) \tag{7.21}$$

It is easy to show that the flow equations (7.9) imply that

$$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 - 2\mathcal{W}_5 = 0\,, \tag{7.22}$$

namely that a solution of the flow equations is conformally Calabi-Yau. It is clear that deviations from this conditions are parametrized by the modes ξ^a. Indeed at first-order we find:

$$\mathcal{W}_1 = \mathcal{W}_3 = 0 \tag{7.23}$$
$$\mathcal{W}_2 = -2e^{-4A_0-p_0+\frac{3x_0}{2}-y_0}\left(e^{2y_0}g_1\wedge g_2 + g_3\wedge g_4\right)\tilde{\xi}_2 \tag{7.24}$$
$$3\mathcal{W}_4 - 2\mathcal{W}_5 = -\frac{1}{2}e^{-4(A_0+p_0)}\left(9\tilde{\xi}_1 + 5\tilde{\xi}_3 + 2\tilde{\xi}_4\right)d\tau. \tag{7.25}$$

We note that since the mode $\tilde{\xi}_2$ is parametrized by seven integration constants

$$\tilde{\xi}_2 = Span(X_1,\ldots,X_7)\,, \tag{7.26}$$

Manifold	Torsion classes
Complex	$\mathcal{W}_1 = \mathcal{W}_2 = 0$
Symplectic	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
Half-flat	$\text{Im}\ \mathcal{W}_1 = \text{Im}\ \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
Special Hermitean	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
Nearly Kähler	$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
Almost Kähler	$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5$
Kähler	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$
Calabi-Yau	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$
Conformal Calabi-Yau	$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 - 2\mathcal{W}_5$

Table 7.1: Special SU(3) structure manifolds correspond to the vanishing of certain torsion classes.

almost all solutions will have a non zero $\mathcal{W}_2$, which means that the perturbations are not complex. An exception is the GKP case, in which all the modes $\tilde{\xi}^a$ are zero. We stress that the vanishing of $\mathcal{W}_1$ and $\mathcal{W}_3$ is not a particular characteristic of our perturbation scheme, but it is a consequence of the $\mathbb{Z}_2$ symmetry of the KS background. More general Ansätze will lead to a more general pattern of torsion classes.

7.3.1 Soft supersymmetry breaking

Here we study an explicit example, a non–supersymmetric gravity dual of supersymmetry breaking induced by gaugino mass terms in the Klebanov–Strassler theory. This solution was first studied in [122] (see also [7, 4, 69] for related works), but only a one–parameter family of solution describing the KS background perturbed by a operator of dimension 3 was constructed. One can be more general, since we can actually turn on two dimension 3 operators, as we discussed in section (4.6.2). From our deformation space, we can fish out a two parameter family of solutions parametrized by two integration constants (X_2, X_7) which generalize the solution of [122]. While to impose the desired boundary conditions one has to study the behavior of the perturbation modes ϕ^a, here we omit this derivation and we simply focus on the result for the susy-breaking modes ξ^a, for which the solution is analytic and particularly simple. These modes are given by:

$$\tilde{\xi}_1 = \tilde{\xi}_3 = \tilde{\xi}_4 = 0\,, \tag{7.27}$$

$$\begin{aligned}\tilde{\xi}_2 = &\frac{X_2}{4}\text{csch}^3(\tau)(\sinh(2\tau) - 2\tau)^2 \\ &- PX_7\Big(\text{csch}(\tau) + \tau(\cosh(\tau) - 2\coth(\tau)\text{csch}(\tau) + \tau\text{csch}^3(\tau))\Big)\,,\end{aligned} \tag{7.28}$$

$$\tilde{\xi}_5 = \frac{X_2}{P} - \frac{3X_7}{2}\,, \tag{7.29}$$

$$\tilde{\xi}_6 = \frac{X_2}{P}\tau\text{csch}(\tau) + X_7\left(-\frac{1}{2}\cosh(\tau) - \tau\text{csch}(\tau)\right), \tag{7.30}$$

$$\tilde{\xi}_7 = \frac{X_2}{P}\Big(\tau\coth(\tau) - 1\Big)\text{csch}(\tau) + X_7\left(\text{csch}(\tau) - \tau\coth(\tau)\text{csch}(\tau) + \frac{1}{2}\sinh(\tau)\right), \tag{7.31}$$

$$\tilde{\xi}_8 = -(X_2 - PX_7)\left(\coth(\tau) - \tau - 2\tau\text{csch}^2(\tau) + \tau^2\coth(\tau)\text{csch}^2(\tau)\right). \tag{7.32}$$

The solution of [122] corresponds to the subset $X_2 = PX_7 = X$. By using the result (7.23) we see that the geometry of this family of backgrounds is characterized by the following torsion classes:

$$\mathcal{W}_1 = \mathcal{W}_3 = 3\mathcal{W}_4 - 2\mathcal{W}_5 = 0\,, \tag{7.33}$$

$$\mathcal{W}_2 \sim \left(e^{2y}g_1 \wedge g_2 + g_3 \wedge g_4\right)\Big[f_1(\tau)\,X_2 + f_2(\tau)\,X_7\Big], \tag{7.34}$$

where f_1 and f_2 can be easily read off from (7.28). We stress that this result is valid at the linearized level. At full non-linear order we expect the mode $\tilde{\xi}_1$ to be non-zero, and thus very likely the combination $3\mathcal{W}_4 - 2\mathcal{W}_5$ will be in general non-zero as well.

7.4 Discussion

In this chapter we studied how the pure spinor equations, which describe $\mathcal{N} = 1$ supersymmetric flux compactifications, are modified for a class of non-supersymmetric solutions obtained as a first-order deformation of the Klebanov-Strassler deformed conifold solution. These results are a first step toward a more systematic understanding of a first-order description of non-supersymmetric backgrounds. As we discussed in the introduction 7.1 and in Section 3.2, for cone-like solutions supersymmetry provides the natural variables to implement a first-order formalism. If we break supersymmetry perturbatively, in a series expansion around a given supersymmetric background, then one can define supersymmetry breaking modes ξ^a which satisfy a decoupled first-order system of ordinary differential equations. In the one dimensional case, where a solution can be thought as a path in a given moduli space (see figure 7.1), these modes measure deviations from the supersymmetric solutions. In fact, at first-order in supersymmetry breaking, the equations for the ξ^a modes are analogous to the geodesic deviation equation (or Jacobi equation).

Besides the interest in studying the geometry of our class of non-supersymmetric solutions, the aim of this chapter was to take a first step in extending the above first-order formalism in the language of generalized geometry. One could use such a formalism to study perturbations around supersymmetric background that have less symmetries and the corresponding solution depends on more then one variables. Even in the one-dimensional case, it would be interesting to understand the relation between the geodesic deviation equations for the ξ^a modes and the equations for the polyforms Ξ and Υ. While a full treatment of this problem is outside the scope of this speculative section, we would like to sketch the steps that can leads to such equations.

The first important step is to understand the relation between the supersymmetric equations and the equations of motion. This is not an easy task, since expressions in terms of pure spinors of basic quantities such as the Riemann tensor are not known. Luckily, it has been understood how to rewrite the effective supergravity action in terms of generalized geometric objects. As in the one dimensional case, one obtains an action which is basically the "square" of the supersymmetric equations. This BPS rewriting has been obtained in [48, 127]. With the restrictions of [48], the effective potential describing the dynamics of six-dimensional quantities has the following schematic structure:

$$\begin{aligned}\mathcal{V}_{eff} = &\int_{\mathcal{M}_6} d\mathrm{Vol}_6\Big[(\mathrm{Re}\ \Xi)^2 + (\mathrm{Im}\ \Xi)^2 + (\Upsilon)^2\Big] \\ &+ \int_{M_6} \Big[\,|\langle\Phi_+, \Upsilon\rangle|^2 + \left|\langle\bar{\Phi}_+, \Upsilon\rangle\right|^2\,\Big] \\ &+ \int_{\mathcal{M}_6} \Big\langle e^{4\tilde{A}-\phi}\mathrm{Re}\ \Phi_+ - C^{el}, d_H F + j_{tot}\Big\rangle + S^{loc.}\,. \end{aligned} \tag{7.35}$$

Here $\langle , \rangle$ is the Mukai pairing, a natural bilinear operation on the space of polyforms, defined as follows:

$$\langle\omega\wedge\eta\rangle = \omega\wedge\sigma(\eta)\big|_6\ , \tag{7.36}$$

where σ reverses the order of indices of a given form. By varying this action with respect to the metric g_{mn}, one can obtain the internal Einstein equations in terms of polyforms. It also clear that setting $\Xi = \Upsilon = 0$ automatically solves the equations. Thus, we are in the same situation as in the simple one-dimensional example, where the polyforms Ξ and Υ have the same role of the deviations ξ^a. We thus expect that at first-order in a supersymmetry breaking parameter, these polyforms will satisfy a decoupled system of first-order differential equations.

When specialized to our class of supersymmetry breaking deformations around the deformed conifold, the equations for Ξ and Υ should reproduce the equations for the modes ξ^a, and expressions (7.12), (7.13) should provide a solution of such equations. We hope to come back to this problem in the future.

Chapter 8

Conclusions and outlook

In this thesis we constructed and studied various non-supersymmetric cone-like compactifications of type IIB and M-theory. These solutions find applications in the gauge/gravity duality, where they provide gravity duals to different supersymmetry breaking scenario in the gauge theories, and in phenomenological aspects such as the construction of de Sitter vacua and models of brane-antibrane inflation.

In the first chapters, we rederived various supersymmetric supergravity solutions dual to confining $\mathcal{N} = 1$ gauge theory (the Klebanov-Strassler [116] and BGMPZ [45] solutions) by using generalized geometry techniques, in particular by an explicit computation of the pure spinor equations. We believe that formal techniques borrowed from the study of flux compactifications are extremely useful to construct solutions relevant for holography. It would be interesting to extend the analysis done for type IIB solutions for the M-theory backgrounds, like the cone based on the Stenzel space $V_{5,2}$ discussed in chapter 6. This would first need a complete reformulation of supersymmetry conditions in terms of differential forms. Another line of research is to begin in this framework the study of more complicated cone-like solutions, for example in cases where some of the symmetries are broken and the fields also depend on some angular coordinates of the base. This is for example the case of $Y^{p,q}$ cones [103]. More ambitiously, one could try to use pure spinor techniques to construct part of the solutions corresponding to polarized branes, which we discussed in Chapter 5. It is important to keep in mind that the Polchinski-Strassler solution [146] is not known beyond linear order, although some closely related M-theory background have been constructed (at least when supersymmetry is not broken) [17, 31].

In Chapter 3 we described a general first-order method to study non-supersymmetric solutions perturbatively around a given supersymmetric and known solution. This is a generalization of the ideas introduced in [41]. We then applied this method to the construction of an explicit solution in closed form for the linearized perturbations around the KS solution preserving the $SU(2) \times SU(2) \times \mathbb{Z}_2$ symmetries of the deformed conifold.

In Chapter 4 we integrated numerically the above solutions and discussed the infrared and ultraviolet boundary conditions corresponding to a stack of anti-D3 branes placed at the tip of the KS geometry. We computed various quantities such as the ratio between the deformation modes of the conifold for the non-supersymmetric and supersymmetric vacua, checking that the solution does not have any other non-normalizable modes in the UV, which is the gravity counterpart of having spontaneous supersymmetry breaking in the field theory. We proved however that the three-form flux of the solution have singular energy density.

