From Classical Gravity to Quantum Amplitudes (lecture 3a)

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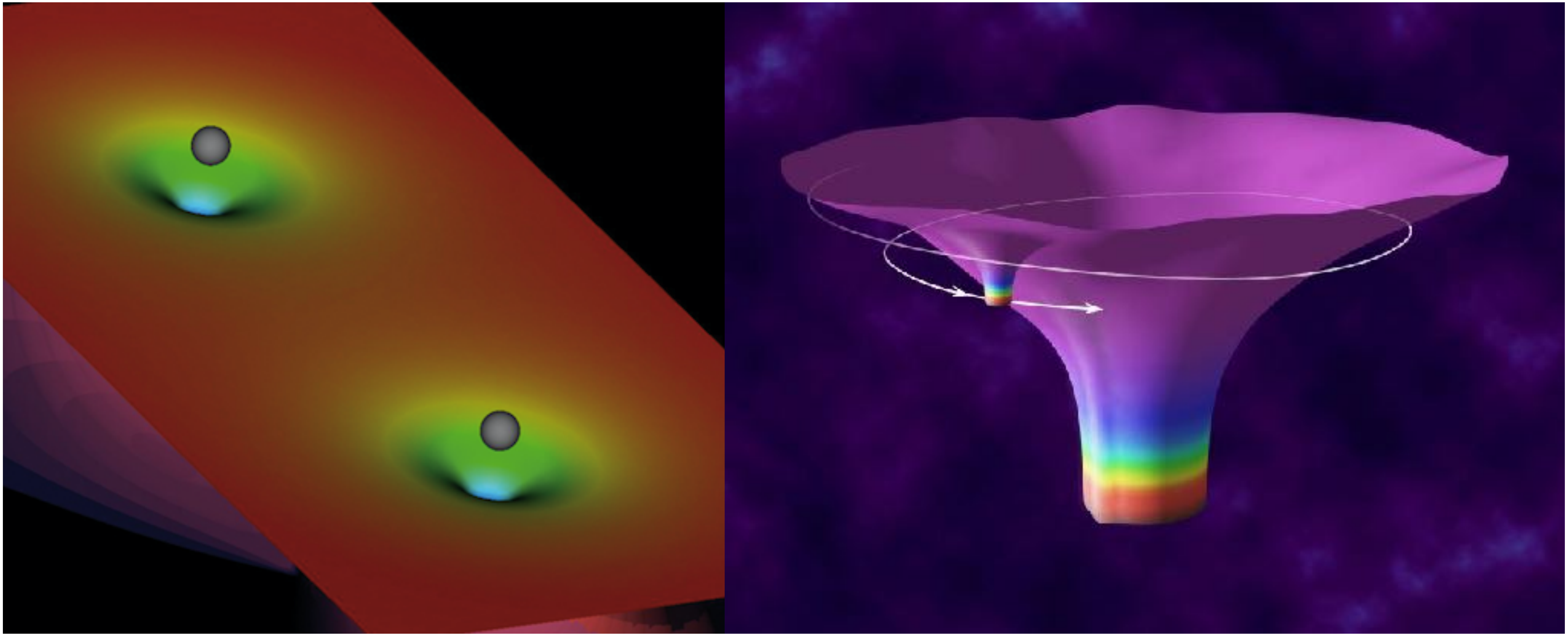
EOB: resumming the dynamics of a two-body system (m_1,m_2,S_1,S_2) in terms of the dynamics of a particle of mass mu and spin S* moving in some effective metric g(M,S)



Effective metric for non-spinning bodies: a nu-deformation of Schwarzschild

$$M = m_1 + m_2 \qquad \mu = \frac{m_1 m_2}{m_1 + m_2} \qquad \nu = \frac{\mu}{M} = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$ds^2_{\text{eff}} = -A(r;\nu)\, dt^2 + B(r;\nu)\, dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

Technical aspects of EOB

the coefficients of the spherically-symmetric effective metric,

$$g_{\mu\nu}^{\rm eff}\, dx^\mu\, dx^\nu = -A(R;\nu)\, c^2\, dT^2 + B(R;\nu)\, dR^2 + R^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,.$$

e looked for as usual-type PN expansions:

Metric

$$A(R;\nu) = 1 + \tilde{a}_1 \frac{GM}{c^2 R} + \tilde{a}_2 \left(\frac{GM}{c^2 R}\right)^2 + \tilde{a}_3 \left(\frac{GM}{c^2 R}\right)^3 + \tilde{a}_4 \left(\frac{GM}{c^2 R}\right)^4 + \cdots ;$$

$$B(R;\nu) = 1 + \tilde{b}_1 \frac{GM}{c^2 R} + \tilde{b}_2 \left(\frac{GM}{c^2 R}\right)^2 + b_3 \left(\frac{GM}{c^2 R}\right)^3 + \cdots ,$$

Energy Map

$$f\left(\frac{E_{\rm real}}{\mu c^2}\right) = 1 + \frac{E_{\rm real}^{\rm binding}}{\mu c^2}\left(1 + \alpha_1 \frac{E_{\rm real}^{\rm binding}}{\mu c^2} + \alpha_2 \left(\frac{E_{\rm real}^{\rm binding}}{\mu c^2}\right)^2 + \alpha_3 \left(\frac{E_{\rm real}^{\rm binding}}{\mu c^2}\right)^3 + \ldots\right).$$

System of algebraic eqs for unknown coefficients: a_n, b_n, alpha_n; impose b1=2:

-> unique solution at 2PN (Buonanno-Damour'99)

$$\alpha_1 = \frac{\nu}{2}, \quad \alpha_2 = 0.$$

—>

$$\frac{\mathcal{E}_0}{m_0 c^2} \equiv \frac{\mathcal{E}_{\rm real}^2 - m_1^2 c^4 - m_2^2 c^4}{2 m_1 m_2 c^4}.$$

$$\hat{a}_2 = 0, \quad \hat{a}_3 = 2\nu, \quad \hat{b}_2 = 4 - 6\nu.$$

—> 2PN effective metric

EOB results at 1PN and 2PN

Theorem 1: The Lorentz-Droste-Einstein-Infeld-Hoffmann 1PN dynamics, considered in the center-of-mass frame, is mapped (at 1PN accuracy) onto the geodesic motion of a particle of mass $\mu = m_1 m_2/(m_1 + m_2)$ in a Schwarzschild background of mass $M = m_1 + m_2$, modulo the very simple (but non trivial) energy map

$$\frac{\mathcal{E}_{\rm eff}}{\mu c^2} = \frac{(\mathcal{E}_{\rm real}^{\rm tot})^2 - m_1^2\, c^4 - m_2^2\, c^4}{2\, m_1\, m_2\, c^4} \quad \text{(at the 1PN, 2PN, 3PN, and 4PN levels).}$$

