From Classical Gravity to Quantum Amplitudes (lecture 2a)

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Cours de Physique Théorique

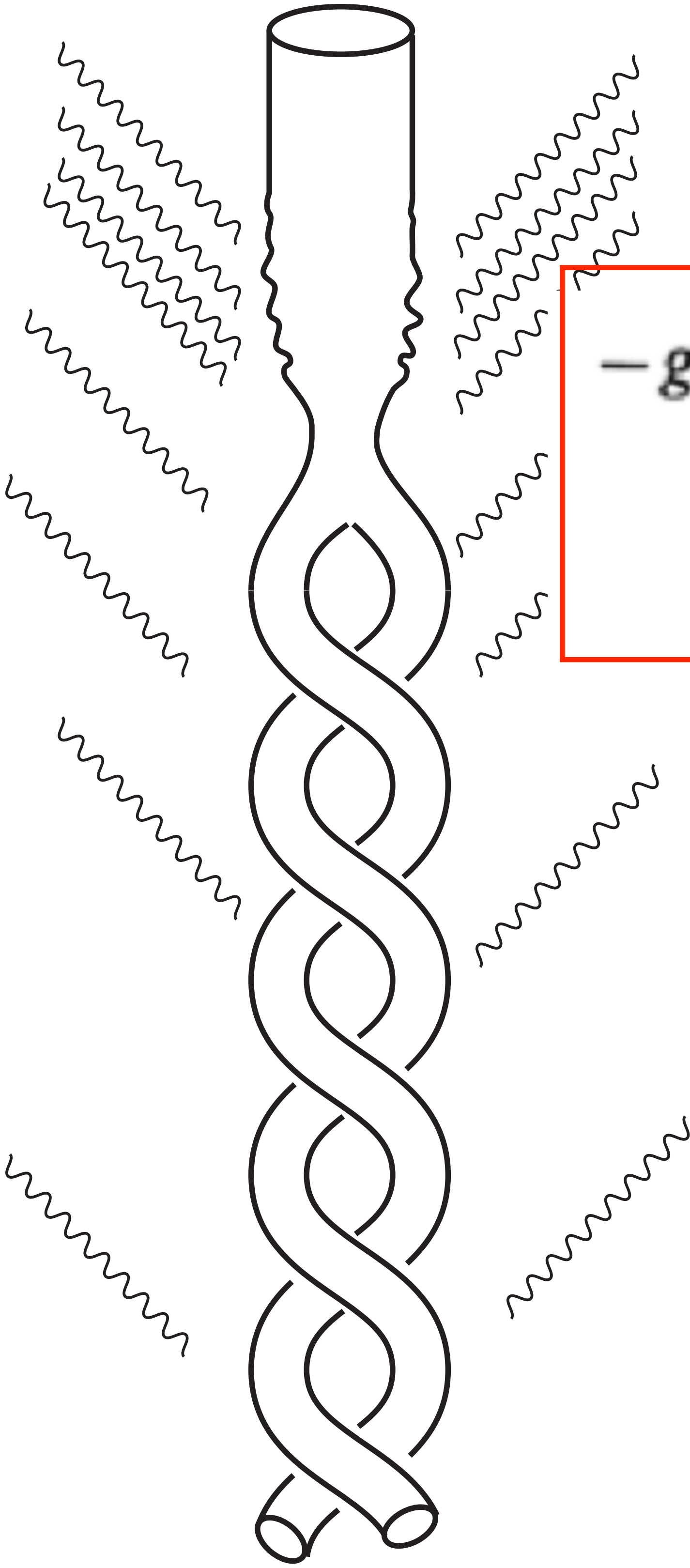
IPhT, Fridays 5, 12 October 2018 (10:00 to 12:15),

and Friday 19 October (10:00 to 12:15, and 14:15 to 16:30)

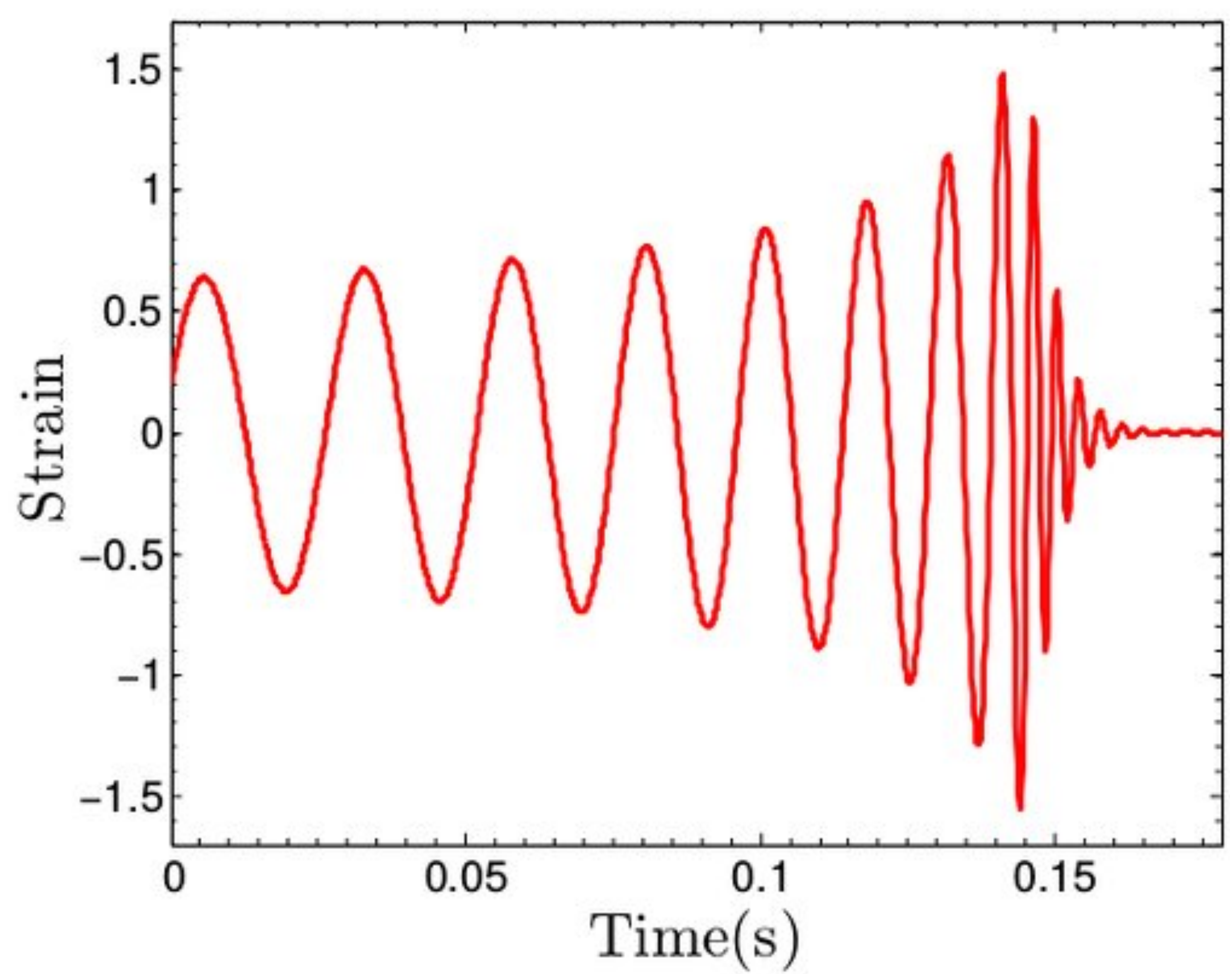
$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