In Chapter 5 we solved the full non-linear equations of motion for the KS system in the infrared region, near the anti-brane source, and we proved that the singularity is not an artifact of our previous linearization method. We then studied the most natural way to resolve the singularity in string theory, namely by brane polarization [142] à la Polchinski-Strassler [146]. The result we obtain in the full backreaction seems to contradict the probe analysis of [111], namely we don't find any polarization channel that could resolve the singularity. We should keep in mind that we worked in the approximation of smeared sources, so the original KPV channel $\overline{\text{D3}} \to$ NS5 wrapping an S^2 on the S^3 at the tip cannot be checked explicitly. However, as also discussed in [59], one expects the fully backreacted solution in the near-brane region to be an AdS_5 throat with relevant flux perturbations, as in the original Polchinski-Strassler setup. If this is the case, then we necessarily have different polarization channels, the NS5 together with the D5 channel that we checked in our computation. Thus, our result is an indication that the NS5 channel will be absent too, indicating that anti-branes in warped throats develop perturbative instabilities and cannot be used to uplift AdS to a dS compactification. If confirmed from a localized anti-D3 solution, it would be extremely interesting to explore further the precise nature of this instability. In type IIA engineering of SQCD metastable states [16], the probe analysis fails because of a logarithm bending of the branes once their backreaction is taken into account. This causes a large deviation in the ultraviolet region from the supersymmetric configuration, meaning that the breaking of supersymmetry cannot be spontaneous. It is suggestive that such logarithm modes are found in different but related contexts. For example, it was shown in [53] that logarithm backreaction in the NS5/$\overline{\text{NS}}$5 configuration used in models of axion monodromy inflation, "climb up" to the bulk of the compactification, setting the energy scale of the brane/antibrane interaction to be the UV scale and not the IR one.

It could perhaps be interesting to investigate relations also with recent works by Polyakov (see e.g. [150]), where an infrared divergency and an IR/UV relation is conjectured to cause an instability of the de Sitter space. It is conceivable that when trying to construct a de Sitter compactification by pumping positive energy to the system, as in the anti-D3 model, one encounters the same problem and the system develops instabilities. While this could invalidate the idea of a string landscape, it can lead to models of dynamical screening of the cosmological constant. It is also important to keep in mind that the perturbative decay of anti-D3s into supersymmetric D3s at the south pole of the S^3 on the deformed conifold (mediated by the NS5 nucleation) can potentially be used as a slow-roll inflationary model, as discussed in [59].

In Chapter 6.5 we constructed an analytic family of non-supersymmetric solutions of eleven dimensional supergravity, by linearizing the equations of motion in a perturbation series around the cone-like supersymmetric solution found in [54] and based on the warped Stenzel space. In particular we focused on the backreaction of anti-M2 branes on the geometry. This configuration is the M-theory analog of anti-D3 branes on the deformed conifold, even if there are important differences, in particular the absence of a logarithmic running of the charge. The supergravity solution has similar features to the anti-D3 one, in particular it shows the same kind of singularity in the energy density of the four-form flux.

While it is very reasonable that the singularity will be present in the full nonlinear solution as well, it would be interesting to study possible resolution by brane polarization in this context. This would clarify, for example, if the presence of the logarithm mode has a role in the physics of anti-branes in warped throats.

In Chapter 7 we studied some geometrical aspects of the non-supersymmetric solutions discussed above. As we shown in Chapter 2, the supersymmetric first-order equations describing supersymmetric and conformally Calabi-Yau cones can be derived by using pure spinor techniques. By using the explicit solutions for non-supersymmetric deformations around the Klebanov-Strassler solution, we then computed how the pure spinor equations are modified for non-supersymmetric backgrounds. We also computed $SU(3)$ structure data such as the intrinsic torsion. We then discussed how to implement the first-order formalism of Section 3.2 in the language of generalized geometry. These results are a starting point to try to explore more general non-supersymmetric solutions and to understand the geometrical tools needed to study de Sitter compactifications in the context of generalized geometry.

Appendix A

Notations

In this Appendix we collect a number of useful definitions that we use in the main text and the relations to various conventions used in the literature. To study the deformed conifold with topology $\mathbb{R} \times S^2 \times S^3$ we introduce the following 1-forms. The forms $\{e_1, e_2\}$ correspond to the S^2, parametrized by the angle (θ_1, ϕ_1). On the S^3, parametrized by ψ and (θ_2, ϕ_2), we introduce left invariant forms $\{\epsilon_1, \epsilon_2, \epsilon_3\}$: $d\epsilon_i = -(1/2)\epsilon_{ijk}\epsilon_j \wedge \epsilon_k$. We also define some forms $\tilde{\epsilon}_i$ which are useful for the PT Ansatz:

$$\begin{aligned}
&e_1 = d\theta_1 && \\
&e_2 = -\sin\theta_1 d\phi_1 && \\
&\tilde{\epsilon}_1 = \epsilon_1 - a(\tau)e_1 && \epsilon_1 = \sin\psi \sin\theta_2 d\phi_2 + \cos\psi d\theta_2 \\
&\tilde{\epsilon}_2 = \epsilon_2 - a(\tau)e_2 && \epsilon_2 = \cos\psi \sin\theta_2 d\phi_2 - \sin\psi d\theta_2 \\
&\tilde{\epsilon}_3 = \epsilon_3 + \cos\theta_1 d\phi_1 && \epsilon_3 = d\psi + cos\theta_2 d\phi_2 \,.
\end{aligned}$$

Here we denote by τ the radial direction of the cone. In the text we will mainly use the forms g_i of KS [116], defined as:

$$\begin{aligned}
g_1 &= \frac{e_2 - \epsilon_2}{\sqrt{2}} \quad g_3 = \frac{e_2 + \epsilon_2}{\sqrt{2}} \\
g_2 &= \frac{e_1 - \epsilon_1}{\sqrt{2}} \quad g_4 = \frac{e_1 + \epsilon_1}{\sqrt{2}} \\
g_5 &= \tilde{\epsilon}_3.
\end{aligned} \tag{A.1}$$

We also give the basis used in [45]:

$$\begin{aligned}
E_1 &= e^{\frac{x+g}{2}} e_1 \,, \qquad E_2 = e^{\frac{x+g}{2}} e_2 \,, \qquad E_3 = e^{\frac{x-g}{2}} \tilde{\epsilon}_1 \,, \\
E_4 &= e^{\frac{x-g}{2}} \tilde{\epsilon}_2 \qquad E_5 = e^{-3p-x/2} d\tau \,, \qquad E_6 = e^{-3p-x/2} \tilde{\epsilon}_3 \,.
\end{aligned}$$

It is useful to define a rotated basis G_i

$$G_1 = E_1 \,, \qquad G_2 = \mathcal{A}E_2 + \mathcal{B}E_4 \,, \qquad G_5 = E_5$$

$$G_3 = E_3 \,, \qquad G_4 = \mathcal{B}E_2 - \mathcal{A}E_4 \,, \qquad G_6 = E_6 \,,$$

which is useful for manipulation of $SU(3)$ structure representation. Here $\mathcal{A}$ and $\mathcal{B}$ are rotations in a 2-plane and thus satisfy $\mathcal{A}^2 + \mathcal{B}^2 = 1$. The change of basis between [116] g_i and the BGMPZ [45] G_i is the following:

$$\begin{aligned}
g_1 &= \frac{1}{\sqrt{2}} e^{-\frac{1}{2}(x+g)} \Big[e^g (\mathcal{A}G_4 - \mathcal{B}G_2) - (a(\tau) - 1)(\mathcal{A}G_2 + \mathcal{B}G_4) \Big] \\
g_2 &= \frac{1}{\sqrt{2}} e^{-\frac{1}{2}(g+x)} \Big[(1 - a(\tau))G_1 - e^g G_3 \Big] \\
g_3 &= \frac{1}{\sqrt{2}} e^{-\frac{1}{2}(g+x)} \Big[e^g (\mathcal{B}G_2 - \mathcal{A}G_4) + (1 + a(\tau))(\mathcal{A}G_2 + \mathcal{B}G_4) \Big] \\
g_4 &= \frac{1}{\sqrt{2}} e^{-\frac{1}{2}(g+x)} \Big[(1 + a(\tau))G_1 + e^g G_3 \Big] \\
d\tau &= e^{3p+\frac{x}{2}} G_5 \,, \qquad g_5 = e^{3p+\frac{x}{2}} G_6 \,.
\end{aligned}$$

The inverse is:

$$\begin{aligned}
G_1 &= \frac{1}{\sqrt{2}} e^{\frac{1}{2}(x+g)} \Big[g_2 + g_4 \Big] \\
G_2 &= \frac{1}{\sqrt{2}} e^{\frac{1}{2}(x-g)} \Big[e^g \mathcal{A}(g_1 + g_3) - \mathcal{B}(g_1 - g_3 + a(\tau)(g_1 + g_3)) \Big] \\
G_3 &= \frac{1}{\sqrt{2}} e^{\frac{1}{2}(x-g)} \Big[g_4 - g_2 - a(\tau)(g_2 + g_4) \Big] \\
G_4 &= \frac{1}{\sqrt{2}} e^{-\frac{1}{2}(x+g)} \Big[e^g \mathcal{B}(g_1 + g_3) + \mathcal{A}(g_1 - g_3 + a(\tau)(g_1 + g_3)) \Big] \\
G_5 &= e^{-3p-\frac{x}{2}} d\tau \,, \qquad G_6 = e^{-3p-\frac{x}{2}} g_5 \,.
\end{aligned} \tag{A.2}$$

From the $\mathbb{Z}_2$ breaking PT Ansatz, we can specialize to a $\mathbb{Z}_2$ symmetric one by imposing:

$$e^{2g} = 1 - a^2 \,, \qquad a = \tanh y \,, \qquad g = \log \left[\frac{1}{\cosh y} \right] , \qquad \chi = 0 \,. \tag{A.3}$$

The $SU(3)$ structure functions $\mathcal{A}$ and $\mathcal{B}$ for KS are

$$\mathcal{A} = e^g \,, \qquad \mathcal{B} = -a \,. \tag{A.4}$$

The relation between the fields used in [116] and [144] is:

$$\begin{aligned}
f(\tau) &= h_1(\tau) - h_2(\tau) \\
k(\tau) &= h_1(\tau) + h_2(\tau) \\
b(\tau) &= \frac{F(\tau)}{P} - 1 \,.
\end{aligned} \tag{A.5}$$

Appendix B

Equations of motion for the KS fields

The equations of motion for the KS scalars are the Euler–Lagrange equations from the effective action:

$$\mathcal{S} = \int \left[\frac{1}{2} G_{ab} \frac{d\phi^a}{d\tau}\frac{d\phi^b}{d\tau} + V(\phi)\right] d\tau \, , \tag{B.1}$$

which are

$$G^{ad}(\phi)\partial_d V(\phi) = \Gamma^a_{bc}(\phi)\frac{d\phi^b}{d\tau}\frac{d\phi^c}{d\tau} + \frac{d^2\phi^a}{d\tau^2} \, . \tag{B.2}$$

Explicitly, these are:

$$\begin{aligned}
x'' &= e^{-4(3p+x)}\Big[2e^{6p+2x}\cosh y - e^{2(6p+x+y)+\phi}P(P-F) - 1\Big] \\
&\quad - \frac{1}{8}e^{-4x}\Big[2e^{2x-2y+\phi}(1+e^{4y})F^2 + e^{2x-\phi}(f-k)^2 + 4K^2\Big] \\
&\quad - \frac{1}{4}e^{2(x+y)-\phi}(f'^2 + 2e^{2(y+\phi)}F'^2 + e^{4y}k'^2) - 4(A'+p')x' \, , \\
y'' &= 2e^{-2x+2y+\phi}P(P-F) + \frac{1}{2}e^{-2(x+y)+\phi}(e^{4y}-1)F^2 \\
&\quad + (\cosh y - 2e^{-6p-2x})\sinh y + \frac{1}{2}e^{-2(x+y)-\phi}(e^{4y}k'^2 - f'^2) - 4(A'+p')y' \, , \\
p'' &= \frac{1}{3}e^{-4(3p+x)}\Big[e^{6p+2x}\cosh y + e^{2(6p+x+y)+\phi}P(P-F) - 1\Big] \\
&\quad + \frac{1}{24}e^{-4x}\Big[2K^2 + e^{2x}(4e^{\phi}F^2\cosh 2y + e^{-\phi}(f-k)^2 + 4e^{2x}\sinh^2 y\Big] \\
&\quad + \frac{1}{12}e^{-2(x+y)-\phi}\Big[f'^2 + 2e^{2(y+\phi)}F'^2 + e^{4y}k'^2\Big] \\
&\quad - \frac{1}{12}\Big[24(A'+p')^2 - 4x'^2 - 2y'^2 - \phi'^2\Big] \, ,
\end{aligned} \tag{B.3}$$