Theorem 2: The full 2PN dynamics (whose general-frame Hamiltonian contains thirteen 2PN-level independent terms besides the five 1PN-level ones), when considered in the center-of-mass frame, is mapped (at 2PN accuracy) onto the geodesic motion of a particle of mass μ in the following simple ν-deformation of the Schwarzschild metric of mass M,

$$(ds^2_{\rm eff})^{\rm 2PN} = -\left(1 - 2\,\frac{GM}{c^2 R} + 2\nu\left(\frac{GM}{c^2 R}\right)^3\right) c^2 dt^2 + \frac{1 - 6\nu\left(\frac{GM}{c^2R}\right)^2}{1 - 2\frac{GM}{c^2R}} dR^2 + R^2 (d\theta^2 + \sin^2\theta d\varphi^2), \quad (65)$$

modulo the same energy map (54) that appeared at the 1PN level.

energymap

2-body Taylor-expanded 3PN Hamiltonian [JS 98, DJS 01]

$$
\begin{aligned}
c^6 H_{3\mathrm{PN}}(\mathbf{x}_a, \mathbf{p}_a) = & -\frac{5}{128}\frac{(\mathbf{p}_1^2)^4}{m_1^7} + \frac{1}{32}\frac{G m_1 m_2}{r_{12}}\Bigg(-14\frac{(\mathbf{p}_1^2)^3}{m_1^6} + 4\frac{((\mathbf{p}_1\cdot\mathbf{p}_2)^2 + 4\mathbf{p}_1^2\mathbf{p}_2^2)\mathbf{p}_1^2}{m_1^4 m_2^2} + 6\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^4 m_2^2} \\
& - 10\frac{(\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2 + \mathbf{p}_2^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2)\mathbf{p}_1^2}{m_1^4 m_2^2} + 24\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^4 m_2^2} \\
& + 2\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3 m_2^3} + \frac{(7\mathbf{p}_1^2\mathbf{p}_2^2 - 10(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2^3} \\
& + \frac{(\mathbf{p}_1^2\mathbf{p}_2^2 - 2(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2^3} + 15\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3 m_2^3} \\
& - 18\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3 m_2^3} + 5\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3 m_2^3}\Bigg) + \frac{G^2 m_1 m_2}{r_{12}^2}\Bigg(\frac{1}{16}(m_1 - 27m_2)\frac{(\mathbf{p}_1^2)^2}{m_1^4} \\
& - \frac{115}{16}m_1\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2} - \frac{1}{48}m_2\frac{25(\mathbf{p}_1\cdot\mathbf{p}_2)^2 + 371\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2 m_2^2} + \frac{17}{16}\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^3} + \frac{5}{12}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^3} \\
& - \frac{1}{8}m_1\frac{(15\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2) + 11(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1))(\mathbf{n}_{12}\cdot\mathbf{p}_1)}{m_1^3 m_2} - \frac{3}{2}m_1\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2} \\
& + \frac{125}{12}m_2\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2 m_2^2} + \frac{10}{3}m_2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} \\
& - \frac{1}{48}(220m_1 + 193m_2)\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2}\Bigg) + \frac{G^3 m_1 m_2}{r_{12}^3}\Bigg(-\frac{1}{48}\left(425m_1^2 + \left(473 - \frac{3}{4}\pi^2\right)m_1 m_2 + 150m_2^2\right)\frac{\mathbf{p}_1^2}{m_1^2} \\
& + \frac{1}{16}\left(77(m_1^2 + m_2^2) + \left(143 - \frac{1}{4}\pi^2\right)m_1 m_2\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1 m_2} + \frac{1}{16}\left(20m_1^2 - \left(43 + \frac{3}{4}\pi^2\right)m_1 m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2} \\
& + \frac{1}{16}\left(21(m_1^2 + m_2^2) + \left(119 + \frac{3}{4}\pi^2\right)m_1 m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1 m_2}\Bigg) \\
& + \frac{1}{8}\frac{G^4 m_1 m_2^3}{r_{12}^4}\left(\left(\frac{227}{3} - \frac{21}{4}\pi^2\right)m_1 + m_2\right) + (1 \leftrightarrow 2).
\end{aligned}
$$

3PN EOB

$$S = -\mu \int ds_{\rm eff} \left[1 + A_{\mu\nu\kappa\lambda}(x)\, u^\mu\, u^\nu\, u^\kappa\, u^\lambda + \ldots\right].$$

$$\boxed{g_{\rm eff}^{\mu\nu}\, P'_\mu\, P'_\nu + \mu^2\, c^2 + Q(P'_\mu) = 0\,,}$$

Theorem 3: The full 3PN dynamics (whose general-frame Hamiltonian contains ~ 40 3PN-level terms), when considered in the center-of-mass frame, is mapped (at 3PN accuracy), via the simple energy map (54) that appeared at 1PN, onto the motion of a particle of mass μ submitted to the mass-shell condition (38) where $g_{\mu\nu}^{\rm eff}$ is given by Eq. , with

$$A^{\rm 3PN}(u) = 1 - 2u + 2\nu\, u^3 + \left(\frac{94}{3} - \frac{41}{32}\,\pi^2\right) \nu u^4, \tag{71}$$

$$\overline{D}^{\rm 3PN}(u) = 1 + 6\nu\, u^2 + (52\nu - 6\nu^2)\, u^3, \tag{72}$$

and where

$$\widehat{Q}^{\rm 3PN} \equiv \frac{Q}{\mu^2 c^2} = (8\nu - 6\nu^2)\, u^2\, \frac{p_r^4}{c^4}. \tag{73}$$

Here $\overline{D} \equiv (AB)^{-1}$, we used the scaled momentum $p_r \equiv P_R^{\rm EOB}/\mu$ (with dimension $[p] = [\text{velocity}]$ so that p/c is dimensionless, and we introduced the convenient dimensionless EOB variable

$$\boxed{u \equiv \frac{GM}{c^2 R_{\rm EOB}}.} \tag{74}$$

DJS gauge

2-body Taylor-expanded 4PN Hamiltonian [DJS, 2014]

$$c^8 H^{\rm local}_{\rm 4PN}(\mathbf{x}_a,\mathbf{p}_a) = \frac{7(\mathbf{p}_1^2)^5}{256 m_1^9} + \frac{G m_1 m_2}{r_{12}} H_{48}(\mathbf{x}_a,\mathbf{p}_a) + \frac{G^2 m_1 m_2}{r_{12}^2} m_1 H_{46}(\mathbf{x}_a,\mathbf{p}_a)$$
$$+ \frac{G^3 m_1 m_2}{r_{12}^3}\left(m_1^2 H_{441}(\mathbf{x}_a,\mathbf{p}_a) + m_1 m_2 H_{442}(\mathbf{x}_a,\mathbf{p}_a)\right)$$
$$+ \frac{G^4 m_1 m_2}{r_{12}^4}\left(m_1^3 H_{421}(\mathbf{x}_a,\mathbf{p}_a) + m_1^2 m_2 H_{422}(\mathbf{x}_a,\mathbf{p}_a)\right)$$
$$+ \frac{G^5 m_1 m_2}{r_{12}^5} H_{40}(\mathbf{x}_a,\mathbf{p}_a) + (1 \leftrightarrow 2), \quad \text{(A3)}$$