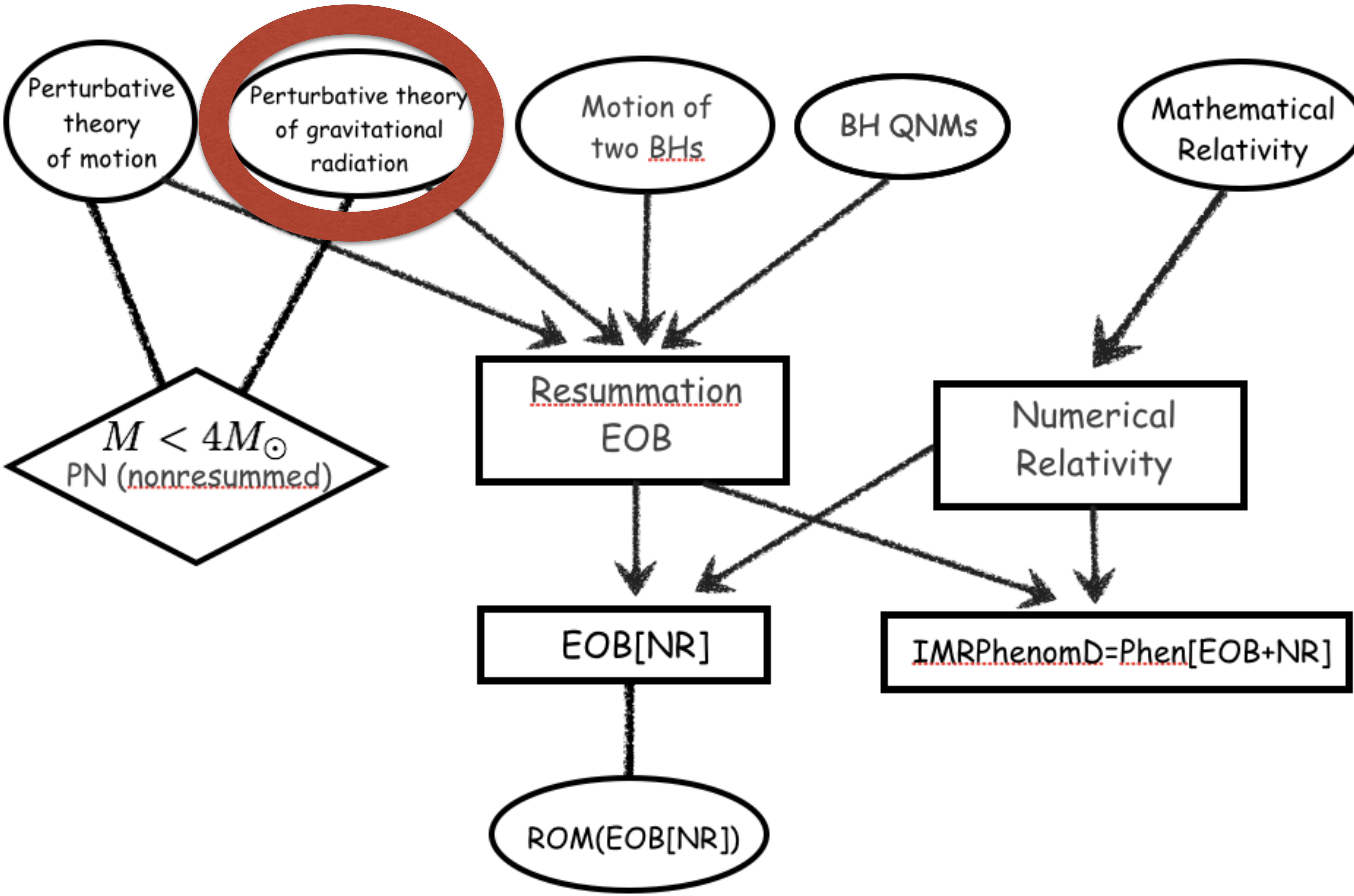
$$R_{\mu\nu} = 0$$

$$ds^2 = g_{\mu\nu}(x^\lambda)\, dx^\mu dx^\nu$$



$$-g^{\mu\nu} g_{\alpha\beta,\,\mu\nu} + g^{\mu\nu} g^{\rho\sigma} \big(g_{\alpha\mu,\,\rho}\, g_{\beta\nu,\,\sigma} - g_{\alpha\mu,\,\rho}\, g_{\beta\sigma,\,\nu} + g_{\alpha\mu,\,\rho}\, g_{\nu\sigma,\,\beta} + g_{\beta\mu,\,\rho}\, g_{\nu\sigma,\,\alpha} - \frac{1}{2} g_{\mu\rho,\,\alpha}\, g_{\nu\sigma,\,\beta}\big) = 0$$





Perturbative theory of motion
Perturbative theory of gravitational radiation
Motion of two BHs
BH QNMs
Mathematical Relativity
$M < 4M_\odot$
PN (nonresummed)
Resummation EOB
Numerical Relativity
EOB[NR]
IMRPhenomD=Phen[EOB+NR]
ROM(EOB[NR])

Basics of Gravitational Waves

In linearized GR (Einstein 1916,1918):

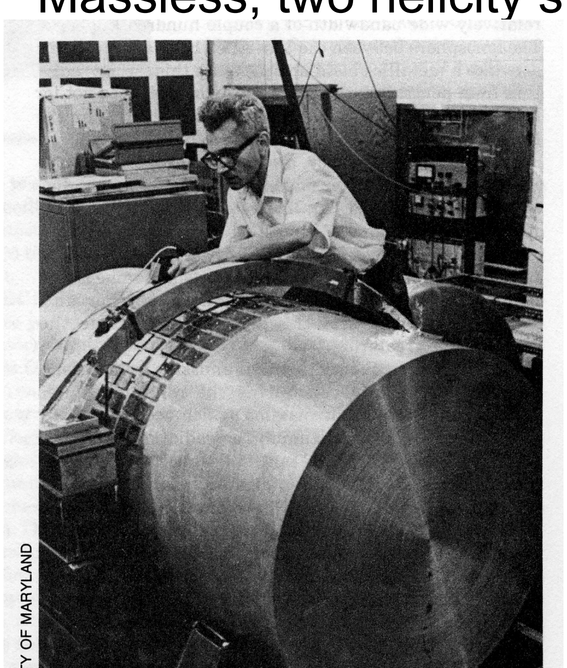
$$\Box h_{\mu\nu} + \partial_\mu H_\nu + \partial_\nu H_\mu = -16\pi G\left(T_{\mu\nu} - \frac{1}{D-2}\eta_{\mu\nu}T\right)$$

$$H_\mu = \frac{1}{2}\partial_\mu h - \partial^\nu h_{\mu\nu} \qquad \partial^\nu T_{\mu\nu} = 0$$

$$ds^2 = -c^2 dt^2 + (\delta_{ij} + h_{ij})dx^i dx^j$$

$$h_{ij} \simeq \frac{2G}{c^4 r}\ddot{Q}_{ij}^{TT}(t - r/c)$$

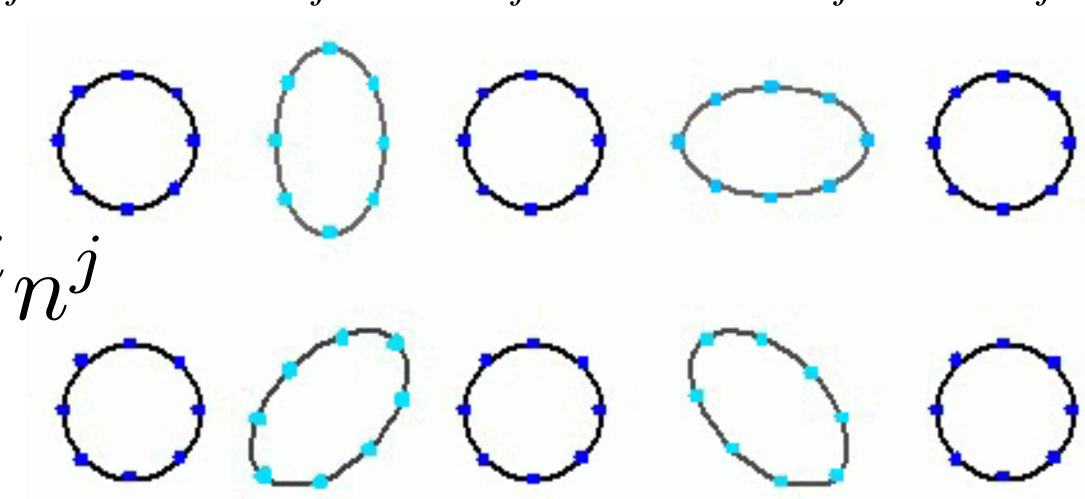
Massless, two helicity states s=±2, i.e. two Transverse-Traceless (TT) tensor polarizations propagating at v=c



Joseph Weber (1919-2000)

$$g_{ij} = \delta_{ij} + h_{ij} \quad h_{ij} = h_+(x_i x_j - y_i y_j) + h_\times(x_i y_j + y_i x_j)$$

$$\frac{\delta L}{L} = \frac{1}{2}h_{ij}n^i n^j$$



Perturbative Theory of the Generation of Gravitational Radiation

Einstein '16, '18 (+ Landau-Lifshitz 41, and Fock '55) : h_+, h_x and quadrupole formula

Relativistic, multipolar extensions of LO quadrupole radiation :

Sachs-Bergmann '58, Sachs '61, Mathews '62, Peters-Mathews '63, Pirani '64

Campbell-Morgan '71,

Campbell et al '75,

nonlinear effects:

Bonnor-Rotenberg '66,

Epstein-Wagoner-Will '75-76

Thorne '80, .., Will et al 00

MPM Formalism:

Blanchet-Damour '86,

Damour-Iyer '91,

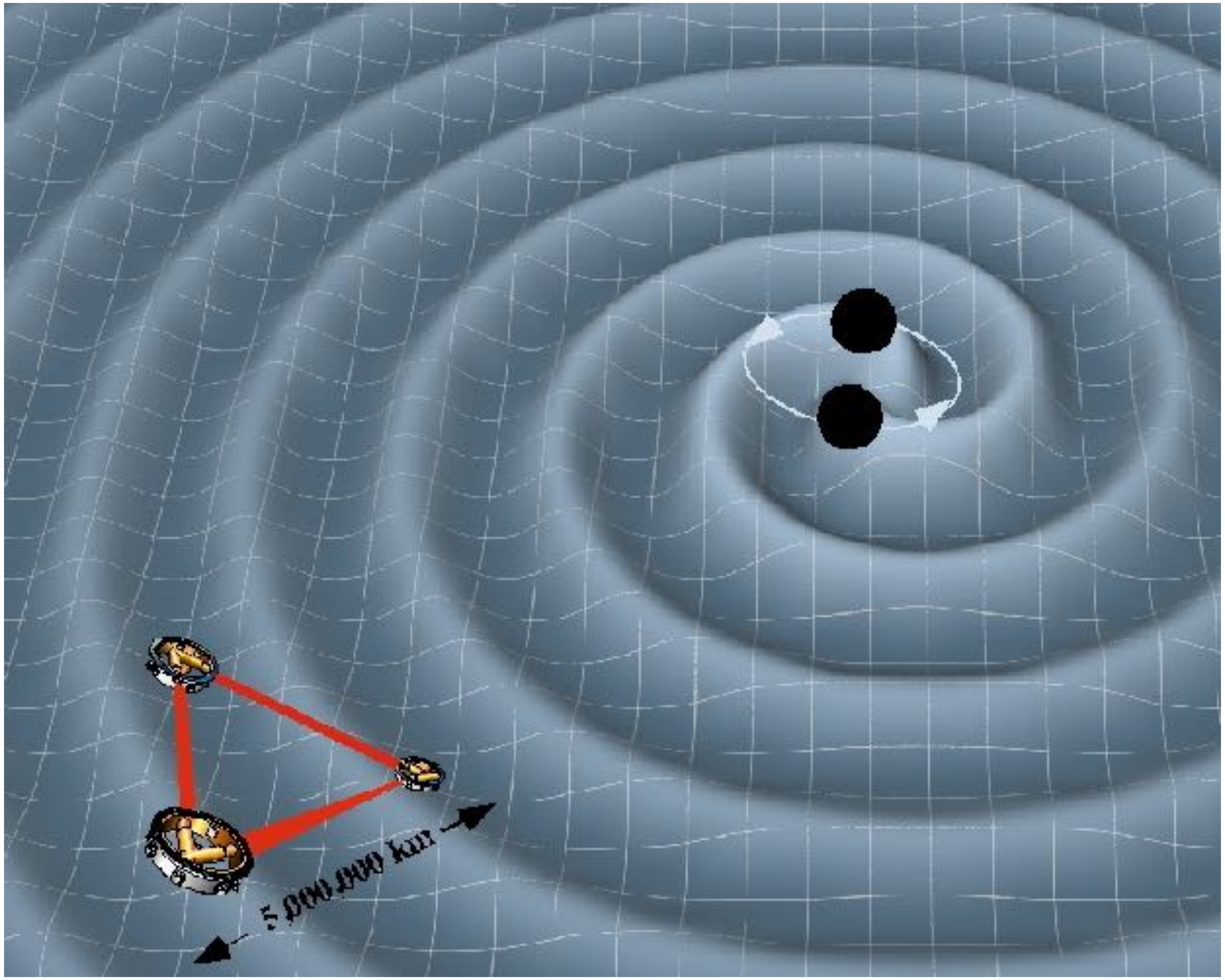
Blanchet '95 '98

Combines multipole exp. ,

Post Minkowkian exp.,

analytic continuation,

and PN matching



Multipolar Expansions Using STF Tensors

A convenient form for (2l+1)-dim irrep of SO(3): STF tensors

multi-index notation (Blanchet-Damour '86)

$$\widehat{T}_L = T_{\langle i_1 i_2 \cdots i_l \rangle} \quad T_L S_L \quad \partial_L = \partial_{i_1 i_2 \cdots i_l}$$

Multipolar expansions of Newtonian potentials with STF tensors

$$\Delta\phi = -4\pi\rho$$

can be written either as

$$\phi(\mathbf{X}) = 4\pi \sum_{l\geq 0} \sum_{-l\leq m\leq l} \frac{Q_{lm}}{2l+1} \frac{Y_{lm}(\Theta,\Phi)}{R^{l+1}}$$

or

$$\phi(\mathbf{X}) = \sum_{l\geq 0} \frac{(-)^l}{l!} Q_{i_1\cdots i_l} \partial_{i_1\cdots i_l}\left[\frac{1}{R}\right],$$

where, respectively,

$$Q_{lm} = \int d^3x\, Y^*_{lm}(\theta,\varphi) r^l \rho(\mathbf{x})$$

or

$$Q_{i_1\cdots i_l} = \int d^3x\, x^{\langle i_1} \cdots x^{i_l\rangle} \rho(\mathbf{x}),$$

Multipolar expansions of retarded potentials with STF tensors

Fundamental spherically symmetric solution of $\Box\phi(x) = 0$

$$\phi(t, r) = \frac{F(t - r/c)}{r}$$

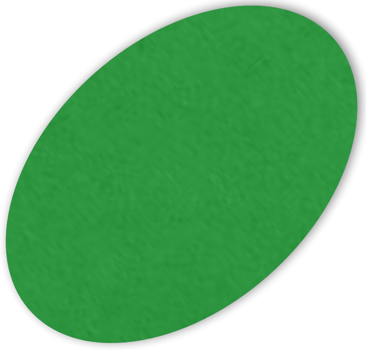
(exterior solution)

Irreducible multipolar outgoing wave solution of $\Box\phi(x) = 0$

$$\phi^{(\ell)}(t, x^i) = \partial_{i_1 i_2 \cdots i_\ell} \left(\frac{F_{i_1 i_2 \cdots i_\ell}(t - r/c)}{r} \right)$$

where F_L is STF

F_L is source-rooted and, when the source rotates under R_ij, it rotates by

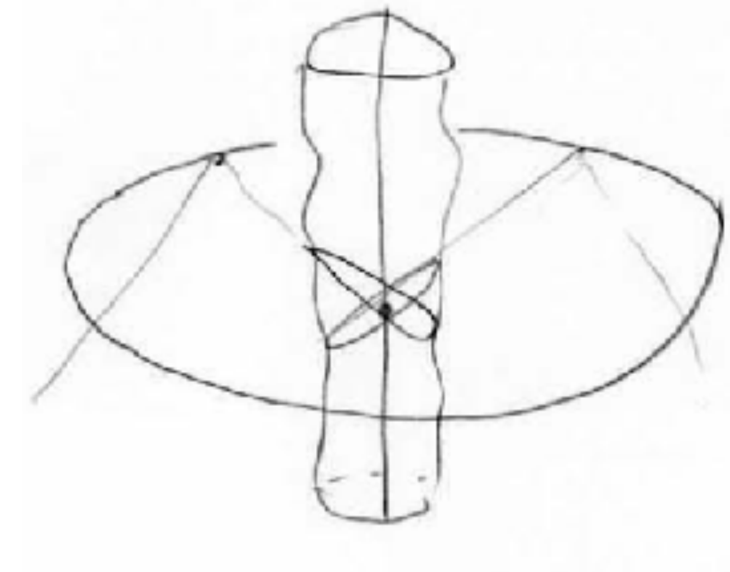


$$F'_{i_1 i_2 \cdots i_\ell} = R_{i_1 j_1} R_{i_2 j_2} \cdots R_{i_\ell j_\ell} F_{j_1 j_2 \cdots j_\ell}$$

STF Multipolar Analysis of Linearized Gravity (Damour-Iyer '91)

Two infinite sequences of « mass -type » M_L and « spin-type » S_L multipole moments

$$\Box \bar{h}_{\mu\nu}(\mathbf{X},T)=-\frac{16\pi G}{c^4}T_{\mu\nu}(\mathbf{X},T)\ . \qquad (\partial_\nu \bar{h}^{\mu\nu}=0$$



$$\bar{h}^{\alpha\beta}_{\text{can}}[\mathbb{M}]=\bar{h}^{\alpha\beta}[\mathbb{M},\mathbb{W}]+\partial^\alpha w^\beta[\mathbb{W}]+\partial^\beta w^\alpha[\mathbb{W}]-f^{\alpha\beta}\partial_\mu w^\mu[\mathbb{W}]\ ,$$

$$\bar{h}^{00}_{\text{can}}(\mathbf{X},T)=+\frac{4}{c^2}\sum_{l\geq 0}\frac{(-)^l}{l!}\partial_L[R^{-1}M_L(U)]\ ,$$

$$\bar{h}^{0i}_{\text{can}}(\mathbf{X},T)=-\frac{4}{c^3}\sum_{l\geq 1}\frac{(-)^l}{l!}\partial_{L-1}[R^{-1}\dot{M}_{iL-1}(U)]-\frac{4}{c^3}\sum_{l\geq 1}\frac{(-)^l l}{(l+1)!}\epsilon_{iab}\partial_{aL-1}[R^{-1}S_{bL-1}(U)]\ ,$$

$$\bar{h}^{ij}_{\text{can}}(\mathbf{X},T)=+\frac{4}{c^4}\sum_{l\geq 2}\frac{(-)^l}{l!}\partial_{L-2}[R^{-1}\ddot{M}_{ijL-2}(U)]+\frac{8}{c^4}\sum_{l\geq 2}\frac{(-)^l l}{(l+1)!}\partial_{aL-2}[R^{-1}\epsilon_{ab(i}\dot{S}_{j)bL-2}(U)]\ .$$

$$M_L(U)=G\int d^3x\int_{-1}^{1}dz\left[\delta_l\hat{x}_L\tilde{\sigma}-\frac{4(2l+1)}{c^2(l+1)(2l+3)}\delta_{l+1}\hat{x}_{aL}\frac{\partial}{\partial U}\tilde{\sigma}^a+\frac{2(2l+1)}{c^4(l+1)(l+2)(2l+5)}\delta_{l+2}\hat{x}_{abL}\frac{\partial^2}{\partial U^2}\tilde{T}^{ab}\right]$$

$$S_L(U)=G\,\underset{L}{\mathrm{STF}}\int d^3x\int_{-1}^{1}dz\left[\delta_l\hat{x}_{L-1}\epsilon_{i_l ab}x^a\tilde{\sigma}^b-\frac{2l+1}{c^2(l+2)(2l+3)}\delta_{l+1}\epsilon_{i_l ab}\hat{x}_{acL-1}\frac{\partial}{\partial U}\tilde{T}^{bc}\right],$$