$$
\begin{aligned}
A'' =& \frac{1}{24}e^{-12p-4x-2y-\phi}\Big[2e^{2(6p+x+\phi)}\left(F^2+e^{4y}(F-2P)^2\right)+e^{2(6p+x+y)}(f-k)^2 \\
&+2e^{2y+\phi}\left(2-8e^{6p+2x}\cosh y+e^{12p}(K^2+2e^{4x}\sinh^2 y)\right)\Big]+ \\
&+\frac{1}{12}e^{-2x-2y-\phi}\left[f'^2+2e^{2(y+\phi)}F'^2+e^{4y}k'^2\right] \\
&+2A'^2+4A'p'+p'^2+\frac{1}{3}x'^2+\frac{1}{6}y'^2+\frac{1}{12}\phi'^2\,, \\
f'' =& \frac{1}{2}e^{-2x}\Big[2e^{2y+\phi}(2P-F)K+e^{2x+2y}(f-k) \\
&+2e^{2x}f'(2(x'+y')+\phi'-4A'-4p')\Big]\,, \\
k'' =& \frac{1}{2}e^{-2(x+y)}\Big[f(2e^{\phi}(2P-F)F-e^{2x})+(e^{2x}+2e^{\phi}F^2)k \\
&-2e^{2(x+y)}k'(4A'+4p'-2x'+2y'-\phi')\Big]\,, \\
F'' =& \frac{1}{2}e^{-2x-\phi}\Big[(k-f)K-2e^{2x+\phi}(e^{2y}P\cosh(2y)F+F'(4A'+4p'-2x'+\phi'))\Big]\,, \\
\phi'' =& \frac{1}{2}e^{-2(x+y)+\phi}\Big[F^2+e^{4y}(F-2P)^2+2e^{2y}F'^2\Big] \\
&+\frac{1}{4}e^{-2(x+y)-\phi}\Big[-e^{2y}(f-k)^2-2f'^2-2e^{4y}k'^2\Big]-4(A'+p')\phi'\,.
\end{aligned}
$$

Appendix C

Supersymmetric conditions for the PT Ansatz

In the following we show the expressions for the left-hand side of the pure spinor equations for the PT Ansatz, discussed in section 2.3. We concisely write $G_{a_1\ldots a_n} = G_{a_1}\wedge\cdots\wedge G_{a_n}$. For simplicity, we also set $e^{\theta_-}=1$.

$$
\begin{aligned}
&\boxed{e^{-3\tilde{A}+\phi}d_H\left[e^{3\tilde{A}-\phi}\Phi_-\right]} = e^{-g+3p}\Big[ie^{-6p-\frac{3x}{2}}\Big(2e^{g}a\mathcal{A}+\mathcal{B}+e^{2g}\mathcal{B}-a^2\mathcal{B}\Big)G_{1234}\\
&-ie^{\frac{x}{2}}G_{3456}\Big(-e^{g}\mathcal{B}-\mathcal{B}^2a'+e^{g}\mathcal{B}\mathcal{A}'-e^{g}\mathcal{A}\mathcal{B}'+e^{g}\mathcal{A}\mathcal{B}g'\Big)\\
&+ie^{\frac{x}{2}}G_{1256}\Big(2a\mathcal{A}+e^{g}\mathcal{B}+a'+\mathcal{A}^2a'-e^{g}\mathcal{B}\mathcal{A}'+e^{g}\mathcal{A}\mathcal{B}'+e^{g}\mathcal{A}\mathcal{B}g'\Big)\\
&+\frac{1}{2}ie^{\frac{x}{2}}G_{2356}\Big(-2e^{g}\mathcal{A}-2\mathcal{A}\mathcal{B}a'+6e^{g}\tilde{A}'+(-1+\mathcal{A}^2-\mathcal{B}^2)e^{g}g'-6e^{g}p'+e^{g}x'-2e^{g}\phi'\Big)\\
&+\frac{1}{2}e^{\frac{x}{2}}(G_{2456}-G_{1356})\Big(2e^{g}\mathcal{A}-2a\mathcal{B}-6e^{g}\tilde{A}'+6e^{g}p'-e^{g}x'+2e^{g}\phi'\Big)\\
&-\frac{1}{2}ie^{\frac{x}{2}}G_{1456}\Big(2e^{g}\mathcal{A}-4a\mathcal{B}-2\mathcal{A}\mathcal{B}a'-6e^{g}\tilde{A}'-(1-\mathcal{A}^2+\mathcal{B}^2)e^{g}g'+6e^{g}p'-e^{g}x'+2e^{g}\phi'\Big)\\
&+ie^{-\frac{x}{2}}G_{123456}\Big(2e^{g}h_2+(2e^{g}a\mathcal{A}-\mathcal{B}+e^{2g}\mathcal{B}-a^2\mathcal{B})h_1'+(2e^{g}\mathcal{A}-2a\mathcal{B})h_2'\\
&+(-2e^{g}a\mathcal{A}-\mathcal{B}'-e^{2g}\mathcal{B}+a^2\mathcal{B})\chi'\Big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
&\boxed{e^{-3\tilde{A}+\phi}d_H\left[e^{3\tilde{A}-\phi}\Phi_+\right]+d\tilde{A}\wedge\bar{\Phi}_++\star\lambda F}\\
&=e^{-g-3p-\frac{3x}{2}}\Big\{-G_5e^{6p+g-i\theta_+}\Big[2e^{i\theta_++\phi}P(h_1+bh_2)-e^{2x}(-\tilde{A}'+e^{2i\theta_+}(-3\tilde{A}'-i\theta_+'+\phi'))\Big]\\
&e^{6p+x}(G_{136}-G_{246})\Big[ie^{x+i\theta_+}a\mathcal{A}+ie^{g+x}\mathcal{B}+e^{g}h_2-e^{g+\phi}Pb'\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+G_{235}\Big[e^{6p+x+\phi}P(e^g\mathcal{A}(a-b)-(1+a^2-2ab)\mathcal{B})+e^{i\theta_p}(ie^g(a\mathcal{A}+e^g\mathcal{B}))\\
&\quad+ie^{6p+2x}\Big(\mathcal{A}^2a'+e^g(\mathcal{A}\mathcal{B}'+\mathcal{B}(-\mathcal{A}'+\mathcal{A}g'))\Big)\\
&\quad+e^{g+6p+x}\Big(\mathcal{A}(h_2'+a(h_1'-\chi')+e^g\mathcal{B}(h_1'-\chi'))\Big)\Big]\\
&-G_{12345}\Big[e^{i\theta_+}\Big((a^2-e^{2g}-1)\mathcal{A}+2e^ga\mathcal{B}\Big)+e^{g+6p+2x-i\theta_+}\tilde{A}'\\
&\quad+e^{g+6p+2x+i\theta_+}\Big(2\mathcal{A}\mathcal{A}'+3\tilde{A}'+2\mathcal{B}\mathcal{B}'+2x'+i\theta_+'-\phi'\Big)-i2e^{g+6p+x+i\theta_+}\mathcal{B}(h_2'+a(h_1'-\chi'))\\
&\quad+ie^{6p+x+i\theta_+}\Big(\mathcal{A}(-h_1'(a^2-1)-2ah_2'+e^{2g}(h_1'-\chi')+(a^2-1)\chi')\Big)\Big]\\
&+\frac{1}{2}G_{345}e^{-i\theta_+}\Big[-2e^{6p+x+i\theta_++\phi}P\mathcal{A}(1+a^2-2ab)+ie^{g+6p+2x+2i\theta_+}\mathcal{A}^2g'\\
&\quad+e^g\Big(-2e^{6p+x+i\theta_++\phi}Pa\mathcal{B}+e^{6p+x}(2e^{i\theta_++\phi}Pb\mathcal{B}+2ie^x\tilde{A}'\\
&\quad+e^{2i\theta_+}(-2\mathcal{B}h_2'-ie^x(6\tilde{A}'+2\mathcal{B}\mathcal{B}'-g'+\mathcal{B}^2g'+2x'+2i\theta_+'-2\phi')))\\
&\quad-2e^{2i\theta_+}a\mathcal{B}(i+e^{6p+x}(h_1'-\chi'))\Big)+2e^{2i\theta_+}\mathcal{A}(ie^{2g}-ie^{6p+2x}(\mathcal{B}a'+e^g\mathcal{A}')+e^{2g+6p+x}(h_1-\chi'))\Big]\\
&+\frac{1}{2}G_{125}e^{-i\theta_+}\Big[2e^{2g+6p+x+i\theta_++\phi}P\mathcal{A}-ie^{g+6p+2x+2i\theta_+}\mathcal{A}^2g'-e^g\Big(e^{6p+x}(-2e^{i\theta_++\phi}Pb\mathcal{B}-2ie^x\tilde{A}'\\
&\quad+e^{2i\theta_+}(2\mathcal{B}h_2'+ie^x(6\tilde{A}'+2\mathcal{B}\mathcal{B}'+g'-\mathcal{B}^2g'+2x'+2i\theta_p'-2\phi')))\\
&\quad+2e^{i\theta_+}a\mathcal{B}(e^{6p+x+\phi}P+e^{i\theta_p}(i+e^{6p+x}(h_1'-\chi')))\Big)\\
&\quad-2e^{2i\theta_+}\mathcal{A}\Big(-i+ia^2+e^{6p+x}(-ie^x(\mathcal{B}a'-e^g\mathcal{A}')+h_1')+e^{6p+x}(2ah_2'+a^2(h_1'-\chi')+\chi')\Big)\Big]\\
&+G_{145}\Big[e^{g+6p+x+\phi}P(\mathcal{A}(a-b)+e^g\mathcal{B})+e^{i\theta_+}Big(i(e^ga\mathcal{A}+\mathcal{B}-a^2\mathcal{B})\\
&\quad+ie^{6p+2x}((1+\mathcal{B}^2)a'+e^g(\mathcal{A}\mathcal{B}'-\mathcal{B}(\mathcal{A}'+\mathcal{A}g')))\\
&\quad+e^{6p+x}(e^g\mathcal{A}(h_2'+a(h_1'-\chi'))-\mathcal{B}((1+a^2)h_1'+2ah_2'-(a^2-1)\chi'))\Big)\Big]\Big\}.
\end{aligned}
$$

Appendix D

IR and UV expansions of analytic solutions

In this Appendix we show the IR and UV expansions for the whole space of linearized deformations around the Klebanov-Strassler solution, obained in chapter 3 and 4.

D.1 IR expansions

The IR behavior of the modes is obtained by Taylor expanding h, j and the integrands in the solutions for the $\tilde{\phi}^a$ modes shown in section 3.3.2, performing the indefinite integral over τ (instead of the integral from 1 to τ), and adding an integration constant Y_a^{IR} (since the conjugate momenta ξ_a do not involve integrals other than h and j, we do not have to introduce a second set of integration constants X^{IR} different from the one used in (3.55)-(3.61)).