$$H_{48}(\mathbf{x}_a,\mathbf{p}_a) = \frac{45(\mathbf{p}_1^2)^4}{128 m_1^8} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2}{64 m_1^6 m_2^2} + \frac{15(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^3}{64 m_1^6 m_2^2} - \frac{9(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{16 m_1^6 m_2^2}$$
$$- \frac{3(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{32 m_1^6 m_2^2} + \frac{15(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{64 m_1^6 m_2^2} - \frac{21(\mathbf{p}_1^2)^3\mathbf{p}_2^2}{64 m_1^6 m_2^2} - \frac{35(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{256 m_1^5 m_2^3}$$
$$+ \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2}{128 m_1^5 m_2^3} + \frac{33(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3(\mathbf{p}_1^2)^2}{256 m_1^5 m_2^3} - \frac{85(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2^3}$$
$$- \frac{45(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{128 m_1^5 m_2^3} - \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2^3} + \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{64 m_1^5 m_2^3}$$
$$+ \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{64 m_1^5 m_2^3} - \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{64 m_1^5 m_2^3} - \frac{3\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{64 m_1^5 m_2^3} + \frac{55(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3}$$
$$- \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2\mathbf{p}_2^2}{128 m_1^5 m_2^3} - \frac{25(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{256 m_1^5 m_2^3} - \frac{23(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3}$$
$$+ \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{128 m_1^5 m_2^3} - \frac{7(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{256 m_1^5 m_2^3} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4\mathbf{p}_1^2}{64 m_1^4 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4(\mathbf{p}_1^2)^2}{64 m_1^4 m_2^4}$$
$$- \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{4 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{16 m_1^4 m_2^4} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{64 m_1^4 m_2^4} + \frac{21(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2\mathbf{p}_2^2}{64 m_1^4 m_2^4}$$
$$- \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{32 m_1^4 m_2^4} - \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{4 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{16 m_1^4 m_2^4} + \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{16 m_1^4 m_2^4}$$
$$- \frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{32 m_1^4 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_2^2)^2}{64 m_1^4 m_2^4} - \frac{3(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_2^2)^2}{32 m_1^4 m_2^4} - \frac{7(\mathbf{p}_1^2)^2(\mathbf{p}_2^2)^2}{128 m_1^4 m_2^4}, \quad \text{(A4a)}$$

$$H_{46}(\mathbf{x}_a,\mathbf{p}_a) = \frac{369(\mathbf{n}_{12}\cdot\mathbf{p}_1)^6}{160 m_1^6} - \frac{889(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4\mathbf{p}_1^2}{192 m_1^6} + \frac{49(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1^2)^2}{16 m_1^6} - \frac{63(\mathbf{p}_1^2)^3}{64 m_1^6} - \frac{549(\mathbf{n}_{12}\cdot\mathbf{p}_1)^5(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{128 m_1^5 m_2}$$
$$+ \frac{67(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{16 m_1^5 m_2} - \frac{167(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1^2)^2}{128 m_1^5 m_2} + \frac{1547(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2} - \frac{851(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{128 m_1^5 m_2}$$
$$+ \frac{1099(\mathbf{p}_1^2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{256 m_1^5 m_2} + \frac{3263(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{1280 m_1^4 m_2^2} + \frac{1067(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{480 m_1^4 m_2^2} - \frac{4567(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1^2)^2}{3840 m_1^4 m_2^2}$$
$$- \frac{3571(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{320 m_1^4 m_2^2} + \frac{3073(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{480 m_1^4 m_2^2} + \frac{4349(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{1280 m_1^4 m_2^2}$$
$$- \frac{3461\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{3840 m_1^4 m_2^2} + \frac{1673(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4\mathbf{p}_2^2}{1920 m_1^4 m_2^2} - \frac{1999(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2\mathbf{p}_2^2}{3840 m_1^4 m_2^2} + \frac{2081(\mathbf{p}_1^2)^2\mathbf{p}_2^2}{3840 m_1^4 m_2^2} - \frac{13(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{8 m_1^3 m_2^3}$$
$$+ \frac{191(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3\mathbf{p}_1^2}{192 m_1^3 m_2^3} - \frac{19(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2^3} - \frac{5(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2^3}$$
$$+ \frac{11(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{192 m_1^3 m_2^3} + \frac{77(\mathbf{p}_1\cdot\mathbf{p}_2)^3}{96 m_1^3 m_2^3} + \frac{233(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_2^2}{96 m_1^3 m_2^3} - \frac{47(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2\mathbf{p}_2^2}{32 m_1^3 m_2^3}$$
$$+ \frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{384 m_1^3 m_2^3} - \frac{185\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{384 m_1^3 m_2^3} - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4}{4 m_1^2 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_2)^4\mathbf{p}_1^2}{4 m_1^2 m_2^4}$$
$$- \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3(\mathbf{p}_1\cdot\mathbf{p}_2)}{2 m_1^2 m_2^4} + \frac{21(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{16 m_1^2 m_2^4} + \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{6 m_1^2 m_2^4} + \frac{49(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2\mathbf{p}_2^2}{48 m_1^2 m_2^4}$$
$$- \frac{133(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)\mathbf{p}_2^2}{24 m_1^2 m_2^4} - \frac{77(\mathbf{p}_1\cdot\mathbf{p}_2)^2\mathbf{p}_2^2}{96 m_1^2 m_2^4} + \frac{197(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_2^2)^2}{96 m_1^2 m_2^4} - \frac{173\mathbf{p}_1^2(\mathbf{p}_2^2)^2}{48 m_1^2 m_2^4} + \frac{13(\mathbf{p}_2^2)^3}{8 m_2^6}, \quad \text{(A4b)}$$

$$H_{441}(\mathbf{x}_a,\mathbf{p}_a) = \frac{5027(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{384 m_1^4} - \frac{22993(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2}{960 m_1^4} - \frac{6695(\mathbf{p}_1^2)^2}{1152 m_1^4} - \frac{3191(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{640 m_1^3 m_2}$$
$$+ \frac{28561(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{1920 m_1^3 m_2} + \frac{8777(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{384 m_1^3 m_2} - \frac{752969\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{28800 m_1^3 m_2}$$
$$- \frac{16481(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{960 m_1^2 m_2^2} + \frac{94433(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{4800 m_1^2 m_2^2} - \frac{103957(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{2400 m_1^2 m_2^2}$$
$$+ \frac{791(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{400 m_1^2 m_2^2} + \frac{26627(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_2^2}{1600 m_1^2 m_2^2} - \frac{118261\mathbf{p}_1^2\mathbf{p}_2^2}{4800 m_1^2 m_2^2} + \frac{105(\mathbf{p}_2^2)^2}{32 m_2^4}, \quad \text{(A4c)}$$