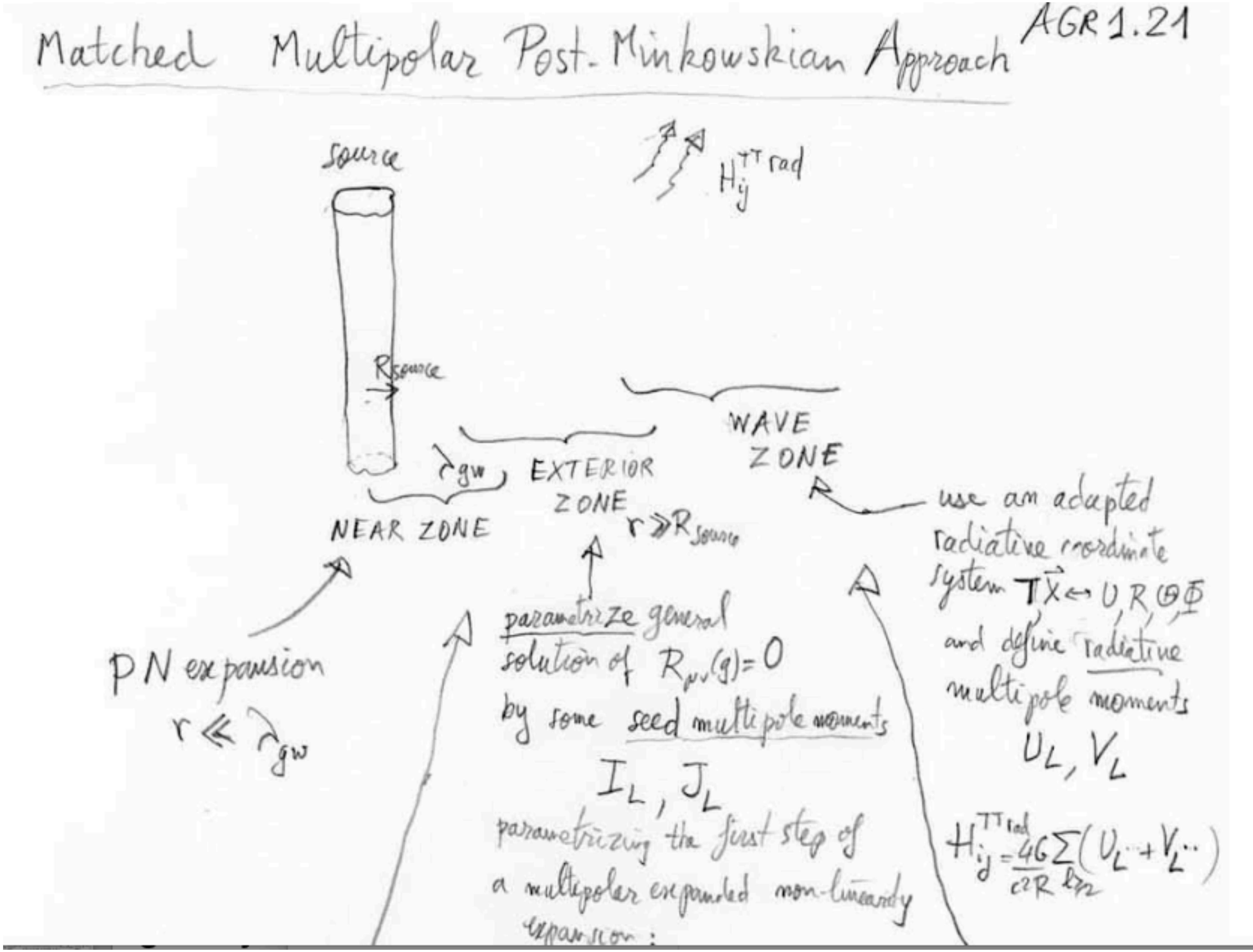
linearized gravity

$$\sigma\equiv\frac{T^{00}+T^{ss}}{c^2}\ ,$$

$$\sigma^a\equiv\frac{T^{0a}}{c}\ ,$$

$$\tilde{T}^{\mu\nu}=T^{\mu\nu}\left(\vec{x},U+\frac{rz}{c}\right)$$

generalizing the scalar field case (Blanchet-Damour'89)



Radiative multipole moments

$$H_{ij}^{\rm TT}(U,\mathbf{X}) = \frac{4G}{c^2 R}\mathcal{P}_{ijab}(\mathbf{N})\sum_{\ell=2}^{+\infty}\frac{1}{c^\ell \ell!}\left\{N_{L-2}\mathrm{U}_{abL-2}(U) - \frac{2\ell}{c(\ell+1)}N_{cL-2}\epsilon_{cd(a}\mathrm{V}_{b)dL-2}(U)\right\} + \mathcal{O}\left(\frac{1}{R^2}\right).$$

MULTIPOLAR POST-MINKOWSKIAN FORMALISM (BLANCHET-DAMOUR-IYER)

Decomposition of space-time in various overlapping regions:

1. **near-zone: r << lambda : PN theory**
2. **exterior zone: r >> r_source: MPM expansion**
3. **far wave-zone: Bondi-type expansion**

followed by matching between the zones

in exterior zone, iterative solution of Einstein's vacuum field equations by means of a double expansion in non-linearity and in multipoles, with crucial use of analytic continuation (complex B) for dealing with formal UV divergences at r=0

$$g = \eta + G h_1 + G^2 h_2 + G^3 h_3 + \ldots,$$
$$\Box h_1 = 0,$$
$$\Box h_2 = \partial\partial h_1 h_1,$$
$$\Box h_3 = \partial\partial h_1 h_1 h_1 + \partial\partial h_1 h_2,$$
$$h_1 = \sum_\ell \partial_{i_1 i_2 \ldots i_\ell} \left(\frac{M_{i_1 i_2 \ldots i_\ell}(t - r/c)}{r} \right) + \partial\partial\ldots\partial \left(\frac{\epsilon_{j_1 j_2 k} S_{k j_3 \ldots j_\ell}(t - r/c)}{r} \right)$$
$$h_2 = FP_B \Box^{-1}_{\text{ret}} \left(\left(\frac{r}{r_0} \right)^B \partial\partial h_1 h_1 \right) + \ldots,$$
$$h_3 = FP_B \Box^{-1}_{\text{ret}} \ldots.$$

TOOLS TO SOLVE ITERATIVELY EINSTEIN'S EOS IN MPM (BLANCHET-DAMOUR, BLANCHET)

$$\Box h^{\alpha\beta}_{(n)} = \Lambda^{\alpha\beta}_{(n)}[h_{(1)}, h_{(2)}, \ldots, h_{(n-1)}] ,$$
$$\partial_\mu h^{\alpha\mu}_{(n)} = 0 .$$

Analytic continuation wrt to B -> 0: take the Finite Part of Laurent expansion

$$I^{\alpha\beta}(B) \equiv \Box^{-1}_{\mathrm{ret}} \left[\widetilde{r}^B \Lambda^{\alpha\beta}_{(n)} \right] \qquad \widetilde{r} \equiv \frac{r}{r_0}$$

Decomposition in orbital spherical harmonics (i.e. in \hat n^L) and use of integration formulas:

$$\Box^{-1}_{\mathrm{ret}} \left[\frac{\hat{n}_L}{r^2} \mathrm{F}(t-r) \right] = -\hat{n}_L \int_1^{+\infty} \mathrm{d}x \, Q_\ell(x) \mathrm{F}(t-rx)$$

Use of LL-improved definition of « source multipole moments » in linearized-gravity multipole expansion:

$$\tau^{\alpha\beta} = |g| T^{\alpha\beta} + \frac{c^4}{16\pi G} \Lambda^{\alpha\beta} . \quad \text{—>} \quad \Box h^{\alpha\beta} = \frac{16\pi G}{c^4} \tau^{\alpha\beta} ,$$

Matching to the PN_expanded near-zone solution

Nonlinearities in harmonic coordinates

(Blanchet-Damour'88,'89,'92)

$$h_2^{\alpha\beta} = \Box_R^{-1}(r^{-2}Q^{\alpha\beta}) + \Box_R^{-1}(r^{-3}N_3^{\alpha\beta} + \cdots) + \sum_{\ell=0,1} \partial_L(r^{-1}T_L^{\alpha\beta})$$

A priori « bad », slow asymptotic decay

$$Q^{\alpha\beta}(u,\mathbf{n}) = \frac{k^\alpha k^\beta}{c^2}\Pi + \frac{4M}{c^4}\frac{d^2 z^{\alpha\beta}}{du^2}$$

$$\Pi(u,\mathbf{n}) = \frac{16\pi}{Gc^3}\frac{dL^{\text{grav}}}{d\Omega}\bigg|_{h_1} = \frac{16\pi}{Gc^3}\frac{dE^{\text{grav}}}{du\,d\Omega}\bigg|_{h_1}$$

M/r light-cone deviation

Both cured by coord. transformation

$$X^\alpha = x^\alpha + G\xi^\alpha + G^2\lambda^\alpha + O(G^3)$$

Linked to the « null »,or « no stiffening », property of Einstein eqs.

$$\xi^\alpha \equiv -\frac{2M}{c^2}\delta_0^\alpha \ln(r/cP^{\text{rad}}) \qquad \lambda^\alpha \equiv \Box_R^{-1}\left(\frac{k^\alpha}{2cr^2}\Pi^{(-1)}(u,\mathbf{n})\right)$$

Hereditary (tail and memory) effects in GW reaction and generation (Blanchet-Damour '88,'89,'92,....)