The IR expansions of h and j are given by

$$\begin{aligned} h_{IR} &= h_0 - \frac{16}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}} P^2\tau^2 + \mathcal{O}(\tau^3)\,, \\ j_{IR} &= -\frac{1}{\tau}\left(\frac{3}{2}\right)^{\frac{2}{3}} + j_0 - \frac{1}{5}\left(\frac{2}{3}\right)^{\frac{1}{3}} \tau + \mathcal{O}(\tau^3)\,, \end{aligned} \tag{D.1}$$

where

$$h_0 = 18.2373P^2, \qquad j_0 = 0.836941\ . \tag{D.2}$$

In the order that those equations were solved and to the order of expansions that we need, the IR asymptotics of the $\tilde{\phi}^a$ modes are given by

$$\tilde{\phi}_8 = \frac{1}{\tau}\frac{32}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}} \left(-h_0X_1 + 3PX_6 + 9X_8\right) + Y_8^{IR} + \mathcal{O}(\tau)\,, \tag{D.3}$$

$$\tilde{\phi}_2 = \frac{1}{\tau} Y_2^{IR} + \frac{\log\tau}{\tau}\left(\frac{16}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}}(h_0 X_1 - 3(X_4 + 2PX_6))\right)$$
$$+ 8\left(\frac{2}{3}\right)^{\frac{1}{3}}(-6X_2 + 4X_3 + 6PX_5 + 9PX_7) + \mathcal{O}(\tau)\,, \tag{D.4}$$
$$\tilde{\phi}_3 = \frac{3Y_3^{IR}}{4\tau^3} + \frac{1}{\tau}\left(\frac{Y_2^{IR}}{2} - \frac{3Y_3^{IR}}{20} + \frac{4}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}} h_0 X_1 + 8\left(\frac{2}{3}\right)^{\frac{1}{3}} PX_6\right)$$
$$+ \frac{\log\tau}{\tau}\left(\frac{8}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}}(h_0X_1 - 3(X_4 + 2PX_6))\right) + \mathcal{O}(\tau)\,, \tag{D.5}$$
$$\tilde{\phi}_1 = -\frac{1}{\tau^3}\frac{Y_3^{IR}}{2} + \frac{1}{\tau}\Big(- 2Y_2^{IR} + \frac{Y_3^{IR}}{10} - \frac{4}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}}(4h_0X_1 - 3(5X_4 + 12PX_6))\Big)$$
$$+ \frac{\log\,\tau}{\tau}\Big(- \frac{32}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}}(h_0X_1 - 3(X_4 + 2PX_6))\Big) + Y_1^{IR}$$
$$+ \log\tau\left(\frac{40}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}}(-6X_2 + 4X_3 + 6PX_5 + 9PX_7)\right) + \mathcal{O}(\tau)\,, \tag{D.6}$$
$$\tilde{\phi}_5 = \frac{Y_6^{IR}}{2} + Y_7^{IR} + \tau^2\Bigg(- \frac{PY_2^{IR}}{2} - \frac{Y_6^{IR}}{8} + \frac{1}{36\,P}h_0^2X_1 - 4\Big(\frac{2}{3}\Big)^{\frac{1}{3}}PX_4$$
$$+ \frac{1}{6}\big(- 32\,2^{\frac{1}{3}}3^{\frac{2}{3}}P^2 + h_0\big)X_6 - 8\;2^{\frac{1}{3}}3^{\frac{2}{3}}PX_8\Bigg)$$
$$+ \tau^2\log\tau\left(-\frac{8}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}}P\,(h_0X_1 - 3(X_4 + 2PX_6))\right) + \mathcal{O}(\tau^3)\,, \tag{D.7}$$
$$\tilde{\phi}_6 = \frac{1}{\tau^2}\Big(- 2Y_6^{IR} + \frac{8}{3}\left(\frac{1}{6P}h_0^2X_1 + h_0X_6\right)\Big)$$
$$+ \Big(\frac{Y_6^{IR}}{6} + Y_7^{IR} - \frac{2PY_2^{IR}}{3} - \frac{128}{9}\left(\frac{2}{3}\right)^{\frac{1}{3}} h_0PX_1 + \frac{2}{27P}h_0^2X_1$$
$$+ 16\left(\frac{2}{3}\right)^{\frac{1}{3}}PX_4 + \left(-\frac{64}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}}P^2 + \frac{4}{9}h_0\right)X_6 - 32\left(\frac{2}{3}\right)^{\frac{1}{3}}PX_8\Big)$$
$$- \log\tau\Big(\frac{32}{9}\left(\frac{2}{3}\right)^{\frac{1}{3}}P\,(h_0X_1 - 3(X_4 + 2PX_6))\Big) + \mathcal{O}(\tau)\,, \tag{D.8}$$
$$\tilde{\phi}_7 = \frac{1}{\tau}\left(-Y_6^{IR} - \frac{2}{3}\left(\frac{1}{6P}h_0^2X_1 + h_0X_6\right)\right) + \tau\left(\frac{PY_2^{IR}}{3} + \frac{Y_6^{IR}}{6} + \frac{64}{9}\left(\frac{2}{3}\right)^{\frac{1}{3}}h_0PX_1\right.$$
$$\left. + \frac{1}{54P}h_0^2X_1 - \frac{8}{3}\left(\frac{2}{3}\right)^{1/3}PX_4 + \frac{1}{9}h_0X_6 - 16\left(\frac{2}{3}\right)^{\frac{1}{3}}PX_8\right)$$

$$
+\tau\log\tau\left(\frac{16}{9}\left(\frac{2}{3}\right)^{\frac{1}{3}}P\left(h_0X_1-3(X_4+2PX_6)\right)\right)+\mathcal{O}(\tau^2)\,, \tag{D.9}
$$

$$
\begin{aligned}
\tilde{\phi}_4 = &\frac{1}{\tau}\Big(\frac{8}{9}\frac{P}{h_0}\left(\frac{2}{3}\right)^{\frac{1}{3}}\left(-6PY_3^{IR}-18Y_6^{IR}-27Y_7^{IR}+\frac{7}{P}h_0^2X_1-12h_0X_6\right)\Big)\\
&+Y_4^{IR}+\mathcal{O}(\tau)\,.
\end{aligned} \tag{D.10}
$$

Note that the constant term in $\tilde{\phi}_2$ and the logarithmic term in $\tilde{\phi}_1$ are identically vanishing once we impose the zero-energy condition (6.11). We omit for simplicity the relation between the constants (X, Y^{IR}) used here and those that first appeared in [25]. We refer to [21] for more details.

D.2 UV expansions

The UV asymptotics of $h(\tau)$ and $j(\tau)$ are

$$
\begin{aligned}
h_{UV} &= 12\,2^{1/3}P^2(4\tau-1)e^{-4\tau/3}-\frac{128}{125}\,2^{1/3}P^2(12-85\tau+25\tau^2)e^{-10\tau/3}+\mathcal{O}(e^{-16\tau/3})\\
j_{UV} &= -\frac{3}{2^{2/3}}e^{-4\tau/3}-\frac{4}{25}\,2^{1/3}(3+10\tau)e^{-10\tau/3}+\mathcal{O}(e^{-16\tau/3})\,.
\end{aligned} \tag{D.11}
$$

The UV expansions for the fields $\tilde{\phi}_a$ are obtained by performing an indefinite integration of the UV series of the integrands as in the IR case. We call Y_a^{UV} the 0th-order term in the expansion for the field $\tilde{\phi}_a$ (or Λ_a if the former is written as a product of the homogeneous solution times Λ_a)

$$
\tilde{\phi}_8 = Y_8^{UV}+12\cdot 2^{1/3}\,e^{-4\tau/3}\Big(P(-1+4\tau)(2X_5+X_7)+8X_8\Big)+\mathcal{O}(e^{-8\tau/3})\,, \tag{D.12}
$$

$$
\begin{aligned}
\tilde{\phi}_2 = &-8\cdot 2^{1/3}\,e^{-\tau/3}\Big(6X_2+(6-4\tau)X_3+2X_4+9PX_7-6P\tau X_7\Big)+2\,e^{-\tau}Y_2^{UV}\\
&+\mathcal{O}(e^{-7\tau/3})\,,
\end{aligned} \tag{D.13}
$$

$$
\begin{aligned}
\tilde{\phi}_3 = &-5\cdot 2^{1/3}\,X_3\,e^{2\tau/3}-\frac{4}{3}\cdot 2^{1/3}e^{-4\tau/3}\Big(108X_2+(336-137\tau)X_3+48X_4\\
&-108P(-3+\tau)X_7\Big)+\mathcal{O}(e^{-2\tau})\,,
\end{aligned} \tag{D.14}
$$

$$
\begin{aligned}
\tilde{\phi}_1 = &\,Y_1^{UV}-10\cdot 2^{1/3}\,X_3\,e^{2\tau/3}+\frac{2}{3}\cdot 2^{1/3}e^{-4\tau/3}\Big(324X_2+(528-316\tau)X_3+114X_4\\
&+81P(7-4\tau)X_7\Big)+\mathcal{O}(e^{-2\tau})\,,
\end{aligned} \tag{D.15}
$$

$$
\begin{aligned}
\tilde{\phi}_5 = &-\frac{Y_5^{UV}}{2}\,e^{\tau}-Y_5^{UV}+Y_7^{UV}+\tau(2Y_5^{UV}-PY_8^{UV})\\
&+6\cdot 2^{1/3}\,e^{-\tau/3}P\Big(6X_2+(21-4\tau)X_3+2X_4+21PX_7\Big)\\
&+\frac{1}{2}e^{-\tau}\Big((5-4\tau)Y_5^{UV}+4Y_6^{UV}-2P(-1+2\tau)(Y_2^{UV}-Y_8^{UV})\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+ 12 \cdot 2^{1/3} e^{-4\tau/3} P\Big(- 12(1+\tau) X_2 - 15 X_3 - 4 X_4 + 2\tau (X_3 + 4\tau X_3 - 2X_4 + 6PX_5) \\
&+ 3P(-3+\tau+4\tau^2) X_7 + 6(PX_5 + X_8)\Big) + \mathcal{O}(e^{-2\tau})\,, \qquad \text{(D.16)}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\phi}_6 = {}& \frac{Y_5^{UV}}{2}\, e^{\tau} - Y_5^{UV} + Y_7^{UV} + \tau(2Y_5^{UV} - PY_8^{UV}) \\
&- 6 \cdot 2^{1/3}\, e^{-\tau/3} P\Big(6X_2 + (21-4\tau)X_3 + 2X_4 + 21PX_7\Big) \\
&+ \frac{1}{2} e^{-\tau}\Big((-5+4\tau)Y_5^{UV} - 4Y_6^{UV} + 2P(-1+2\tau)(Y_2^{UV} - Y_8^{UV})\Big) \\
&+ 12 \cdot 2^{1/3} e^{-4\tau/3} P\Big(- 12(1+\tau) X_2 - 15 X_3 - 4 X_4 + 2\tau (X_3 + 4\tau X_3 - 2X_4 + 6PX_5) \\
&+ 3P(-3+\tau+4\tau^2) X_7 + 6(PX_5 + X_8)\Big) + \mathcal{O}(e^{-2\tau})\,, \qquad \text{(D.17)}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\phi}_7 = {}& -\frac{Y_5^{UV}}{2}\, e^{\tau} + 18 \cdot 2^{1/3} e^{-\tau/3} P\Big(- 6X_2 + (-9+4\tau)X_3 - 2(X_4 + P(5-2\tau)X_7)\Big) \\
&+ e^{-\tau}\Big((-\frac{1}{2} + 2\tau)Y_5^{UV} - 2Y_6^{UV} + P(Y_2^{UV} + 2\tau Y_2^{UV} - Y_8^{UV})\Big) + \mathcal{O}(e^{-7\tau/3})\,, \\
& \qquad \text{(D.18)}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\phi}_4 = {}& \frac{Y_4^{UV}}{12 \cdot 2^{1/3}(4\tau - 1)} e^{4\tau/3} - \frac{8 \cdot 2^{1/3}(2\tau+1)\, X_3}{4\tau - 1} e^{2\tau/3} + \frac{2Y_1^{UV}}{5} - \frac{Y_5^{UV}}{P} + \frac{Y_8^{UV}}{2} \\
&- \frac{2Y_7^{UV}}{P(4\tau-1)} + \frac{4 \cdot 2^{2/3}(12 - 85\tau + 25\tau^2)Y_4^{UV}}{1125(4\tau-1)^2} e^{-2\tau/3} + \frac{2^{1/3}}{(4\tau-1)} e^{-4\tau/3}\Big(18(7+8\tau)X_2 \\
&+ 32(2\tau+1)X_4 - 18P(7+8\tau)X_5 - 9P(23+8\tau+32\tau^2)X_7 - 72X_8 \\
&+ \frac{40803 - 170884\tau + 161120\tau^2 - 332800\tau^3)X_3}{375(4\tau - 1)}\Big) + \mathcal{O}(e^{-2\tau})\,. \qquad \text{(D.19)}
\end{aligned}
$$