$$H_{442}(\mathbf{x}_a,\mathbf{p}_a) = \left(\frac{2749\pi^2}{8192} - \frac{211189}{19200}\right)\frac{(\mathbf{p}_1^2)^2}{m_1^4} + \left(\frac{63347}{1600} - \frac{1059\pi^2}{1024}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2\mathbf{p}_1^2}{m_1^4} + \left(\frac{375\pi^2}{8192} - \frac{23533}{1280}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^4}$$
$$+ \left(\frac{10631\pi^2}{8192} - \frac{1918349}{57600}\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} + \left(\frac{13723\pi^2}{16384} - \frac{2492417}{57600}\right)\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2 m_2^2}$$
$$+ \left(\frac{1411429}{19200} - \frac{1059\pi^2}{512}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2\mathbf{p}_1^2}{m_1^2 m_2^2} + \left(\frac{248991}{6400} - \frac{6153\pi^2}{2048}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^2 m_2^2}$$
$$- \left(\frac{30383}{960} + \frac{36405\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} + \left(\frac{1243717}{14400} - \frac{40483\pi^2}{16384}\right)\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2}$$
$$+ \left(\frac{2369}{60} + \frac{35655\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2} + \left(\frac{43101\pi^2}{16384} - \frac{391711}{6400}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)\mathbf{p}_1^2}{m_1^3 m_2}$$
$$+ \left(\frac{56955\pi^2}{16384} - \frac{1646983}{19200}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2}, \quad \text{(A4d)}$$

$$H_{421}(\mathbf{x}_a,\mathbf{p}_a) = \frac{64861\mathbf{p}_1^2}{4800 m_1^2} - \frac{91(\mathbf{p}_1\cdot\mathbf{p}_2)}{8 m_1 m_2} + \frac{105\mathbf{p}_2^2}{32 m_2^2} - \frac{9841(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{1600 m_1^2} - \frac{7(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{2 m_1 m_2}, \quad \text{(A4e)}$$

$$H_{422}(\mathbf{x}_a,\mathbf{p}_a) = \left(\frac{1937033}{57600} - \frac{199177\pi^2}{49152}\right)\frac{\mathbf{p}_1^2}{m_1^2} + \left(\frac{176033\pi^2}{24576} - \frac{2864917}{57600}\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1 m_2} + \left(\frac{282361}{19200} - \frac{21837\pi^2}{8192}\right)\frac{\mathbf{p}_2^2}{m_2^2}$$
$$+ \left(\frac{698723}{19200} + \frac{21745\pi^2}{16384}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2} - \left(\frac{63641\pi^2}{24576} - \frac{2712013}{19200}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1 m_2}$$
$$+ \left(\frac{3200179}{57600} - \frac{28691\pi^2}{24576}\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_2^2}, \quad \text{(A4f)}$$

$$H_{40}(\mathbf{x}_a,\mathbf{p}_a) = -\frac{m_1^4}{16} + \left(\frac{6237\pi^2}{1024} - \frac{169799}{2400}\right) m_1^3 m_2 + \left(\frac{44825\pi^2}{6144} - \frac{609427}{7200}\right) m_1^2 m_2^2. \quad \text{(A4g)}$$

$$H^{\rm nonloc}_{\rm 4PN}(t) = -\frac{1}{5}\frac{G^2 M}{c^8} I^{(3)}_{ij}(t) \times \mathrm{Pf}_{2r_{12}/c}\int_{-\infty}^{+\infty}\frac{dv}{|v|} I^{(3)}_{ij}(t+v).$$

Resummed (non-spinning) 4PN EOB interaction potentials

$$M = m_1 + m_2\,; \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}\,, \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2} = \frac{\mu}{M} \qquad u \equiv \frac{GM}{R\,c^2}$$

$$ds^2_{\rm eff} = -A(r;\nu)\,dt^2 + B(r;\nu)\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right) \qquad \bar{D} \equiv (A\,B)^{-1}$$

$$A(u) = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41\pi^2}{32}\right)\nu u^4 + \left(\left(\frac{2275\pi^2}{512} - \frac{4237}{60} + \frac{128}{5}\gamma_{\rm E} + \frac{256}{5}\ln 2\right)\nu + \left(\frac{41\pi^2}{32} - \frac{221}{6}\right)\nu^2 + \frac{64}{5}\nu\ln u\right)u^5,$$

$$A^{\rm EOB}(u) = {\rm Pade}^1_4[A^{PN}(u)]$$

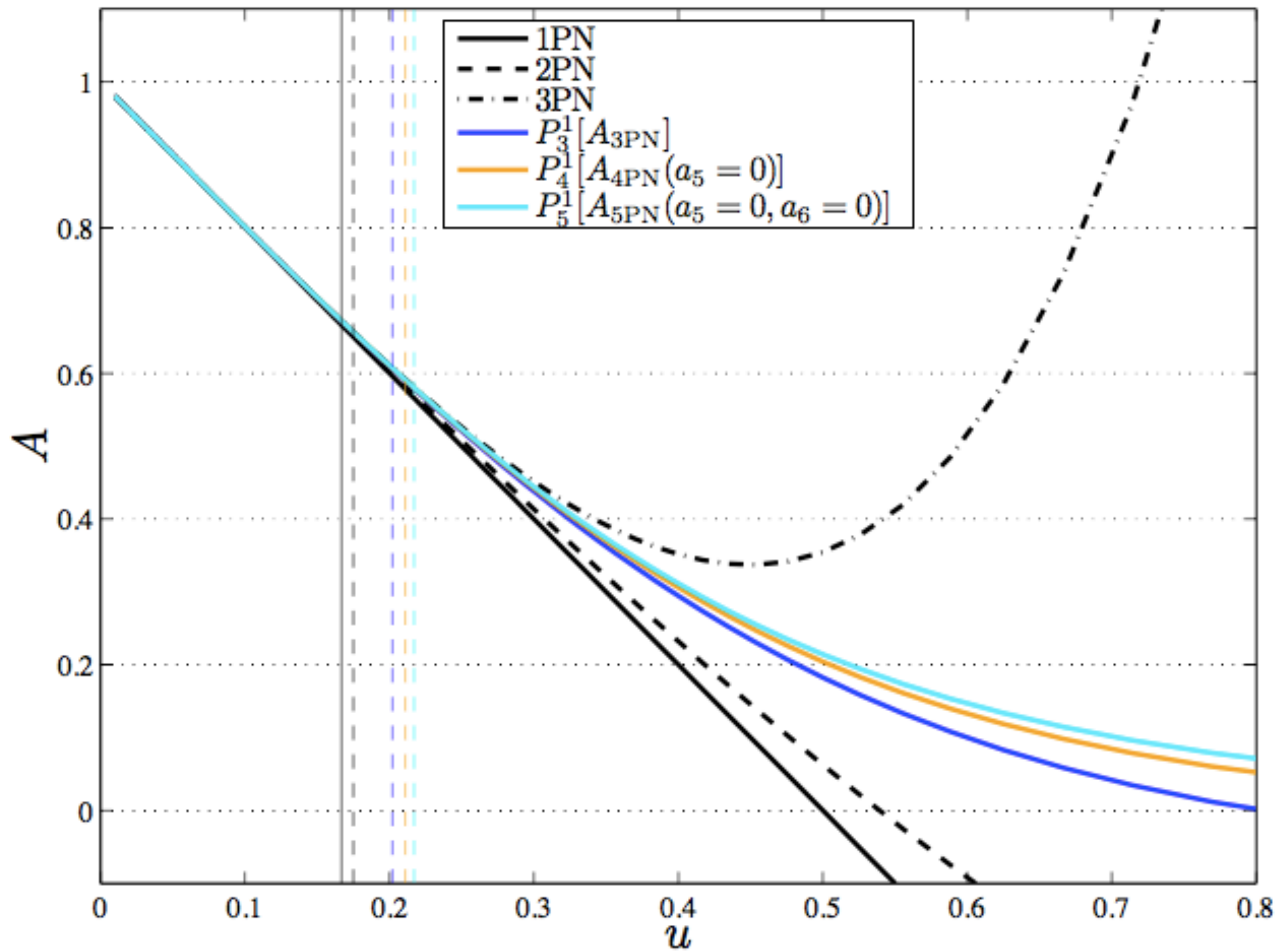
$$\bar{D}(u) = 1 + 6\nu u^2 + (52\nu - 6\nu^2)u^3 + \left(\left(-\frac{533}{45} - \frac{23761\pi^2}{1536} + \frac{1184}{15}\gamma_{\rm E} - \frac{6496}{15}\ln 2 + \frac{2916}{5}\ln 3\right)\nu + \left(\frac{123\pi^2}{16} - 260\right)\nu^2 + \frac{592}{15}\nu\ln u\right)u^4,$$