Hereditary tail effect (mass x quadrupole) in near-zone

$$(\delta g_{00}^{\mathrm{near-zone}})^{\mathrm{hereditary}} = \frac{1}{c^{10}}\left[-\frac{8}{5}x_{ab}I(t)\int_0^{+\infty}dv\,\ln\left[\frac{v}{2P}\right]{}^{(7)}I_{ab}(t-v)+\right.$$

dependence on infinite past

Memory and tail effects in wave-zone (radiative) GW multipoles

(Blanchet-Damour '89,92, Christodoulou'91,....)

tail-transported hereditary effect

depends on multipoles of energy flux

$$I_L^{\mathrm{rad}[\ell]}(U) = M_L^{(\ell)}(U) + \frac{Gc^{\ell+1}\ell!}{2(\ell+1)(\ell+2)}\int_{-\infty}^{U}dV\,\Pi_L(V) + \frac{2GM}{c^3}\int_0^{+\infty}dY\ln\left(\frac{Y}{2P^{\mathrm{rad}}}\right)M_L^{(\ell+2)}(U-Y)$$
$$+GS'_{2L}(U) + O(G^2)\,,$$
$$J_L^{\mathrm{rad}[\ell]}(U) = S_L^{(\ell)}(U) + \frac{2GM}{c^3}\int_0^{+\infty}dY\ln\left(\frac{Y}{2P^{\mathrm{rad}}}\right)S_L^{(\ell+2)}(U-Y) + GS''_{2L}(U) + O(G^2)\,,$$

2-body Taylor-expanded 4PN Hamiltonian [DJS, 2014, JS 2015]

$$\begin{aligned} c^8 H^{\text{loc}}_{4\text{PN}}(\mathbf{x}_a,\mathbf{p}_a) = {}& \frac{7(\mathbf{p}_1^2)^5}{256 m_1^9} + \frac{G m_1 m_2}{r_{12}} H_{48}(\mathbf{x}_a,\mathbf{p}_a) + \frac{G^2 m_1 m_2}{r_{12}^2} m_1 H_{46}(\mathbf{x}_a,\mathbf{p}_a) \\ &+ \frac{G^3 m_1 m_2}{r_{12}^3}\left(m_1^2 H_{441}(\mathbf{x}_a,\mathbf{p}_a) + m_1 m_2 H_{442}(\mathbf{x}_a,\mathbf{p}_a)\right) \\ &+ \frac{G^4 m_1 m_2}{r_{12}^4}\left(m_1^3 H_{421}(\mathbf{x}_a,\mathbf{p}_a) + m_1^2 m_2 H_{422}(\mathbf{x}_a,\mathbf{p}_a)\right) \\ &+ \frac{G^5 m_1 m_2}{r_{12}^5} H_{40}(\mathbf{x}_a,\mathbf{p}_a) + (1 \leftrightarrow 2), \end{aligned} \tag{A3}$$

$$\begin{aligned} H_{48}(\mathbf{x}_a,\mathbf{p}_a) = {}& \frac{45(\mathbf{p}_1^2)^4}{128 m_1^8} - \frac{9(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2}{64 m_1^6 m_2^2} + \frac{15(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^3}{64 m_1^6 m_2^2} - \frac{9(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{16 m_1^6 m_2^2} \\ &- \frac{3(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{32 m_1^6 m_2^2} + \frac{15(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{64 m_1^6 m_2^2} - \frac{21(\mathbf{p}_1^2)^3\mathbf{p}_2^2}{64 m_1^6 m_2^2} - \frac{35(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{256 m_1^5 m_2^3} \\ &+ \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2}{128 m_1^5 m_2^3} + \frac{33(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3(\mathbf{p}_1^2)^2}{256 m_1^5 m_2^3} - \frac{85(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2^3} \\ &- \frac{45(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{128 m_1^5 m_2^3} - \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2^3} + \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{64 m_1^5 m_2^3} \\ &+ \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{64 m_1^5 m_2^3} - \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{64 m_1^5 m_2^3} - \frac{3\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{64 m_1^5 m_2^3} + \frac{55(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3} \\ &- \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2\mathbf{p}_2^2}{128 m_1^5 m_2^3} - \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{256 m_1^5 m_2^3} - \frac{23(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3} \\ &+ \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{128 m_1^5 m_2^3} - \frac{7(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4\mathbf{p}_1^2}{64 m_1^4 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4(\mathbf{p}_1^2)^2}{64 m_1^4 m_2^4} \\ &- \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{4 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{16 m_1^4 m_2^4} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{64 m_1^4 m_2^4} + \frac{21(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2\mathbf{p}_2^2}{64 m_1^4 m_2^4} \\ &- \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{32 m_1^4 m_2^4} - \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{4 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{16 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{16 m_1^4 m_2^4} \\ &- \frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{32 m_1^4 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_2^2)^2}{64 m_1^4 m_2^4} - \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_2^2)^2}{32 m_1^4 m_2^4} - \frac{7(\mathbf{p}_1^2)^2(\mathbf{p}_2^2)^2}{128 m_1^4 m_2^4}, \end{aligned} \tag{A4a}$$

$$\begin{aligned} H_{46}(\mathbf{x}_a,\mathbf{p}_a) = {}& \frac{369(\mathbf{n}_{12}\cdot\mathbf{p}_1)^6}{160 m_1^6} - \frac{889(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4\mathbf{p}_1^2}{192 m_1^6} + \frac{49(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1^2)^2}{16 m_1^6} - \frac{63(\mathbf{p}_1^2)^3}{64 m_1^6} - \frac{549(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{128 m_1^5 m_2} \\ &+ \frac{67(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{16 m_1^5 m_2} - \frac{167(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2}{128 m_1^5 m_2} + \frac{1547(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2} - \frac{851(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{128 m_1^5 m_2} \\ &+ \frac{1099(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2} + \frac{3263(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{1280 m_1^4 m_2^2} + \frac{1067(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{480 m_1^4 m_2^2} - \frac{4567(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2}{3840 m_1^4 m_2^2} \\ &- \frac{3571(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{320 m_1^4 m_2^2} + \frac{3073(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{480 m_1^4 m_2^2} + \frac{4349(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{1280 m_1^4 m_2^2} \\ &- \frac{3461\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{3840 m_1^4 m_2^2} + \frac{1673(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4\mathbf{p}_2^2}{1920 m_1^4 m_2^2} - \frac{1999(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2\mathbf{p}_2^2}{3840 m_1^4 m_2^2} + \frac{2081(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{3840 m_1^4 m_2^2} - \frac{13(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{8 m_1^3 m_2^3} \\ &+ \frac{191(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2}{192 m_1^3 m_2^3} - \frac{19(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2^3} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2^3} \\ &+ \frac{11(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{192 m_1^3 m_2^3} + \frac{77(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{96 m_1^3 m_2^3} + \frac{233(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_2^2}{96 m_1^3 m_2^3} - \frac{47(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2\mathbf{p}_2^2}{32 m_1^3 m_2^3} \\ &+ \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{384 m_1^3 m_2^3} - \frac{185\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{384 m_1^3 m_2^3} - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4}{4 m_1^2 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4\mathbf{p}_1^2}{4 m_1^2 m_2^4} \\ &- \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3(\mathbf{p}_1\cdot\mathbf{p}_2)}{2 m_1^2 m_2^4} + \frac{21(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{16 m_1^2 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{6 m_1^2 m_2^4} + \frac{49(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2\mathbf{p}_2^2}{48 m_1^2 m_2^4} \\ &- \frac{133(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{24 m_1^2 m_2^4} - \frac{77(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{96 m_1^2 m_2^4} + \frac{197(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_2^2)^2}{96 m_1^2 m_2^4} - \frac{173\mathbf{p}_1^2(\mathbf{p}_2^2)^2}{48 m_1^2 m_2^4} + \frac{13(\mathbf{p}_2^2)^3}{8 m_2^6}, \end{aligned} \tag{A4b}$$

$$\begin{aligned} H_{441}(\mathbf{x}_a,\mathbf{p}_a) = {}& \frac{5027(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{384 m_1^4} - \frac{22993(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2}{960 m_1^4} - \frac{6695(\mathbf{p}_1^2)^2}{1152 m_1^4} - \frac{3191(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{640 m_1^3 m_2} \\ &+ \frac{28561(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{1920 m_1^3 m_2} + \frac{8777(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2} - \frac{752969\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{28800 m_1^3 m_2} \\ &- \frac{16481(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{960 m_1^2 m_2^2} + \frac{94433(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{4800 m_1^2 m_2^2} - \frac{103957(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{2400 m_1^2 m_2^2} \\ &+ \frac{791(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{400 m_1^2 m_2^2} + \frac{26627(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_2^2}{1600 m_1^2 m_2^2} - \frac{118261\mathbf{p}_1^2\mathbf{p}_2^2}{4800 m_1^2 m_2^2} + \frac{105(\mathbf{p}_2^2)^2}{32 m_2^4}, \end{aligned} \tag{A4c}$$