Appendix E

Analytic non-supersymmetric M-theory solutions

Here we show the analytic solutions for the modes $\tilde{\xi}_a$ which can be obtained by explicitly performing the integrations that appear in (6.35)

$$
\begin{aligned}
\tilde{\xi}_4 &= m^2 X_4 H(y)\,, \qquad\qquad (\text{E.1})\\
\tilde{\xi}_1 &= 2m^2 X_4 H(y) + X_1\,,\\
\tilde{\xi}_5 &= -2\sqrt{2}(y^4-3)^{-1}(y^4-1)^{-1/2} L_5(y) - 2^{-1/2}(y^4-3)^{-1}(y^4-1)^{3/2} L_6(y)\,,\\
\tilde{\xi}_6 &= 4\sqrt{2}(y^4-3)^{-1}(y^4-1)^{-3/2}(3y^4-5)L_5(y) - \sqrt{2}(y^4-3)^{-1}(y^4-7)(y^4-1)^{1/2} L_6(y)\,,\\
\tilde{\xi}_3 &= -\frac{3}{2} y^4 (y^4-3)^2 L_3(y)\,,\\
\tilde{\xi}_2 &= \frac{3}{4} X_1 (y^4-1) + \frac{1}{2} X_2 (y^8 - 4y^4 + 3)^{1/2} + \frac{9}{8}(y^4-3)^2 (y^4-1) L_3(y) + \frac{16}{3\sqrt{3}}(y^4-1)^{-2} L_5(y)\\
&\quad - \frac{4}{3\sqrt{3}}(y^4-4) L_6(y) + m^2 X_4 \Big(\frac{3}{2}(y^4-1) H(y) - 2\sqrt{2} y^{-3} (y^4-1)^{-3/2} (y^4-3)\Big)\,,
\end{aligned}
$$

where

$$
\begin{aligned}
L_5(y) &= X_5 + m^2 X_4 G(y)\,, \qquad\qquad (\text{E.2})\\
L_6(y) &= X_6 - m^2 X_4 \Big(G(y) + \frac{\sqrt{3}}{2} H(y)\Big)\\
L_3(y) &= X_3 + m^2 X_4 \Big(\frac{16\sqrt{2}(2y^4-3)}{27 y^3 (y^4-3)(y^4-1)^{3/2}} + \frac{22 G(y)}{27\sqrt{3}} - \frac{13 H(y)}{27}\Big)\,. \qquad\qquad (\text{E.3})
\end{aligned}
$$

We recall that the variable y is defined as

$$
y = (2 + \cosh(2r))^{1/4} \qquad\qquad (\text{E.4})
$$

and the expression for the warp factor $H(y)$ and the Green's function $G(y)$ are given in (6.16) and (6.37).

We now show the expanded form of the solution for the modes $\tilde{\phi}_a$ in terms of the variable y, obtained by replacing the analytic solutions for the modes $\tilde{\xi}_a$ in (6.39). We impose the zero energy condition, so we put $X_2 = 0$.

$$\begin{aligned}
\tilde{\phi}_1 = \frac{1}{(3-4y^4+y^8)^{1/2}}\Bigg[& Y_1 - \frac{9yX_1}{\sqrt{2}(y^4-3)^{1/2}} - \frac{27}{\sqrt{2}}X_3 y\sqrt{y^4-3} + \frac{4\sqrt{2}X_5 y(11-5y^4)}{(y^4-1)\sqrt{3}(y^4-3)^{1/2}} \\
& - \frac{8\sqrt{2}X_6 y}{\sqrt{3}(y^4-3)^{1/2}} - \frac{1}{\sqrt{2}\,3^{3/4}}F\Big(\arcsin(3^{1/4}y^{-1})|-1\Big)\Big(45\sqrt{3}X_1 + 162\sqrt{3}X_3 - 40X_5 - 112X_6\Big) \\
& + \frac{1}{2}m^2 X_4\Bigg(\int^y \frac{3\sqrt{2}(-19-26u^4+13u^8)H(u)}{(u^4-3)^{3/2}}du - \int^y \frac{2\sqrt{6}\sqrt{u^4-3}(11-38u^4+11u^8)G(u)}{(u^4-1)^2}du \\
& - \frac{48y^2(y^4-3)}{(y^8-4y^4+3)^{1/2}} + 48\sqrt{3}E\Big(\arcsin(y^2)|\frac{1}{3}\Big) - 32\sqrt{3}F\Big(\arcsin(y^2)|\frac{1}{3}\Big)\Bigg)\Bigg],
\end{aligned} \tag{E.5}$$

$$\begin{aligned}
\tilde{\phi}_2 = -\frac{3\tilde{\phi}_1}{y^4} + \frac{4}{y^4(y^4-3)^2}\Bigg[& Y_2 + \frac{9X_1 y\sqrt{y^4-1}}{\sqrt{2}} + \frac{9X_3 y\sqrt{y^4-1}}{140\sqrt{2}}(825-639y^4+343y^8-49y^{12}) \\
& + \frac{4\sqrt{6}X_5 y(5y^4-7)}{(y^4-1)^{3/2}} + \frac{4}{7}\sqrt{2}\Big(63X_1 + 432X_3 - 7\sqrt{3}(5X_5+11X_6)\Big)F\Big(\arcsin(y^{-1})|-1\Big) \\
& - 4\sqrt{6}X_6 y\sqrt{y^4-1} + 3m^2X_4\Bigg(\int^y \frac{(-171-342u^4+936u^8-546u^{12}+91u^{16})H(u)}{12\sqrt{2}(u^4-1)^{1/2}}du \\
& - \int^y \frac{(u^4-3)^2(99-342u^4+176u^8-154u^{12}+77u^{16})G(u)}{6\sqrt{6}(u^4-1)^{5/2}}du \\
& + \frac{2y^2(138-119y^4+14y^8)}{9(1-y^4)} + \frac{16}{3}\operatorname{arccoth} y^2\Bigg)\Bigg],
\end{aligned} \tag{E.6}$$

$$\begin{aligned}
\tilde{\phi}_3 = Y_3 & - \frac{3}{8}(y^4-2)\,\tilde{\phi}_1 - \frac{3}{8}\,\tilde{\phi}_2 + \frac{27X_1 y\sqrt{y^4-1}}{8\sqrt{2}(y^4-3)} + \frac{27X_3 y\sqrt{y^4-1}}{4\sqrt{2}} + \frac{\sqrt{3}X_5 y(5y^4-7)}{(y^4-1)^{3/2}} \\
& + \frac{1}{8\sqrt{2}}\Big(135X_1 + 432X_3 - 8\sqrt{3}(5X_5+12X_6)\Big)F\Big(\arcsin(y^{-1})|-1\Big) \\
& + \frac{1}{4}\Big(7\sqrt{3}X_1 - 4(X_5+X_6)\Big)G(y) + 3m^2X_4\Bigg[\int^y \frac{(-51+9u^4+39u^8-13u^{12})H(u)}{4\sqrt{2}(u^4-3)^{3/2}(3-4u^4+u^8)^{1/2}}du \\
+ \int^y & \frac{(-33+125u^4-79u^8+11u^{12})G(u)}{2\sqrt{6}(u^4-1)^{5/2}}du + \frac{5y^2}{4-4y^4} - \frac{\sqrt{3}}{4}\operatorname{arctanh}\left(\frac{y^2}{\sqrt{3}}\right) + \log\left(\frac{1+y^2}{1-y^2}\right)\Bigg].
\end{aligned} \tag{E.7}$$

For the flux perturbation we have

$$\begin{pmatrix}\tilde{\phi}_5\\ \\ \tilde{\phi}_6\end{pmatrix}=\begin{pmatrix}\frac{1}{2\sqrt{2}}(y^4-1)^{-3/2}(y^4-3)^3 & -\frac{1}{2\sqrt{2}}(y^4-7)(y^4-1)^{1/2}\\ \\ -\frac{1}{4\sqrt{2}}(y^4-1)^{-1/2}(y^4-3)(y^4+1) & \frac{1}{4\sqrt{2}}(y^4-1)^{3/2}\end{pmatrix}\begin{pmatrix}\Lambda_5\\ \\ \Lambda_6\end{pmatrix} \tag{E.8}$$

where

$$\begin{aligned}\Lambda_5 &= Y_5+\frac{4-y^4}{6\sqrt{3}}\,\tilde{\phi}_1+\frac{3\sqrt{3}X_1y(7-5y^4)}{5\sqrt{2}(y^4-1)^{3/2}}-\frac{3\sqrt{3}X_3y(-72+45y^4+5y^8)}{10\sqrt{2}(y^4-1)^{3/2}} \\
&-\frac{10\sqrt{2}X_5y}{9(y^4-3)(y^4-1)^{3/2}}+\frac{2\sqrt{2}X_6y(272-279y^4+60y^8)}{45(y^4-3)(y^4-1)^{3/2}}-\frac{1}{108}\Big(27X_1+44\sqrt{3}(X_5+X_6)\Big)G(y) \\
&+\frac{1}{180}\Big(-54\sqrt{3}X_1-297\sqrt{3}X_3+40X_5+106X_6-30X_6y^4+\frac{120(X_5+X_6)}{(y^4-3)^2}\Big)H(y) \\
&+m^2X_4\Bigg[\int^y H(u)\left(\frac{76+85u^4-78u^8+13u^{12}}{2\sqrt{6}(u^4-3)^{3/2}(u^8-4u^4+3)^{1/2}}+\frac{2}{3}y^3G(y)+\frac{y^3}{\sqrt{3}}H(y)+\frac{8y^3H(y)}{\sqrt{3}(y^4-3)^3}\right)du \\
&+\int^y\frac{(44-163u^4+82u^8-11u^{12})G(u)}{3\sqrt{2}(u^4-1)^{5/2}}du+\frac{2\sqrt{3}y^2}{y^4-1}-\frac{2}{3}\text{arctanh}\left(\frac{y^2}{\sqrt{2}}\right)-\frac{2}{\sqrt{3}}\log\left(\frac{1+y^2}{1-y^2}\right)\Bigg], \\
\Lambda_6 &= Y_6-\frac{y^4(-3+y^4)^2}{6\sqrt{3}(-1+y^4)^2}\,\tilde{\phi}_1+\frac{3\sqrt{3}X_1y(7-5y^4)}{5\sqrt{2}(y^4-1)^{3/2}}+\frac{3\sqrt{3}X_3y(72-45y^4-5y^8)}{10\sqrt{2}(y^4-1)^{3/2}}+\frac{2\sqrt{2}X_5y(y^4-3)}{3(y^4-1)^{7/2}} \\
&+\frac{2\sqrt{2}X_6y(20y^4-33)}{15(y^4-1)^{3/2}}+H(y)\Big(-\frac{7\sqrt{3}X_1}{40}-\frac{9\sqrt{3}X_3}{10}+\frac{X_6(11-5y^4)}{30}+\frac{X_5(1+6y^4-3y^8)}{6(y^4-1)^2}\Big) \\
&+m^2X_4\Bigg[\int^y H(u)\left(\frac{y^4(-19-26y^4+13y^8)}{2\sqrt{6}(y^4-1)^{5/2}}+\frac{2y^3(y^4-3)(y^8+3)G(y)}{3(y^4-1)^3}+\frac{y^3H(y)}{\sqrt{3}}\right) \\
&-\int^y\frac{u^4(u^4-3)^2(11-38u^4+11u^8)G(u)}{3\sqrt{2}(u^4-1)^{9/2}}+\frac{2y^3(-3-4y^4+3y^8)}{3\sqrt{3}(y^4-1)^3}+\frac{1}{\sqrt{3}}\log\left(\frac{y^2-1}{-1-y^2}\right)\Bigg].\end{aligned} \tag{E.9}$$