$$\hat{Q}(\mathbf{r}',\mathbf{p}') = \left(2(4-3\nu)\nu u^2 + \left(\left(-\frac{5308}{15} + \frac{496256}{45}\ln 2 - \frac{33048}{5}\ln 3\right)\nu - 83\nu^2 + 10\nu^3\right)u^3\right)(\mathbf{n}'\cdot\mathbf{p}')^4 + \left(\left(-\frac{827}{3} - \frac{2358912}{25}\ln 2 + \frac{1399437}{50}\ln 3 + \frac{390625}{18}\ln 5\right)\nu - \frac{27}{5}\nu^2 + 6\nu^3\right)u^2(\mathbf{n}'\cdot\mathbf{p}')^6 + \mathcal{O}[\nu u(\mathbf{n}'\cdot\mathbf{p}')^8].$$

Padé-resumming A(u; nu)

$$u = \frac{GM}{c^2 r}$$

$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$



Spinning EOB effective Hamiltonian

Damour'01, Damour-Jaranowski-Schaefer'08,Barausse-Buonanno'10, Damour-Nagar'14, ……

$$H_{\rm eff} = H_{\rm orb} + H_{\rm so} \quad \rightarrow H_{\rm EOB} = Mc^2\sqrt{1+2\nu\left(\frac{H_{\rm eff}}{\mu c^2}-1\right)}$$

$$\hat{H}^{\rm eff}_{\rm orb} = \sqrt{A\left(1 + B_p \boldsymbol{p}^2 + B_{np}(\boldsymbol{n}\cdot\boldsymbol{p})^2 - \frac{1}{1+\frac{(\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2}{r^2}}\frac{(r^2+2r+(\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2)}{\mathcal{R}^4+\Delta(\boldsymbol{n}\cdot\boldsymbol{\chi}_0)^2}((\boldsymbol{n}\times\boldsymbol{p})\cdot\boldsymbol{\chi}_0)^2 + Q_4\right)}.$$

$$H_{\rm so} = G_S \boldsymbol{L}\cdot\boldsymbol{S} + G_{S^*}\boldsymbol{L}\cdot\boldsymbol{S}^*,$$

$$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}\,;\; \mathbf{S}_* = \frac{m_2}{m_1}\mathbf{S_1} + \frac{m_1}{m_2}\mathbf{S_2}\,,$$

Gyrogravitomagnetic ratios (when neglecting spin^2 effects)

$$r^3 G_S^{\rm PN} = 2 - \frac{5}{8}\nu u - \frac{27}{8}\nu p_r^2 + \nu\left(-\frac{51}{4}u^2 - \frac{21}{2}u p_r^2 + \frac{5}{8}p_r^4\right) + \nu^2\left(-\frac{1}{8}u^2 + \frac{23}{8}u p_r^2 + \frac{35}{8}p_r^4\right)$$

$$r^3 G_{S_*}^{\rm PN} = \frac{3}{2} - \frac{9}{8}u - \frac{15}{8}p_r^2 + \nu\left(-\frac{3}{4}u - \frac{9}{4}p_r^2\right) - \frac{27}{16}u^2 + \frac{69}{16}u p_r^2 + \frac{35}{16}p_r^4 + \nu\left(-\frac{39}{4}u^2 - \frac{9}{4}u p_r^2 + \frac{5}{2}p_r^4\right) + \nu^2\left(-\frac{3}{16}u^2 + \frac{57}{16}u p_r^2 + \frac{45}{16}p_r^4\right)$$

NR-completed resummed 5PN EOB radial A potential

« We think, however, that a suitable "numerically fitted" and, if possible, "analytically extended" EOB Hamiltonian should be able to fit the needs of upcoming GW detectors. » (TD 2001)

here Damour-Nagar-Bernuzzi '13, Nagar-etal '16; alternative: Taracchini et al '14, Bohe et al '17

4PN analytically complete + 5 PN logarithmic term in the A(u, nu) function,
With u = GM/R and nu = m1 m2 / (m1 + m2)^2
[Damour 09, Blanchet et al 10, Barack-Damour-Sago 10, Le Tiec et al 11, Barausse et al 11, Akcay et al 12, Bini-Damour 13, Damour-Jaranowski-Schäfer 14, Nagar-Damour-Reisswig-Pollney 15]