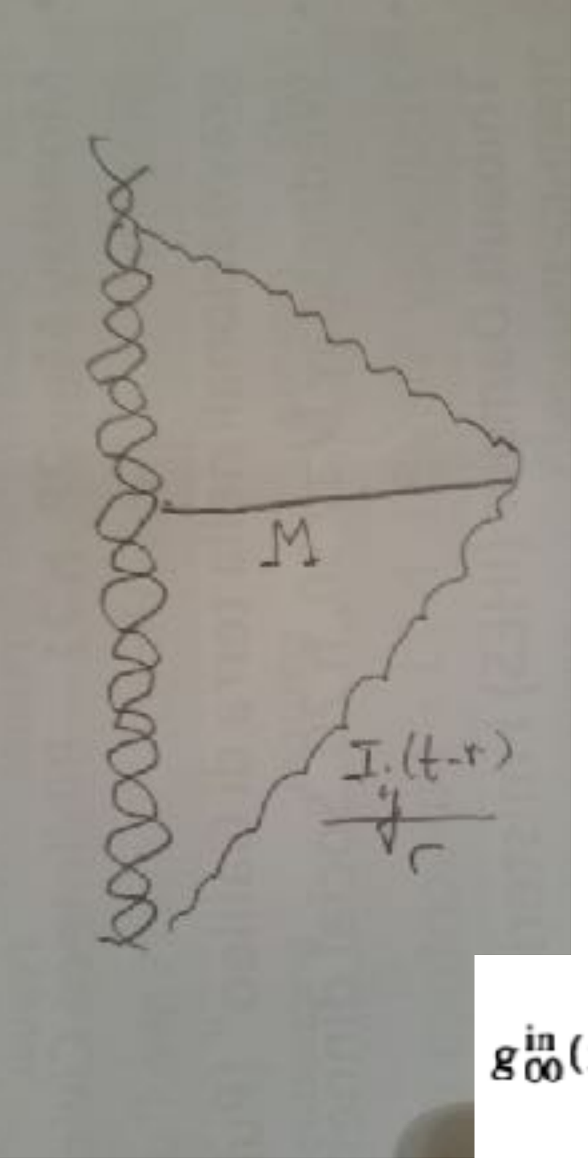
$$\begin{aligned} H_{442}(\mathbf{x}_a,\mathbf{p}_a) = {}& \left(\frac{2749\pi^2}{8192} - \frac{211189}{19200}\right)\frac{(\mathbf{p}_1^2)^2}{m_1^4} + \left(\frac{63347}{1600} - \frac{1059\pi^2}{1024}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2}{m_1^4} + \left(\frac{375\pi^2}{8192} - \frac{23533}{1280}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^4} \\ &+ \left(\frac{10631\pi^2}{8192} - \frac{1918349}{57600}\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} + \left(\frac{13723\pi^2}{16384} - \frac{2492417}{57600}\right)\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2 m_2^2} \\ &+ \left(\frac{1411429}{19200} - \frac{1059\pi^2}{512}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{m_1^2 m_2^2} + \left(\frac{248991}{6400} - \frac{6153\pi^2}{2048}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^2 m_2^2} \\ &- \left(\frac{30383}{960} + \frac{36405\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} + \left(\frac{1243717}{14400} - \frac{40483\pi^2}{16384}\right)\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2} \\ &+ \left(\frac{2369}{60} + \frac{35655\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2} + \left(\frac{43101\pi^2}{16384} - \frac{391711}{6400}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{m_1^3 m_2} \\ &+ \left(\frac{56955\pi^2}{16384} - \frac{1646983}{19200}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2}, \end{aligned} \tag{A4d}$$

$$H_{421}(\mathbf{x}_a,\mathbf{p}_a) = \frac{64861\mathbf{p}_1^2}{4800 m_1^2} - \frac{91(\mathbf{p}_1\cdot\mathbf{p}_2)}{8 m_1 m_2} + \frac{105\mathbf{p}_2^2}{32 m_2^2} - \frac{9841(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{1600 m_1^2} - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{2 m_1 m_2}, \tag{A4e}$$

$$\begin{aligned} H_{422}(\mathbf{x}_a,\mathbf{p}_a) = {}& \left(\frac{1937033}{57600} - \frac{199177\pi^2}{49152}\right)\frac{\mathbf{p}_1^2}{m_1^2} + \left(\frac{176033\pi^2}{24576} - \frac{2864917}{57600}\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1 m_2} + \left(\frac{282361}{19200} - \frac{21837\pi^2}{8192}\right)\frac{\mathbf{p}_2^2}{m_2^2} \\ &+ \left(\frac{698723}{19200} + \frac{21745\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2} - \left(\frac{63641\pi^2}{24576} - \frac{2712013}{19200}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1 m_2} \\ &+ \left(\frac{3200179}{57600} - \frac{28691\pi^2}{24576}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_2^2}, \end{aligned} \tag{A4f}$$

$$H_{40}(\mathbf{x}_a,\mathbf{p}_a) = -\frac{m_1^4}{16} + \left(\frac{6237\pi^2}{1024} - \frac{169799}{2400}\right)m_1^3 m_2 + \left(\frac{44825\pi^2}{6144} - \frac{609427}{7200}\right)m_1^2 m_2^2. \tag{A4g}$$

$$\begin{aligned} H^{\text{nonloc}}_{4\text{PN}}(t) = {}& -\frac{1}{5}\frac{G^2 M}{c^8} I^{(3)}_{ij}(t) \\ &\times \text{Pf}_{2r_{12}/c}\int_{-\infty}^{+\infty}\frac{dv}{|v|} I^{(3)}_{ij}(t+v), \end{aligned}$$



Nonlocality in time: Tail-transported hereditary effects

(Blanchet-Damour ’88)

Hereditary (time-dissymetric) modification of the quadrupolar radiation-damping force, signalling a breakdown of a basic tenet of PN expansion at the 4PN level: (v/c)^8 fractional

$$g_{00}^{\text{in}}(\mathbf{x},t) = -1 + \frac{1}{c^2}\left[2\int \frac{d^3\mathbf{y}\,\rho(\mathbf{y},t)}{|\mathbf{x}-\mathbf{y}|}\right] + \frac{1}{c^4}\left[\partial_t^2 X - 2U^2 + 4\int \frac{d^3\mathbf{y}}{|\mathbf{x}-\mathbf{y}|}\rho\left[\mathbf{v}^2 + U + \frac{\Pi}{2} + \frac{3p}{2\rho}\right]\right]$$
$$+ \frac{1}{c^6}\,{}_6\hat{\Phi}_{00} + \frac{1}{c^7}\left[-\tfrac{2}{5}x_{ab}\,{}^{(5)}I_{ab}(t)\right] + \frac{1}{c^8}\,{}_8\hat{\Phi}_{00} + \frac{1}{c^9}\,{}_9\hat{\Phi}_{00}$$
$$+ \frac{1}{c^{10}}\left[-\tfrac{8}{5}x_{ab}I(t)\int_0^{+\infty} dv \ln\left[\frac{v}{2P}\right]{}^{(7)}I_{ab}(t-v) + {}_{10}\hat{\Phi}_{00}\right] + \cdots .$$

generates a time-symmetric nonlocal-in-time 4PN-level action (Damour-Jaranowski-Schaefer’14) which was uniquely matched to the local-zone metric via the Regge-Wheeler-Zerilli-Mano-Suzuki-Takasugi- based work of Bini-Damour’13

$$H_{4\text{PN}}^{\text{nonloc}}(t) = -\frac{1}{5}\frac{G^2 M}{c^8} I_{ij}^{(3)}(t) \times \text{Pf}_{2r_{12}/c}\int_{-\infty}^{+\infty}\frac{dv}{|v|} I_{ij}^{(3)}(t+v),$$

Link radiative multipoles <-> source variables

(Blanchet-Damour '89'92, Damour-Iyer'91, Blanchet '95…)

$$\mathrm{U}_{ij}(U) = \mathrm{M}^{(2)}_{ij}(U) + \frac{2GM}{c^3}\int_0^{+\infty} d\tau\, \mathrm{M}^{(4)}_{ij}(U-\tau)\left[\ln\left(\frac{c\tau}{2r_0}\right)+\frac{11}{12}\right]$$

← tail

$$+\frac{G}{c^5}\left\{-\frac{2}{7}\int_0^{+\infty} d\tau\, \mathrm{M}^{(3)}_{a\langle i}(U-\tau)\mathrm{M}^{(3)}_{j\rangle a}(U-\tau)\right.$$

← memory

$$\left. -\frac{2}{7}\mathrm{M}^{(3)}_{a\langle i}\mathrm{M}^{(2)}_{j\rangle a} - \frac{5}{7}\mathrm{M}^{(4)}_{a\langle i}\mathrm{M}^{(1)}_{j\rangle a} + \frac{1}{7}\mathrm{M}^{(5)}_{a\langle i}\mathrm{M}_{j\rangle a} + \frac{1}{3}\varepsilon_{ab\langle i}\mathrm{M}^{(4)}_{j\rangle a}\mathrm{S}_b\right\}$$

← instant.

$$+\frac{2G^2\mathrm{M}^2}{c^6}\int_0^{+\infty} d\tau\, \mathrm{M}^{(5)}_{ij}(U-\tau)\left[\ln^2\left(\frac{c\tau}{2r_0}\right)+\frac{57}{70}\ln\left(\frac{c\tau}{2r_0}\right)+\frac{124627}{44100}\right]$$

← tail-of-tail

$$+\mathcal{O}\left(\frac{1}{c^7}\right).$$

$$\mathrm{M}_{ij} = \mathrm{I}_{ij} - \frac{4G}{c^5}\left[\mathrm{W}^{(2)}\mathrm{I}_{ij} - \mathrm{W}^{(1)}\mathrm{I}^{(1)}_{ij}\right] + \mathcal{O}\left(\frac{1}{c^7}\right)$$

$$\Sigma = \frac{\overline{\tau}^{00}+\overline{\tau}^{ii}}{c^2},$$

$$\Sigma_i = \frac{\overline{\tau}^{0i}}{c},$$

$$\Sigma_{ij} = \overline{\tau}^{ij}$$

$$\mathrm{I}_L(u) = \mathcal{FP}\int d^3\mathbf{x}\int_{-1}^{1} dz\left\{\delta_l\hat{x}_L\Sigma - \frac{4(2l+1)}{c^2(l+1)(2l+3)}\delta_{l+1}\hat{x}_{iL}\Sigma^{(1)}_i\right.$$

$$\left.+\frac{2(2l+1)}{c^4(l+1)(l+2)(2l+5)}\delta_{l+2}\hat{x}_{ijL}\Sigma^{(2)}_{ij}\right\}(\mathbf{x}, u+z|\mathbf{x}|/c), \qquad (85)$$

$$\mathrm{J}_L(u) = \mathcal{FP}\int d^3\mathbf{x}\int_{-1}^{1} dz\, \varepsilon_{ab\langle i_l}\left\{\delta_l\hat{x}_{L-1\rangle a}\Sigma_b - \frac{2l+1}{c^2(l+2)(2l+3)}\delta_{l+1}\hat{x}_{L-1\rangle ac}\Sigma^{(1)}_{bc}\right\}(\mathbf{x}, u+z|\mathbf{x}|/c).$$