Appendix F

Brane/antibrane potential

In this Appendix we review the calculation of the force on a probe antibrane in the Stenzel background which has been performed in [19]. We consider a stack of M2 branes at a position $r = r_0$ in the transverse geometry and we want to compute the force exerted on a probe anti–M2 brane placed at the tip $r = 0$, due to the backreaction of the M2 branes. To compute the full backreacted geometry we only need to add a harmonic function $\delta H(r)$ to the background warp factor $H_0(r)$ [90]. This function is given by the Green's function on the warped Stenzel space [151] and since we are considering smeared branes, we only need to solve the radially symmetric Laplace equation. The Laplacian is given by

$$\Delta\delta H = \frac{1}{\sqrt{G}}\frac{\partial}{\partial x^l}\left(\sqrt{G}g^{lm}\frac{\partial\delta H}{\partial x^m}\right) , \tag{F.1}$$

where we are labeling the eleven dimensional coordinates by x_l $(l, m = 0, \ldots, 10)$ and $G = \det g_{lm}$. From (6.1) we easily get

$$\sqrt{G} = -e^{z+3\alpha+3\beta+2\gamma} , \qquad g^{rr} = e^{-z-2\gamma} , \tag{F.2}$$

and so imposing $\Delta\delta H = 0$ we get the following equation for $\delta H'(r)$

$$e^{3(\alpha_0+\beta_0)}\delta H'(r) = \text{const} . \tag{F.3}$$

So the two solutions of the Laplace equation are, using (6.15)

$$H_1 = d_1 \tag{F.4}$$

$$H_2 = d_2 \int^r \frac{\operatorname{csch}^3 u}{(2+\cosh 2u)^{3/4}} dr , \tag{F.5}$$

and we should set $\delta H = H_1$ for $r < r_0$ and $\delta H = H_2$ for $r > r_0$. The constant d_1 is then fixed from the matching condition $d_1 = H_2(r_0)$ and the constant d_2 is related to the number of M2 branes from (6.62). In fact, we have

$$N = \frac{1}{(2\pi l_p)^6}\int_{\delta\mathcal{M}} \star_{11} G_4 = \frac{2^{11}\, m^2\, \mathrm{Vol}_{V_{5,2}}}{3^4\,(2\pi l_p)^6}\, H_2'\, e^{3(\alpha_0+\beta_0)} , \tag{F.6}$$

where $\delta\mathcal{M}$ is a small shell around r_0. If we use H_2 given by (F.5), the above equation thus fixes d_2 in terms of N

$$d_2 = \frac{324\,\pi^2\, l_p^{-6}\, N}{m^2}\,. \tag{F.7}$$

We now compute the force exerted on the probe anti–M2 brane by looking at the variation in the potential when we move the stack of M2 branes away from $r = r_0$. For anti–M2 branes, $V_{DBI} = V_{WZ}$ and since we have

$$V_{DBI} \sim (g_{00}g_{11}g_{22})^{1/2} = e^{-3z} = \frac{1}{m^2\,H}\,, \tag{F.8}$$

the potential is just proportional to $2H^{-1}$. Expanding this we get

$$V = \frac{2}{m^2\,H} \approx \frac{2}{m^2 H_0}\left(1 - \frac{\delta H}{H_0}\right)\,, \tag{F.9}$$

and so we easily get the force:

$$\begin{aligned} F_{M2} &= -\frac{2}{m^2}\frac{\partial V}{\partial r_0}\Big|_{r=0} \\ &= \frac{1}{m^2\,H_0^2}\frac{2\,d_2\,\mathrm{csch}^3 r}{(2+\cosh 2r)^{3/4}}\,. \end{aligned} \tag{F.10}$$

This result agrees with the computation of the force exerted on a probe M2 brane due to the backreaction of a stack of anti–M2 branes (6.97).

Bibliography

[1] M. Aganagic, C. Beem, J. Seo, and C. Vafa, "Geometrically Induced Metastability and Holography," *Nucl.Phys.* **B789** (2008) 382–412, `arXiv:hep-th/0610249 [hep-th]`.

[2] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, "Large N field theories, string theory and gravity," *Phys.Rept.* **323** (2000) 183–386, `arXiv:hep-th/9905111 [hep-th]`.

[3] O. Aharony, A. Hashimoto, S. Hirano, and P. Ouyang, "D-brane Charges in Gravitational Duals of 2+1 Dimensional Gauge Theories and Duality Cascades," *JHEP* **1001** (2010) 072, `arXiv:0906.2390 [hep-th]`.

[4] O. Aharony, E. Schreiber, and J. Sonnenschein, "Stable nonsupersymmetric supergravity solutions from deformations of the Maldacena-Nunez background," *JHEP* **0204** (2002) 011, `arXiv:hep-th/0201224 [hep-th]`.

[5] D. Andriot, E. Goi, R. Minasian, and M. Petrini, "Supersymmetry breaking branes on solvmanifolds and de Sitter vacua in string theory," *JHEP* **1105** (2011) 028, `arXiv:1003.3774 [hep-th]`.

[6] C. Angelantonj and E. Dudas, "Metastable string vacua," *Phys.Lett.* **B651** (2007) 239–245, `arXiv:0704.2553 [hep-th]`.

[7] R. Apreda, "Nonsupersymmetric regular solutions from wrapped and fractional branes," `arXiv:hep-th/0301118 [hep-th]`.

[8] R. Argurio, M. Bertolini, S. Franco, and S. Kachru, "Gauge/gravity duality and meta-stable dynamical supersymmetry breaking," *JHEP* **0701** (2007) 083, `arXiv:hep-th/0610212 [hep-th]`.

[9] R. Argurio, M. Bertolini, S. Franco, and S. Kachru, "Meta-stable vacua and D-branes at the conifold," *JHEP* **0706** (2007) 017, `arXiv:hep-th/0703236 [hep-th]`.

[10] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, "Systematics of moduli stabilisation in Calabi-Yau flux compactifications," *JHEP* **0503** (2005) 007, `arXiv:hep-th/0502058 [hep-th]`.

[11] V. Balasubramanian, J. de Boer, S. El-Showk, and I. Messamah, “Black Holes as Effective Geometries,” *Class.Quant.Grav.* **25** (2008) 214004, `arXiv:0811.0263 [hep-th]`.

[12] D. Baumann, A. Dymarsky, S. Kachru, I. R. Klebanov, and L. McAllister, “D3-brane Potentials from Fluxes in AdS/CFT,” *JHEP* **1006** (2010) 072, `arXiv:1001.5028 [hep-th]`.

[13] D. Baumann, A. Dymarsky, I. R. Klebanov, J. M. Maldacena, L. P. McAllister, *et al.*, “On D3-brane Potentials in Compactifications with Fluxes and Wrapped D-branes,” *JHEP* **0611** (2006) 031, `arXiv:hep-th/0607050 [hep-th]`.

[14] D. Baumann and L. McAllister, “Advances in Inflation in String Theory,” *Ann.Rev.Nucl.Part.Sci.* **59** (2009) 67–94, `arXiv:0901.0265 [hep-th]`.

[15] I. Bena, J. Blaback, U. Danielsson, and T. Van Riet, “Antibranes don’t go black,” `arXiv:1301.7071 [hep-th]`.

[16] I. Bena, E. Gorbatov, S.Hellerman, N. Seiberg, and D. Shih, “A Note on (Meta)stable Brane Configurations in MQCD.” Jhep 0611:088,2006, 2006.

[17] I. Bena and C. Ciocarlie, “Exact N=2 supergravity solutions with polarized branes,” *Phys.Rev.* **D70** (2004) 086005, `arXiv:hep-th/0212252 [hep-th]`.

[18] I. Bena, G. Dall’Agata, S. Giusto, C. Ruef, and N. P. Warner, “Non-BPS Black Rings and Black Holes in Taub-NUT,” *JHEP* **0906** (2009) 015, `arXiv:0902.4526 [hep-th]`.

[19] I. Bena, G. Giecold, M. Grana, and N. Halmagyi, “On The Inflaton Potential From Antibranes in Warped Throats,” *JHEP* **1207** (2012) 140, `arXiv:1011.2626 [hep-th]`.

[20] I. Bena, G. Giecold, M. Grana, N. Halmagyi, and S. Massai, “On Metastable Vacua and the Warped Deformed Conifold: Analytic Results,” `arXiv:1102.2403 [hep-th]`. [1102.2403].

[21] I. Bena, G. Giecold, M. Grana, N. Halmagyi, and S. Massai, “The backreaction of anti-D3 branes on the Klebanov-Strassler geometry,” `arXiv:1106.6165 [hep-th]`.

[22] I. Bena, G. Giecold, M. Graña, N. Halmagyi, and F. Orsi, “Supersymmetric Consistent Truncations of IIB on T(1,1),” 2010.

[23] I. Bena, G. Giecold, M. Grana, N. Halmagyi, and F. Orsi, “Supersymmetric Consistent Truncations of IIB on $T^{1,1}$,” *JHEP* **1104** (2011) 021, `arXiv:1008.0983 [hep-th]`.

[24] I. Bena, G. Giecold, and N. Halmagyi, "The Backreaction of Anti-M2 Branes on a Warped Stenzel Space," *JHEP* **1104** (2011) 120, arXiv:1011.2195 [hep-th].

[25] I. Bena, M. Grana, and N. Halmagyi, "On the Existence of Meta-stable Vacua in Klebanov-Strassler," *JHEP* **1009** (2010) 087, arXiv:0912.3519 [hep-th].

[26] I. Bena, M. Grana, S. Kuperstein, and S. Massai, "Anti-D3's - Singular to the Bitter End," arXiv:1206.6369 [hep-th].

[27] I. Bena, M. Grana, S. Kuperstein, and S. Massai, "Polchinski-Strassler does not uplift Klebanov-Strassler," arXiv:1212.4828 [hep-th].

[28] I. Bena, D. Junghans, S. Kuperstein, T. Van Riet, T. Wrase, *et al.*, "Persistent anti-brane singularities," arXiv:1205.1798 [hep-th].

[29] I. Bena, A. Puhm, and B. Vercnocke, "Metastable Supertubes and non-extremal Black Hole Microstates," *JHEP* **1204** (2012) 100, arXiv:1109.5180 [hep-th].

[30] I. Bena, A. Puhm, and B. Vercnocke, "Non-extremal Black Hole Microstates: Fuzzballs of Fire or Fuzzballs of Fuzz ?," *JHEP* **1212** (2012) 014, arXiv:1208.3468 [hep-th].

[31] I. Bena and N. P. Warner, "A Harmonic family of dielectric flow solutions with maximal supersymmetry," *JHEP* **0412** (2004) 021, arXiv:hep-th/0406145 [hep-th].

[32] I. Bena and N. P. Warner, "One ring to rule them all ... and in the darkness bind them?," *Adv.Theor.Math.Phys.* **9** (2005) 667–701, arXiv:hep-th/0408106 [hep-th].

[33] A. Bergman and C. P. Herzog, "The Volume of some nonspherical horizons and the AdS / CFT correspondence," *JHEP* **0201** (2002) 030, arXiv:hep-th/0108020 [hep-th].

[34] M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda, R. Marotta, *et al.*, "Fractional D-branes and their gauge duals," *JHEP* **0102** (2001) 014, arXiv:hep-th/0011077 [hep-th].

[35] M. Bertolini, "Four lectures on the gauge-gravity correspondence," *Int. J. Mod. Phys.* **A18** (2003) 5647–5712, hep-th/0303160. http://arXiv.org/abs/hep-th/0303160. [hep-th/0303160].

[36] F. Bigazzi, A. Cotrone, M. Petrini, and A. Zaffaroni, "Supergravity duals of supersymmetric four-dimensional gauge theories," *Riv.Nuovo Cim.* **25N12** (2002) 1–70, arXiv:hep-th/0303191 [hep-th].

[37] J. Blaback, U. H. Danielsson, D. Junghans, T. Van Riet, T. Wrase, *et al.*, "Smeared versus localised sources in flux compactifications," *JHEP* **1012** (2010) 043, `arXiv:1009.1877 [hep-th]`.

[38] J. Blaback, U. H. Danielsson, D. Junghans, T. Van Riet, T. Wrase, *et al.*, "The problematic backreaction of SUSY-breaking branes," *JHEP* **1108** (2011) 105, `arXiv:1105.4879 [hep-th]`.

[39] J. Blaback, U. H. Danielsson, D. Junghans, T. Van Riet, T. Wrase, *et al.*, "(Anti-)Brane backreaction beyond perturbation theory," *JHEP* **1202** (2012) 025, `arXiv:1111.2605 [hep-th]`. 12 pages + appendices, 1 figure.

[40] J. Blaback, U. H. Danielsson, and T. Van Riet, "Resolving anti-brane singularities through time-dependence," `arXiv:1202.1132 [hep-th]`.

[41] V. Borokhov and S. S. Gubser, "Nonsupersymmetric deformations of the dual of a confining gauge theory," *JHEP* **0305** (2003) 034, `arXiv:hep-th/0206098 [hep-th]`.