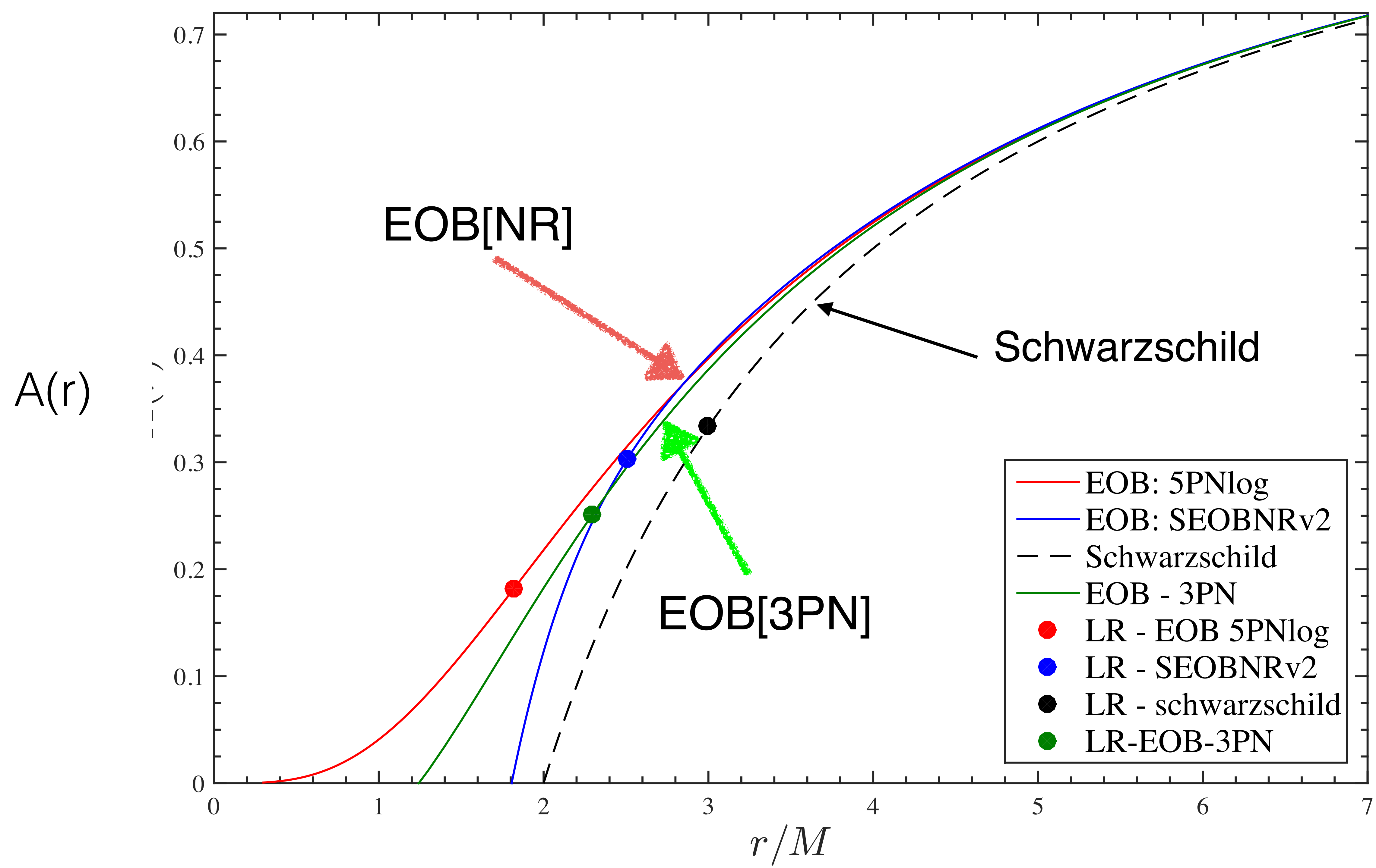
$$A(u;\nu,a_6^c) = P_5^1\Big[1 - 2u + 2\nu\, u^3 + \nu\left(\frac{94}{3} - \frac{41}{32}\pi^2\right) u^4 + \nu\left[-\frac{4237}{60} + \frac{2275}{512}\pi^2 + \left(-\frac{221}{6} + \frac{41}{32}\pi^2\right)\nu + \frac{64}{5}\ln(16 e^{2\gamma} u)\right] u^5 + \nu\left[a_6^c(\nu) - \left(\frac{7004}{105} + \frac{144}{5}\nu\right)\ln u\right] u^6\Big]$$

$$u = \frac{GM}{c^2 R} \qquad \nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$a_6^{c\,\text{NR-tuned}}(\nu) = 81.38 - 1330.6\,\nu + 3097.3\,\nu^2$$

MAIN RADIAL EOB POTENTIAL A(R)

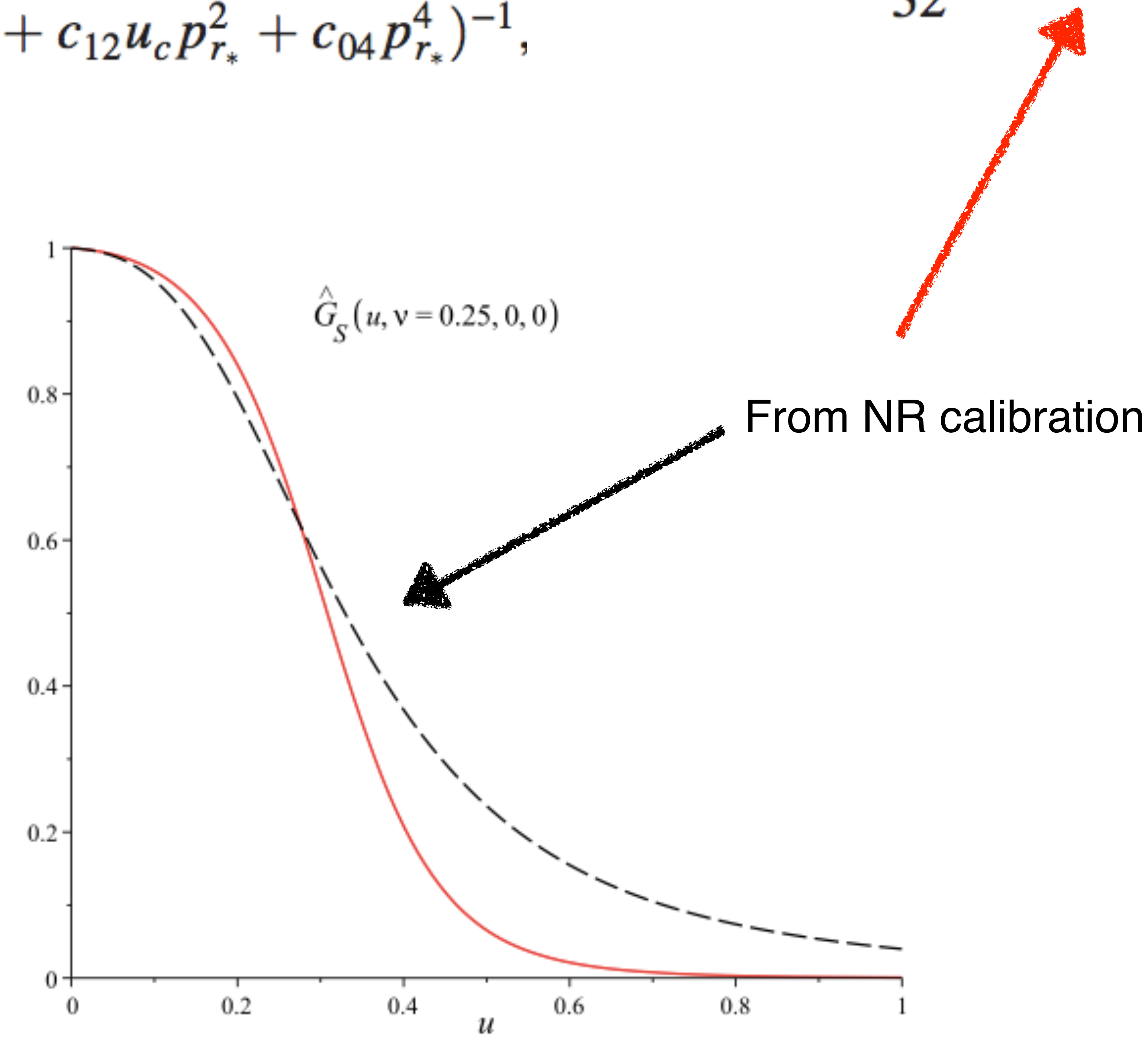
m1=m2 case



NR-COMPLETION OF FIRST EOB GYROGRAVITOMAGNETIC RATIO

$$\hat{G}_S = (1 + c_{10}u_c + c_{20}u_c^2 + c_{30}u_c^3 + c_{02}p_{r_*}^2 + c_{12}u_c p_{r_*}^2 + c_{04}p_{r_*}^4)^{-1},$$