Explicit Source Quadrupole Moment at 3.5 PN for a binary system

(Blanchet-Damour-Esposito-Farese-Iyer'05; Blanchet et al;Faye-Marsat-Blanchet-Iyer'12)

$$\mathrm{I}_{ij} = \mu \left(A\, x_{\langle ij \rangle} + B\, \frac{r^2}{c^2} v^{\langle ij \rangle} + \frac{48}{7}\, \frac{G^2 m^2 \nu}{c^5 r}\, C\, x_{\langle i} v_{j \rangle} \right) + \mathcal{O}\left(\frac{1}{c^8}\right)$$

$$A = 1 + \gamma \left(-\frac{1}{42} - \frac{13}{14}\nu \right) + \gamma^2 \left(-\frac{461}{1512} - \frac{18395}{1512}\nu - \frac{241}{1512}\nu^2 \right)$$

$$+ \gamma^3 \left(\frac{395899}{13200} - \frac{428}{105} \ln\left(\frac{r_{12}}{r_0}\right) + \left[\frac{3304319}{166320} - \frac{44}{3} \ln\left(\frac{r_{12}}{r'_0}\right) \right] \nu + \frac{162539}{16632}\nu^2 + \frac{2351}{33264}\nu^3 \right)$$

$$B = \frac{11}{21} - \frac{11}{7}\nu + \gamma \left(\frac{1607}{378} - \frac{1681}{378}\nu + \frac{229}{378}\nu^2 \right)$$

$$+ \gamma^2 \left(-\frac{357761}{19800} + \frac{428}{105} \ln\left(\frac{r_{12}}{r_0}\right) - \frac{92339}{5544}\nu + \frac{35759}{924}\nu^2 + \frac{457}{5544}\nu^3 \right),$$

$$C = 1 + \gamma \left(-\frac{256}{135} - \frac{1532}{405}\nu \right).$$

Perturbative computation of GW flux from binary system

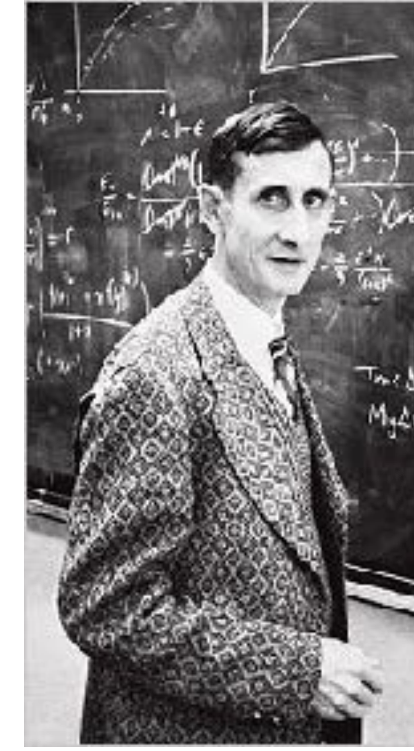
- lowest order : Einstein 1918 Peters-Mathews 63
- 1 + (v^2/c^2) : Wagoner-Will 76
- … + (v^3/c^3) : Blanchet-Damour 92, Wiseman 93
- … + (v^4/c^4) : Blanchet-Damour-Iyer Will-Wiseman 95
- … + (v^5/c^5) : Blanchet 96
- … + (v^6/c^6) : Blanchet-Damour-Esposito-Farèse-Iyer 2004
- … + (v^7/c^7) : Blanchet

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$x = \left(\frac{v}{c}\right)^2 = \left(\frac{G(m_1+m_2)\Omega}{c^3}\right)^{\frac{2}{3}} = \left(\frac{\pi G(m_1+m_2)f}{c^3}\right)^{\frac{2}{3}}$$

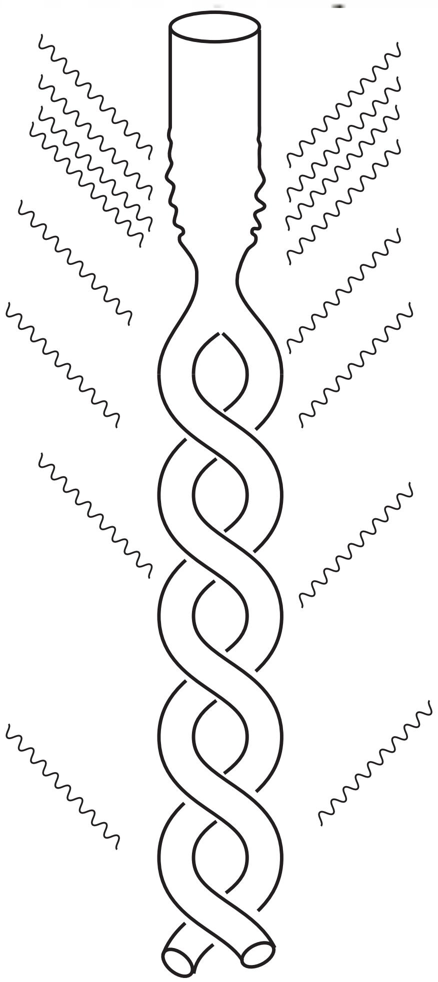
$$\begin{aligned}\mathcal{F} = \frac{32c^5}{5G}\nu^2 x^5 \Bigg\{ & 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x + 4\pi x^{3/2} \\ & + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)x^2 + \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2} \\ & + \Bigg[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}\gamma_E - \frac{856}{105}\ln(16\,x) \\ & \quad + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\Bigg]x^3 \\ & + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Bigg\}.\end{aligned}$$

Pioneering the GWs from coalescing compact binaries



Freeman Dyson 1963, using Einstein 1918 + Landau-Lifshitz 1951 (+ Peters '64)
first vision of an intense GW flash from coalescing binary NS

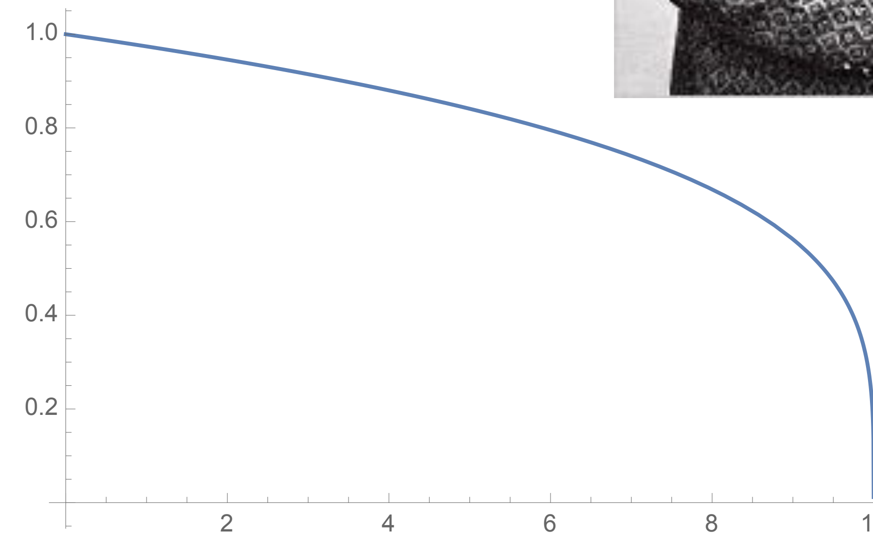
but the final end will be the same. According to (11), the loss of energy by gravitational radiation will bring the two stars closer with ever-increasing speed, until in the last second of their lives they plunge together and release a gravitational flash at a frequency of about 200 cycles and of unimaginable intensity.

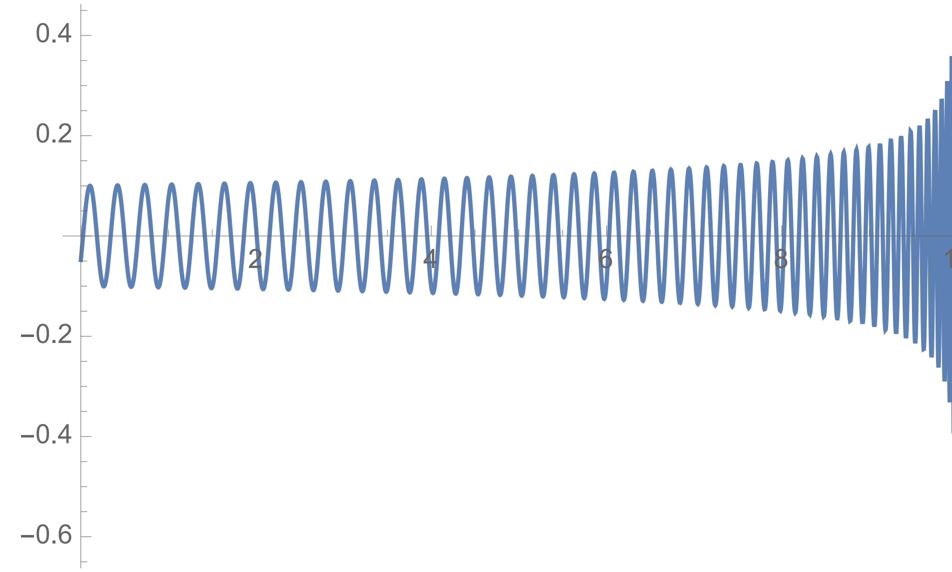


$$E = -\frac{G\, m_1\, m_2}{2r}$$

$$\frac{d}{dt} E = -F$$

$$F = \frac{32\, G^4}{5\, c^5}\, \frac{m_1^2\, m_2^2 (m_1 + m_2)}{r^5}$$





Challenge: describe the intense flash of GWs emitted by the last orbits and the merger of a binary BH, when v~c and r~GM/c^2