[42] R. Bousso and J. Polchinski, "Quantization of four form fluxes and dynamical neutralization of the cosmological constant," *JHEP* **0006** (2000) 006, `arXiv:hep-th/0004134 [hep-th]`.

[43] M. Buican, D. Malyshev, and H. Verlinde, "On the geometry of metastable supersymmetry breaking," *JHEP* **0806** (2008) 108, `arXiv:0710.5519 [hep-th]`.

[44] C. Burgess and L. McAllister, "Challenges for String Cosmology," *Class.Quant.Grav.* **28** (2011) 204002, `arXiv:1108.2660 [hep-th]`.

[45] A. Butti, M. Grana, R. Minasian, M. Petrini, and A. Zaffaroni, "The baryonic branch of Klebanov-Strassler solution: A supersymmetric family of SU(3) structure backgrounds," *JHEP* **03** (2005) 069, `arXiv:hep-th/0412187`. [`hep-th/0412187`].

[46] P. G. Camara and M. Grana, "No-scale supersymmetry breaking vacua and soft terms with torsion," *JHEP* **0802** (2008) 017, `arXiv:0710.4577 [hep-th]`.

[47] P. Candelas and X. C. de la Ossa, "Comments on Conifolds," *Nucl.Phys.* **B342** (1990) 246–268.

[48] D. Cassani, "Reducing democratic type II supergravity on SU(3) x SU(3) structures," *JHEP* **0806** (2008) 027, `arXiv:0804.0595 [hep-th]`.

[49] D. Cassani, G. Dall'Agata, and A. F. Faedo, "BPS domain walls in N=4 supergravity and dual flows," `arXiv:1210.8125 [hep-th]`.

[50] A. Ceresole and G. Dall'Agata, "Flow Equations for Non-BPS Extremal Black Holes," *JHEP* **0703** (2007) 110, arXiv:hep-th/0702088 [hep-th].

[51] A. H. Chamseddine and M. S. Volkov, "NonAbelian BPS monopoles in N=4 gauged supergravity," *Phys.Rev.Lett.* **79** (1997) 3343–3346, arXiv:hep-th/9707176 [hep-th].

[52] M. Cicoli and F. Quevedo, "String moduli inflation: An overview," *Class.Quant.Grav.* **28** (2011) 204001, arXiv:1108.2659 [hep-th].

[53] J. P. Conlon, "Brane-Antibrane Backreaction in Axion Monodromy Inflation," *JCAP* **1201** (2012) 033, arXiv:1110.6454 [hep-th].

[54] M. Cvetic, G. Gibbons, H. Lu, and C. Pope, "Ricci flat metrics, harmonic forms and brane resolutions," *Commun.Math.Phys.* **232** (2003) 457–500, arXiv:hep-th/0012011 [hep-th].

[55] J. de Boer, E. P. Verlinde, and H. L. Verlinde, "On the holographic renormalization group," *JHEP* **0008** (2000) 003.

[56] F. Denef, "Supergravity flows and D-brane stability," *JHEP* **0008** (2000) 050, arXiv:hep-th/0005049 [hep-th].

[57] O. DeWolfe, D. Freedman, S. Gubser, and A. Karch, "Modeling the fifth-dimension with scalars and gravity," *Phys.Rev.* **D62** (2000) 046008, arXiv:hep-th/9909134 [hep-th].

[58] O. DeWolfe, S. Kachru, and M. Mulligan, "A Gravity Dual of Metastable Dynamical Supersymmetry Breaking," *Phys.Rev.* **D77** (2008) 065011, arXiv:0801.1520 [hep-th].

[59] O. DeWolfe, S. Kachru, and H. L. Verlinde, "The Giant inflaton," *JHEP* **0405** (2004) 017, arXiv:hep-th/0403123 [hep-th].

[60] P. Di Vecchia, A. Lerda, and P. Merlatti, "N=1 and N=2 superYang-Mills theories from wrapped branes," *Nucl.Phys.* **B646** (2002) 43–68, arXiv:hep-th/0205204 [hep-th].

[61] M. R. Douglas and S. Kachru, "Flux compactification," *Rev.Mod.Phys.* **79** (2007) 733–796, arXiv:hep-th/0610102 [hep-th].

[62] M. R. Douglas and G. W. Moore, "D-branes, Quivers, and ALE Instantons," hep-th/9603167. http://arXiv.org/abs/hep-th/9603167. [hep-th/9603167].

[63] E. Dudas, J. Mourad, and F. Nitti, "Metastable vacua in brane worlds," *JHEP* **0708** (2007) 057, arXiv:0706.1269 [hep-th].

[64] E. Dudas, J. Mourad, and A. Sagnotti, "Charged and uncharged D-branes in various string theories," *Nucl.Phys.* **B620** (2002) 109–151, `arXiv:hep-th/0107081 [hep-th]`.

[65] E. Dudas, G. Pradisi, M. Nicolosi, and A. Sagnotti, "On tadpoles and vacuum redefinitions in string theory," *Nucl.Phys.* **B708** (2005) 3–44, `arXiv:hep-th/0410101 [hep-th]`.

[66] G. Dvali and S. H. Tye, "Brane inflation," *Phys.Lett.* **B450** (1999) 72–82, `arXiv:hep-ph/9812483 [hep-ph]`.

[67] A. Dymarsky, "On gravity dual of a metastable vacuum in Klebanov-Strassler theory," *JHEP* **1105** (2011) 053, `arXiv:1102.1734 [hep-th]`.

[68] A. Dymarsky, I. R. Klebanov, and N. Seiberg, "On the Moduli Space of the Cascading SU(M+p)xSU(p) Gauge Theory." Jhep 0601:155,2006, 2005.

[69] N. J. Evans, M. Petrini, and A. Zaffaroni, "The Gravity dual of softly broken N=1 superYang-Mills," *JHEP* **0206** (2002) 004, `arXiv:hep-th/0203203 [hep-th]`.

[70] S. Ferrara, G. W. Gibbons, and R. Kallosh, "Black holes and critical points in moduli space," *Nucl.Phys.* **B500** (1997) 75–93, `arXiv:hep-th/9702103 [hep-th]`.

[71] R. Flauger, L. McAllister, E. Pajer, A. Westphal, and G. Xu, "Oscillations in the CMB from Axion Monodromy Inflation," *JCAP* **1006** (2010) 009, `arXiv:0907.2916 [hep-th]`.

[72] S. Franco and A. M. . Uranga, "Dynamical SUSY breaking at meta-stable minima from D-branes at obstructed geometries," *JHEP* **0606** (2006) 031, `arXiv:hep-th/0604136 [hep-th]`.

[73] D. Z. Freedman and J. A. Minahan, "Finite temperature effects in the supergravity dual of the N=1* gauge theory," *JHEP* **0101** (2001) 036, `arXiv:hep-th/0007250 [hep-th]`.

[74] D. Freedman, S. Gubser, K. Pilch, and N. Warner, "Renormalization group flows from holography supersymmetry and a c theorem," *Adv.Theor.Math.Phys.* **3** (1999) 363–417, `arXiv:hep-th/9904017 [hep-th]`.

[75] A. R. Frey and M. Grana, "Type IIB solutions with interpolating supersymmetries," *Phys.Rev.* **D68** (2003) 106002, `arXiv:hep-th/0307142 [hep-th]`.

[76] S. B. Giddings, S. Kachru, and J. Polchinski, "Hierarchies from fluxes in string compactifications," *Phys.Rev.* **D66** (2002) 106006, `arXiv:hep-th/0105097 [hep-th]`.

[77] G. Giecold, "Remark on the Baryonic Branch of the Warped Deformed Conifold," `arXiv:1112.1054 [hep-th]`.

[78] G. Giecold, E. Goi, and F. Orsi, "Assessing a candidate IIA dual to metastable supersymmetry-breaking," `arXiv:1108.1789 [hep-th]`.

[79] G. Giecold, F. Orsi, and A. Puhm, "Insane Anti-Membranes?," `arXiv:1303.1809 [hep-th]`.

[80] L. Girardello, M. Petrini, M. Porrati, and A. Zaffaroni, "Novel local CFT and exact results on perturbations of N=4 superYang Mills from AdS dynamics," *JHEP* **9812** (1998) 022, `arXiv:hep-th/9810126 [hep-th]`.

[81] L. Girardello, M. Petrini, M. Porrati, and A. Zaffaroni, "The Supergravity dual of N=1 superYang-Mills theory," *Nucl.Phys.* **B569** (2000) 451–469, `arXiv:hep-th/9909047 [hep-th]`.

[82] A. Giveon and D. Kutasov, "Gauge Symmetry and Supersymmetry Breaking From Intersecting Branes," *Nucl.Phys.* **B778** (2007) 129–158, `arXiv:hep-th/0703135 [HEP-TH]`.

[83] K. Goldstein and S. Katmadas, "Almost BPS black holes," *JHEP* **0905** (2009) 058, `arXiv:0812.4183 [hep-th]`.

[84] M. Grana, "Flux compactifications in string theory: A comprehensive review," *Phys. Rept.* **423** (2006) 91–158, `arXiv:hep-th/0509003`.

[85] M. Grana, R. Minasian, M. Petrini, and A. Tomasiello, "Supersymmetric backgrounds from generalized Calabi-Yau manifolds," *JHEP* **08** (2004) 046, `arXiv:hep-th/0406137`.

[86] M. Grana, R. Minasian, M. Petrini, and A. Tomasiello, "Type II strings and generalized Calabi-Yau manifolds," *Comptes Rendus Physique* **5** (2004) 979–986, `arXiv:hep-th/0409176`.

[87] M. Grana, R. Minasian, M. Petrini, and A. Tomasiello, "Generalized structures of N=1 vacua," *JHEP* **11** (2005) 020, `arXiv:hep-th/0505212`. [`hep-th/0505212`].

[88] M. Grana, R. Minasian, M. Petrini, and A. Tomasiello, "A scan for new N=1 vacua on twisted tori," *JHEP* **05** (2007) 031, `arXiv:hep-th/0609124`. [`hep-th/0609124`].

[89] M. Grana, R. Minasian, M. Petrini, and D. Waldram, "T-duality, Generalized Geometry and Non-Geometric Backgrounds," *JHEP* **04** (2009) 075, `arXiv:0807.4527 [hep-th]`.

[90] M. Grana and J. Polchinski, "Supersymmetric three form flux perturbations on AdS_5," *Phys.Rev.* **D63** (2001) 026001, `arXiv:hep-th/0009211 [hep-th]`.

[91] M. Gualtieri, "Generalized complex geometry," 2004. `http://www.citebase.org/abstract?id=oai:arXiv.org:math/0401221`.

[92] S. Gubser, I. R. Klebanov, and A. M. Polyakov, "Gauge theory correlators from noncritical string theory," *Phys.Lett.* **B428** (1998) 105–114, `arXiv:hep-th/9802109 [hep-th]`.

[93] S. S. Gubser, "Supersymmetry and F theory realization of the deformed conifold with three form flux," `arXiv:hep-th/0010010 [hep-th]`.

[94] S. S. Gubser, C. P. Herzog, and I. R. Klebanov, "Symmetry Breaking and Axionic Strings in the Warped Deformed Conifold." Jhep0409:036,2004, 2004.

[95] S. S. Gubser, A. A. Tseytlin, and M. S. Volkov, "NonAbelian 4-d black holes, wrapped five-branes, and their dual descriptions," *JHEP* **0109** (2001) 017, `arXiv:hep-th/0108205 [hep-th]`.

[96] S. Gukov, C. Vafa, and E. Witten, "CFT's from Calabi-Yau four folds," *Nucl.Phys.* **B584** (2000) 69–108, `arXiv:hep-th/9906070 [hep-th]`.

[97] N. Halmagyi, J. T. Liu, and P. Szepietowski, "On N = 2 Truncations of IIB on $T^{1,1}$," *JHEP* **1207** (2012) 098, `arXiv:1111.6567 [hep-th]`.

[98] A. Hashimoto, "Comments on domain walls in holographic duals of mass deformed conformal field theories," *JHEP* **1107** (2011) 031, `arXiv:1105.3687 [hep-th]`.