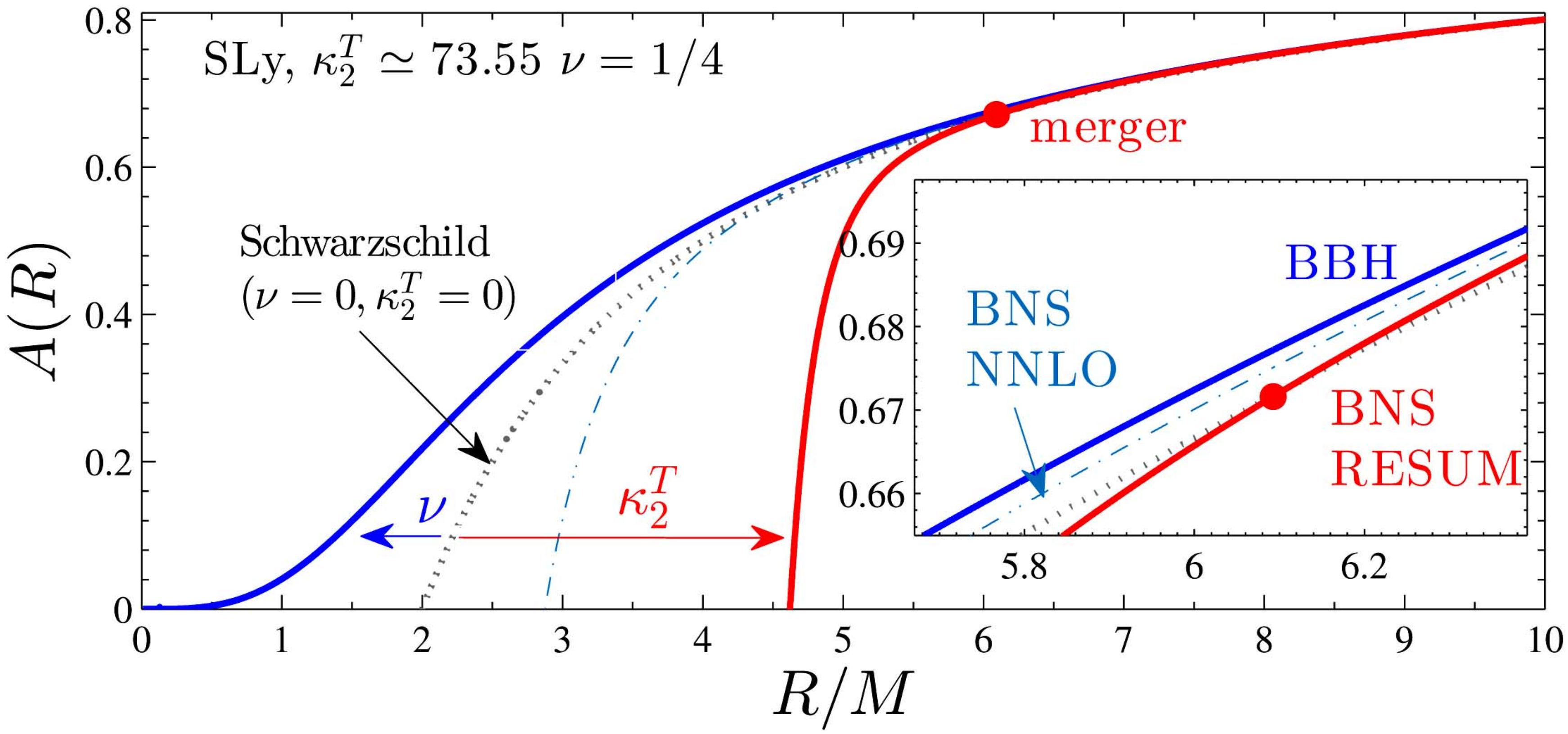
$$c_{30}^* = \frac{135}{32} + \nu c_{3}$$



Tidal extension of EOB (TEOB) [Damour-Nagar 09]

$$A(r) = A_r^0 + A^{\rm tidal}(r)$$
$$A^{\rm tidal}(r) = -\kappa_2^T u^6 \left(1 + \bar{\alpha}_1 u + \bar{\alpha}_2 u^2 + \ldots\right) + \ldots$$

TEOB[NR] A(R) potential (Bernuzzi et al. 2015)

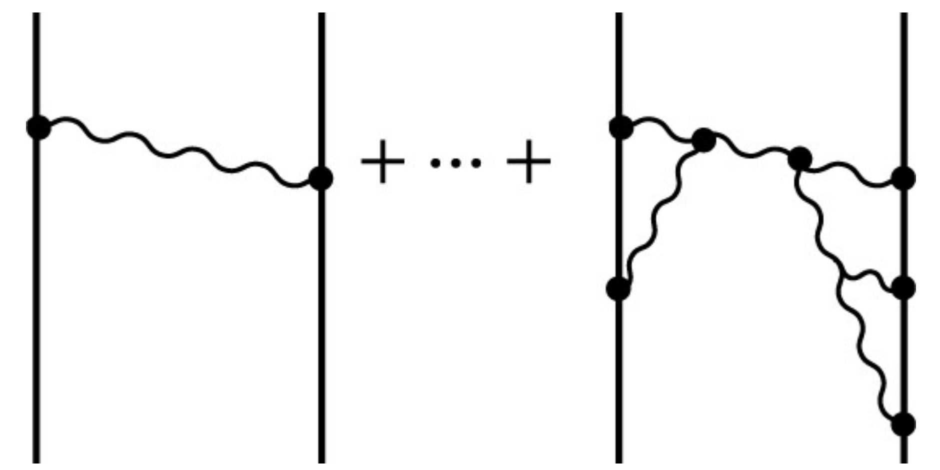


MULTI-MESSENGER + PROBING THE NUCLEAR EOS FROM LATE INSPIRAL TIDAL EFFECTS IN NSNS OR BHNS

(Damour-Nagar-Villain, Agathos-DelPozzo-vandenBroeek, Bernuzzi et al, Hotokezaka et al.,…)

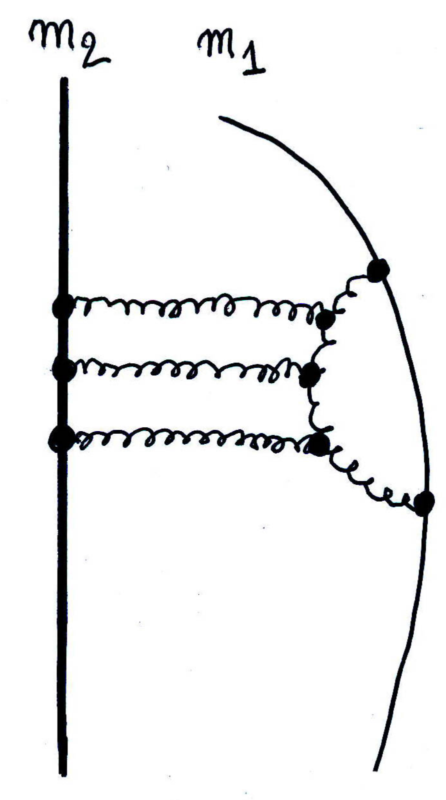
EOB AND GSF: FIRST ORDER IN NU

$$\nu = \frac{\mu}{M} = \frac{m_1 m_2}{(m_1 + m_2)^2}$$



Comparable-mass case: $m_1 \sim m_2$

Gravitational Self-Force Theory : $m_1 << m_2$



- Analytical high-PN results : Blanchet-Detweiler-LeTiec-Whiting '10, Damour '10, Blanchet et al '10, LeTiec et al '12, Bini-Damour '13-15, Kavanagh-Ottewill-Wardell '15 Bini-Damour-Geralico'16, Hopper-Kavanagh-Ottewill'16
- (gauge-invariant) Numerical results : Detweiler '08, Barack-Sago '09, Blanchet-Detweiler-LeTiec-Whiting '10, Barack-Damour-Sago '10, Shah-Friedman-Keidl '12, Dolan et al '14, Nolan et al '15, Akcay-van de Meent '16
- Analytical PN results from high-precision (hundreds to thousands of digits !) numerical results : Shah-Friedman-Whiting '14, Johnson-McDaniel-Shah-Whiting '15