Adiabatic, PN-expanded approach to inspiral phasing

Because of matched-filter technique,

$$\langle output | h_{\rm template} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\rm template}(f)$$

the quantity to estimate with highest accuracy if the frequency-domain phase psi(f)

$$\int dt e^{-2\pi i f t} h(t) = \tilde{h}(f) = a(f) e^{-i\psi(f)}$$

Using the stationary phase approximation, psi(f) is the Legendre transform of the time-domain phase phi(t)

$$\tilde{h}(f) = \frac{a(t_f)}{2\sqrt{\dot{F}(t_f)}} e^{-i[2\pi f t_f - (\pi/4) - \phi(t_f)]}.$$

where t_f is the time when the GW frequency omega(t)=d phi(t)/dt reaches 2pi f

PN-expanded phasing in the adiabatic approximation

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\mathcal{F}\,. \qquad x = \left(\frac{v}{c}\right)^2 = \left(\frac{G(m_1+m_2)\Omega}{c^3}\right)^{\frac{2}{3}} = \left(\frac{\pi G(m_1+m_2)f}{c^3}\right)^{\frac{2}{3}}$$

$$\begin{aligned}
E_{\leq 4\mathrm{PN}}(x;\nu) = -\frac{\mu c^2 x}{2}\Bigg(1 &- \left(\frac{3}{4}+\frac{\nu}{12}\right)x + \left(-\frac{27}{8}+\frac{19\nu}{8}-\frac{\nu^2}{24}\right)x^2 + \left(-\frac{675}{64}+\left(\frac{34445}{576}-\frac{205\pi^2}{96}\right)\nu - \frac{155\nu^2}{96}-\frac{35\nu^3}{5184}\right)x^3 \\
&+ \Bigg(-\frac{3969}{128}+\left(\frac{9037\pi^2}{1536}-\frac{123671}{5760}+\frac{448}{15}(2\gamma_\mathrm{E}+\ln(16x))\right)\nu \\
&+ \left(\frac{3157\pi^2}{576}-\frac{498449}{3456}\right)\nu^2 + \frac{301\nu^3}{1728}+\frac{77\nu^4}{31104}\Bigg)x^4\Bigg). \qquad (5.5)
\end{aligned}$$

$$\begin{aligned}
\mathcal{F} = \frac{32c^5}{5G}\nu^2 x^5\Bigg\{1 &+ \left(-\frac{1247}{336}-\frac{35}{12}\nu\right)x + 4\pi x^{3/2} \\
&+ \left(-\frac{44711}{9072}+\frac{9271}{504}\nu+\frac{65}{18}\nu^2\right)x^2 + \left(-\frac{8191}{672}-\frac{583}{24}\nu\right)\pi x^{5/2} \\
&+ \Bigg[\frac{6643739519}{69854400}+\frac{16}{3}\pi^2-\frac{1712}{105}\gamma_\mathrm{E}-\frac{856}{105}\ln(16\,x) \\
&\qquad + \left(-\frac{134543}{7776}+\frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2-\frac{775}{324}\nu^3\Bigg]x^3 \\
&+ \left(-\frac{16285}{504}+\frac{214745}{1728}\nu+\frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Bigg\}.
\end{aligned}$$

$$\frac{dE(x)}{dt} = \frac{dE(x)}{dx}\frac{dx}{dt} = -\mathcal{F}(x)$$

-> ODE for x(t), i.e. phi(t), solved by quadrature

$$v = x^{1/2} = (\pi G M f / c^3)^{1/3}$$

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\psi(f)},$$

$$\begin{aligned}\psi_{3.5}^{(\mathrm{F2})}(f) &= 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\,\nu\,v^5}\left[1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\nu\right)v^2 - 16\pi v^3 + 10\left(\frac{3058673}{1016064} + \frac{5429}{1008}\,\nu + \frac{617}{144}\,\nu^2\right)v^4\right. \\ &+ \pi\left(\frac{38645}{756} - \frac{65}{9}\nu\right)\left\{1 + 3\log\left(\frac{v}{v_{\mathrm{lso}}}\right)\right\}v^5 + \left\{\frac{11583231236531}{4694215680} - \frac{640}{3}\pi^2 - \frac{6848\,\gamma}{21} - \frac{6848}{21}\log(4\,v)\right. \\ &+ \left.\left.\left(-\frac{15737765635}{3048192} + \frac{2255\,\pi^2}{12}\right)\nu + \frac{76055}{1728}\nu^2 - \frac{127825}{1296}\nu^3\right\}v^6 + \pi\left(\frac{77096675}{254016} + \frac{378515}{1512}\,\nu - \frac{74045}{756}\,\nu^2\right)v^7\right]\end{aligned}$$

consider dimensionless « quality factor » of GW phase, to eliminate the « gauge parameters » t_c and phi_c

$$Q_\omega = f^2 \frac{d^2\psi(f)}{df^2} \approx \frac{\omega^2}{\dot{\omega}}$$

Inaccuracy of LO Quadrupole phasing during late inspiral:
-> necessity of including high-order PN corrections

Dimensionless « quality factor » of GW phase $Q_\omega = f^2 \frac{d^2\psi(f)}{df^2} \approx \frac{\omega^2}{\dot\omega}$

LO quadrupole approximation of Q_omega: $Q_\omega^N(\omega) = \frac{5}{3\nu} 2^{-7/3} \omega^{-5/3}$,

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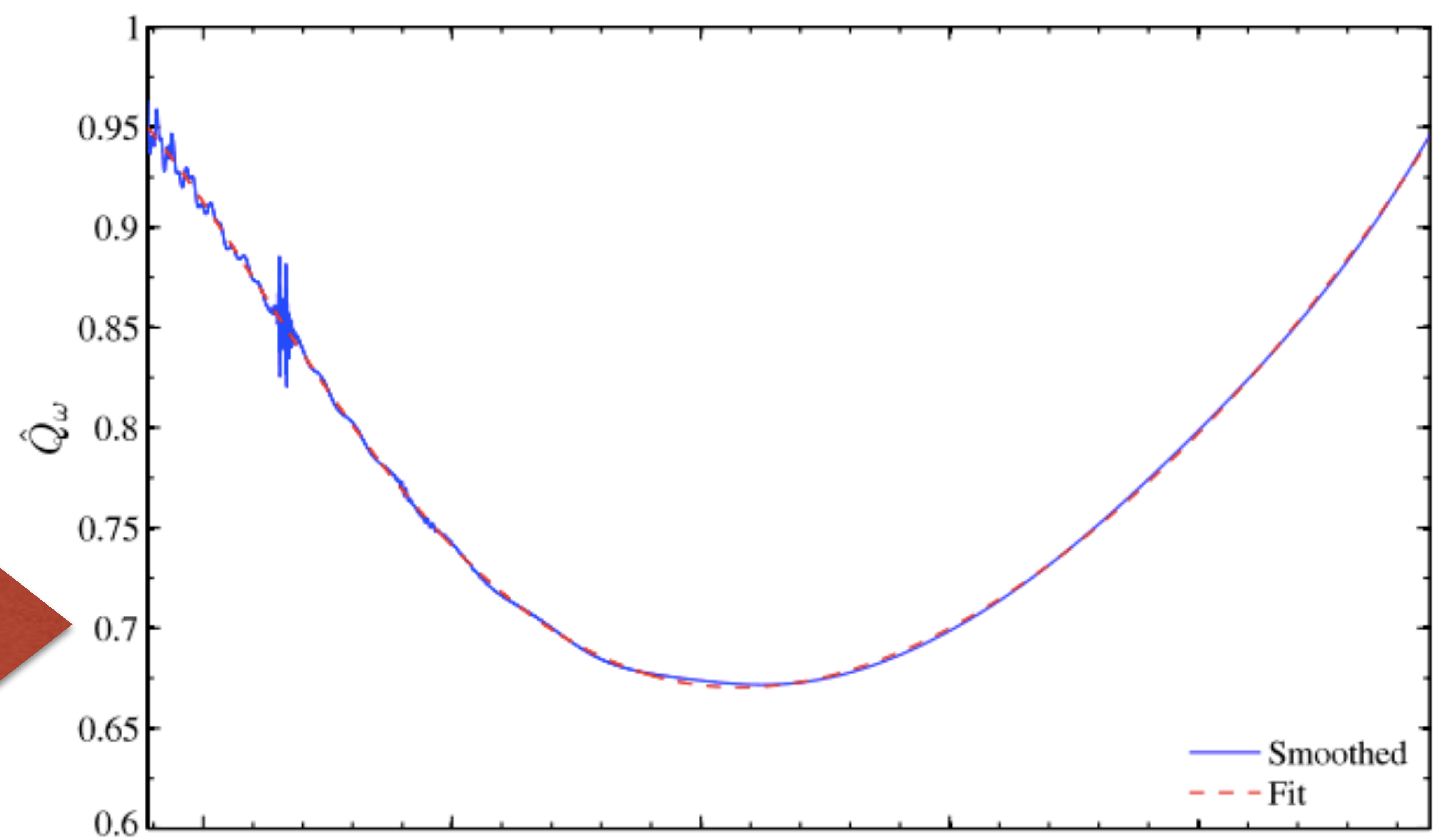
ratio to LO:
$\hat{Q}_\omega(\omega) = Q_\omega / Q_\omega^N$

70%



PN, EOB, NR, PHENOMD

PN accuracy loss during late inspiral

-> necessity to develop an improved description

Dimensionless « quality factor » of GW phase $$Q_\omega = f^2 \frac{d^2\psi(f)}{df^2} \approx \frac{\omega^2}{\dot{\omega}}$$

difference from LO

$$Q_\omega - Q_\omega^N$$

v=1/2 at merger !