[99] A. Hashimoto and P. Ouyang, "Quantization of charges and fluxes in warped Stenzel geometry," *JHEP* **1106** (2011) 124, `arXiv:1104.3517 [hep-th]`.

[100] J. Held, D. Lust, F. Marchesano, and L. Martucci, "DWSB in heterotic flux compactifications," *JHEP* **1006** (2010) 090, `arXiv:1004.0867 [hep-th]`.

[101] C. P. Herzog, I. R. Klebanov, and P. Ouyang, "D-Branes on the Conifold and N=1 Gauge/Gravity Dualities," 2002.

[102] C. P. Herzog, I. R. Klebanov, and P. Ouyang, "Remarks on the Warped Deformed Conifold," 2003.

[103] C. Herzog, Q. Ejaz, and I. Klebanov, "Cascading RG flows from new Sasaki-Einstein manifolds," *JHEP* **0502** (2005) 009, `arXiv:hep-th/0412193 [hep-th]`.

[104] N. Hitchin, "Generalized calabi-yau manifolds," *QUART.J.MATH.OXFORD SER.* **54** (2003) 281.
`http://www.citebase.org/abstract?id=oai:arXiv.org:math/0209099`.

[105] E. Imeroni, "The gauge / string correspondence towards realistic gauge theories," `hep-th/0312070`. `http://arXiv.org/abs/hep-th/0312070`. [`hep-th/0312070`].

[106] K. Intriligator, N. Seiberg, and D. Shih, "Dynamical SUSY Breaking in Meta-Stable Vacua." Jhep 0604:021,2006, 2006.

[107] B. Janssen, P. Smyth, T. Van Riet, and B. Vercnocke, "A First-order formalism for timelike and spacelike brane solutions," *JHEP* **0804** (2008) 007, `arXiv:0712.2808 [hep-th]`.

[108] C. V. Johnson, A. W. Peet, and J. Polchinski, "Gauge theory and the excision of repulson singularities," *Phys.Rev.* **D61** (2000) 086001, `arXiv:hep-th/9911161 [hep-th]`.

[109] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister, and S. P. Trivedi, "Towards Inflation in String Theory." Jcap 0310:013,2003, 2003.

[110] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, "De Sitter vacua in string theory," *Phys.Rev.* **D68** (2003) 046005, `arXiv:hep-th/0301240 [hep-th]`.

[111] S. Kachru, J. Pearson, and H. Verlinde, "Brane/Flux Annihilation and the String Dual of a Non-Supersymmetric Field Theory." Jhep 0206:021,2002, 2006.

[112] S. Kachru and E. Silverstein, "4-D conformal theories and strings on orbifolds," *Phys.Rev.Lett.* **80** (1998) 4855–4858, `arXiv:hep-th/9802183 [hep-th]`.

[113] I. R. Klebanov, "TASI lectures: Introduction to the AdS / CFT correspondence," `arXiv:hep-th/0009139 [hep-th]`.

[114] I. R. Klebanov and N. A. Nekrasov, "Gravity duals of fractional branes and logarithmic RG flow," *Nucl.Phys.* **B574** (2000) 263–274, `arXiv:hep-th/9911096 [hep-th]`.

[115] I. R. Klebanov and S. S. Pufu, "M-Branes and Metastable States," *JHEP* **1108** (2011) 035, `arXiv:1006.3587 [hep-th]`.

[116] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities," *JHEP* **08** (2000) 052, `arXiv:hep-th/0007191`. [`hep-th/0007191`].

[117] I. R. Klebanov and E. Witten, "Superconformal field theory on three-branes at a Calabi-Yau singularity," *Nucl.Phys.* **B536** (1998) 199–218, `arXiv:hep-th/9807080 [hep-th]`.

[118] I. Klebanov and A. Tseytlin, "Gravity Duals of Supersymmetric SU(N) x SU(N+M) Gauge Theories," PUPT-1919, OHSTPY-HEP-T-00-002, `arXiv:hep-th/0002159v2`.

[119] P. Koerber and D. Tsimpis, "Supersymmetric sources, integrability and generalized-structure compactifications," *JHEP* **0708** (2007) 082, `arXiv:0706.1244 [hep-th]`.

[120] C. Krishnan and S. Kuperstein, "The Mesonic Branch of the Deformed Conifold," *JHEP* **0805** (2008) 072, `arXiv:0802.3674 [hep-th]`.

[121] D. Krotov and A. M. Polyakov, "Infrared Sensitivity of Unstable Vacua," *Nucl.Phys.* **B849** (2011) 410–432, `arXiv:1012.2107 [hep-th]`.

[122] S. Kuperstein and J. Sonnenschein, "Analytic non-supersymmetric background dual of a confining gauge theory and the corresponding plane wave theory of Hadrons." Jhep 0402 (2004) 015, 2004. [`hep-th/0309011`].

[123] D. Kutasov and A. Wissanji, "IIA Perspective On Cascading Gauge Theory," `arXiv:1206.0747 [hep-th]`.

[124] O. Lebedev, H. P. Nilles, and M. Ratz, "De Sitter vacua from matter superpotentials," *Phys.Lett.* **B636** (2006) 126–131, `arXiv:hep-th/0603047 [hep-th]`.

[125] H. Lin, O. Lunin, and J. M. Maldacena, "Bubbling AdS space and 1/2 BPS geometries," *JHEP* **0410** (2004) 025, `arXiv:hep-th/0409174 [hep-th]`.

[126] J. T. Liu and P. Szepietowski, "Supersymmetry of consistent massive truncations of IIB supergravity," *Phys.Rev.* **D85** (2012) 126010, `arXiv:1103.0029 [hep-th]`.

[127] D. Lust, F. Marchesano, L. Martucci, and D. Tsimpis, "Generalized non-supersymmetric flux vacua," *JHEP* **11** (2008) 021, `arXiv:0807.4540 [hep-th]`.

[128] J. Maldacena and C. Nunez, "Supergravity description of field theories on curved manifolds and a no go theorem," `arXiv:hep-th/0007018v2`.

[129] J. M. Maldacena, "The Large N limit of superconformal field theories and supergravity," *Adv.Theor.Math.Phys.* **2** (1998) 231–252, `arXiv:hep-th/9711200 [hep-th]`.

[130] J. M. Maldacena, "TASI 2003 lectures on AdS / CFT," arXiv:hep-th/0309246 [hep-th].

[131] D. Marolf, "Chern-Simons terms and the three notions of charge," arXiv:hep-th/0006117 [hep-th].

[132] J. Marsano, K. Papadodimas, and M. Shigemori, "Nonsupersymmetric Brane/Antibrane Configurations in Type IIA and M Theory," *Nucl.Phys.* **B789** (2008) 294–361, arXiv:0705.0983 [hep-th].

[133] D. Martelli and J. Sparks, "AdS_4/CFT_3 duals from M2-branes at hypersurface singularities and their deformations," *JHEP* **0912** (2009) 017, arXiv:0909.2036 [hep-th].

[134] S. Massai, "A comment on anti-brane singularities in warped throats," arXiv:1202.3789 [hep-th].

[135] S. Massai, "Metastable Vacua and the Backreacted Stenzel Geometry," *JHEP* **1206** (2012) 059, arXiv:1110.2513 [hep-th].

[136] S. D. Mathur, "The Fuzzball proposal for black holes: An Elementary review," *Fortsch.Phys.* **53** (2005) 793–827, arXiv:hep-th/0502050 [hep-th].

[137] L. McAllister and E. Silverstein, "String Cosmology: A Review," *Gen.Rel.Grav.* **40** (2008) 565–605, arXiv:0710.2951 [hep-th].

[138] L. McAllister, E. Silverstein, and A. Westphal, "Gravity Waves and Linear Inflation from Axion Monodromy," *Phys.Rev.* **D82** (2010) 046003, arXiv:0808.0706 [hep-th].

[139] J. McGreevy, L. Susskind, and N. Toumbas, "Invasion of the giant gravitons from Anti-de Sitter space," *JHEP* **0006** (2000) 008, arXiv:hep-th/0003075 [hep-th].

[140] P. McGuirk, G. Shiu, and Y. Sumitomo, "Non-supersymmetric infrared perturbations to the warped deformed conifold," *Nucl.Phys.* **B842** (2011) 383–413, arXiv:0910.4581 [hep-th].

[141] R. Minasian and D. Tsimpis, "On the geometry of nontrivially embedded branes," *Nucl.Phys.* **B572** (2000) 499–513, arXiv:hep-th/9911042 [hep-th].

[142] R. C. Myers, "Dielectric branes," *JHEP* **9912** (1999) 022, arXiv:hep-th/9910053 [hep-th].

[143] H. Ooguri and Y. Ookouchi, "Landscape of supersymmetry breaking vacua in geometrically realized gauge theories," *Nucl.Phys.* **B755** (2006) 239–253, arXiv:hep-th/0606061 [hep-th].

[144] G. Papadopoulos and A. A. Tseytlin, "Complex geometry of conifolds and 5-brane wrapped on 2- sphere," *Class. Quant. Grav.* **18** (2001) 1333–1354, `arXiv:hep-th/0012034`. [`hep-th/0012034`].

[145] J. Polchinski, "Introduction to Gauge/Gravity Duality," `arXiv:1010.6134 [hep-th]`.

[146] J. Polchinski and M. J. Strassler, "The String Dual of a Confining Four-Dimensional Gauge Theory," NSF-ITP-00-16,IAS–TH–00/18, `arXiv:hep-th/0003136v2`.

[147] A. M. Polyakov, "String theory and quark confinement," *Nucl.Phys.Proc.Suppl.* **68** (1998) 1–8, `arXiv:hep-th/9711002 [hep-th]`.

[148] A. Polyakov, "De Sitter space and eternity," *Nucl.Phys.* **B797** (2008) 199–217, `arXiv:0709.2899 [hep-th]`.

[149] A. Polyakov, "Decay of Vacuum Energy," *Nucl.Phys.* **B834** (2010) 316–329, `arXiv:0912.5503 [hep-th]`.

[150] A. Polyakov, "Infrared instability of the de Sitter space," `arXiv:1209.4135 [hep-th]`.

[151] S. S. Pufu, I. R. Klebanov, T. Klose, and J. Lin, "Green's Functions and Non-Singlet Glueballs on Deformed Conifolds," *J.Phys.* **A44** (2011) 055404, `arXiv:1009.2763 [hep-th]`.

[152] M. Rummel and A. Westphal, "A sufficient condition for de Sitter vacua in type IIB string theory," *JHEP* **1201** (2012) 020, `arXiv:1107.2115 [hep-th]`.

[153] A. Saltman and E. Silverstein, "The Scaling of the no scale potential and de Sitter model building," *JHEP* **0411** (2004) 066, `arXiv:hep-th/0402135 [hep-th]`.

[154] K. Skenderis and M. Taylor, "The fuzzball proposal for black holes," *Phys.Rept.* **467** (2008) 117–171, `arXiv:0804.0552 [hep-th]`.

[155] K. Skenderis and P. K. Townsend, "Gravitational stability and renormalization group flow," *Phys.Lett.* **B468** (1999) 46–51, `arXiv:hep-th/9909070 [hep-th]`.

[156] M. Stenzel, "Ricci-flat metrics on the complexification of a compact rank one symmetric space," *Manuscripta Math. 80 1* (1993) .

[157] L. Susskind, "The Anthropic landscape of string theory," `arXiv:hep-th/0302219 [hep-th]`.

[158] M. Taylor, "Anomalies, counterterms and the N=0 Polchinski-Strassler solutions," arXiv:hep-th/0103162 [hep-th].

[159] M. Trigiante, T. Van Riet, and B. Vercnocke, "Fake supersymmetry versus Hamilton-Jacobi," *JHEP* **1205** (2012) 078, arXiv:1203.3194 [hep-th].

[160] H. Verlinde, "On metastable branes and a new type of magnetic monopole," arXiv:hep-th/0611069 [hep-th].

[161] S. Weinberg, "Anthropic Bound on the Cosmological Constant," *Phys.Rev.Lett.* **59** (1987) 2607.

[162] E. Witten, "Anti-de Sitter space and holography," *Adv.Theor.Math.Phys.* **2** (1998) 253–291, arXiv:hep-th/9802150 [hep-th].