Gravitational Self-Force

Black-Hole perturbation theory `a la Regge-Wheeler-Zerilli-Mano-Suzuki-Takasugi

$$g_{\mu\nu}(x; m_1, m_2) = g^{(0)}_{\mu\nu}(x; m_2) + m_1 h_{\mu\nu}(x) + O(m_1^2)$$

$$h^R_{uu} := \mathrm{Reg}_{x\to y_1}[h_{\mu\nu}(x) u_1^\mu u_1^\nu]. \qquad h^R_{uu} = \sum_{l=0}^{\infty}(h^{(l)}_{uu} - D_0)$$

$$\mathcal{L}^{(r)}_{(\mathrm{RW})}[R^{(\mathrm{odd})}_{lm\omega}] = S^{(\mathrm{odd})}_{lm\omega}(r).$$

$$\mathcal{L}^{(r)}_{(\mathrm{RW})} = f^2(r)\frac{d^2}{dr^2} + \frac{2M}{r^2} f(r)\frac{d}{dr} + [\omega^2 - V_{(\mathrm{RW})}(r)], \qquad S^{(\mathrm{odd})}_{lm\omega}(r) = s_0^{(\mathrm{o})}\delta(r - r_0) + s_1^{(\mathrm{o})}\delta'(r - r_0),$$

$$R^{(\mathrm{even/odd})}_{lm\omega}(r) = \int dr' G(r, r') f(r')^{-1} S^{(\mathrm{even/odd})}_{lm\omega}(r').$$

Here the *retarded* Green function, $G(r, r') = \frac{1}{W}[X_{(\mathrm{in})}(r)X_{(\mathrm{up})}(r')H(r' - r) + X_{(\mathrm{in})}(r')X_{(\mathrm{up})}(r)H(r - r')]$, is computed from the usual "incoming," $X_{(\mathrm{in})}$, and "upgoing," $X_{(\mathrm{up})}$, *homogeneous* solutions of the RW equation. [$H(x)$ denotes a step function, and W the constant Wronskian of $X_{(\mathrm{in})}$ and $X_{(\mathrm{up})}$.]

EOB, SF, EOB[SF], LISA ETC

Remarkable cancellations in EOB expansions in nu=m1m2/(m1+m2)^2: while

$$E_{\le 4\mathrm{PN}}(x;\nu) = -\frac{\mu c^2 x}{2}\left(1 - \left(\frac{3}{4}+\frac{\nu}{12}\right)x + \left(-\frac{27}{8}+\frac{19\nu}{8}-\frac{\nu^2}{24}\right)x^2 + \left(-\frac{675}{64}+\left(\frac{34445}{576}-\frac{205\pi^2}{96}\right)\nu - \frac{155\nu^2}{96}-\frac{35\nu^3}{5184}\right)x^3 + \left(-\frac{3969}{128}+\left(\frac{9037\pi^2}{1536}-\frac{123671}{5760}+\frac{448}{15}(2\gamma_E+\ln(16x))\right)\nu + \left(\frac{3157\pi^2}{576}-\frac{498449}{3456}\right)\nu^2+\frac{301\nu^3}{1728}+\frac{77\nu^4}{31104}\right)x^4\right). \quad (5.5$$

$$A(u) = 1-2u+2\nu u^3+\left(\frac{94}{3}-\frac{41\pi^2}{32}\right)\nu u^4+\left(\left(\frac{2275\pi^2}{512}-\frac{4237}{60}+\frac{128}{5}\gamma_E+\frac{256}{5}\ln 2\right)\nu+\left(\frac{41\pi^2}{32}-\frac{221}{6}\right)\nu^2+\frac{64}{5}\nu\ln u\right)u^5,$$

Computation of 4PN O(nu) term in A from numerical (Barausse-Buonanno-LeTiec'12) and analytical (Bini-Damour'13) SF computation;
Confirmation of all 4PN O(nu) terms of Damour-Jaranowski-Schaefer'14'15 from SF computations (Barack-Damour-Sago, Bini-Damour-Geralico, van de Meent)

EOB[SF] program: improve the few EOB gauge-invariant potentials by SF-computing (analytically or numerically) the contributions linear in nu. Recently implemented for A, B, Q, gS, gS* (Bini,Damour,Geralico,Kavanagh,Akcay, van de Meent, Hopper,Wardell,Ottewill,…

Aim: define template banks for LISA

GSF : ANALYTICAL HIGH-PN RESULTS

Bini-Damour 15

$$
\begin{aligned}
a_{10}^{c} &= \frac{18605478842060273}{7079830758000}\ln(2) - \frac{1619008}{405}\zeta(3) - \frac{21339873214728097}{1011404394000}\gamma \\
&+ \frac{27101981341}{100663296}\pi^6 - \frac{6236861670873}{125565440}\ln(3) + \frac{360126}{49}\ln(2)\ln(3) + \frac{180063}{49}\ln(3)^2 \\
&- \frac{121494974752}{9823275}\ln(2)^2 - \frac{24229836023352153}{549755813888}\pi^4 + \frac{1115369140625}{124540416}\ln(5) + \frac{96889}{2779} \\
&+ \frac{75437014370623318623299}{18690753201120000} - \frac{60648244288}{9823275}\ln(2)\gamma + \frac{200706848}{280665}\gamma^2 \\
&+ \frac{11980569677139}{2306867200}\pi^2 + \frac{360126}{49}\gamma\ln(3), \\
a_{10}^{\ln} &= -\frac{21275143333512097}{2022808788000} + \frac{200706848}{280665}\gamma - \frac{30324122144}{9823275}\ln(2) + \frac{180063}{49}\ln(3), \\
a_{10}^{\ln^2} &= \frac{50176712}{280665}, \\
a_{10.5}^{c} &= -\frac{185665618769828101}{24473489040000}\pi + \frac{377443508}{77175}\ln(2)\pi + \frac{2414166668}{1157625}\pi\gamma - \frac{5846788}{11025}\pi^3 - 24 \\
a_{10.5}^{\ln} &= \frac{1207083334}{1157625}\pi.
\end{aligned}
$$

Kavanagh et al 15

$$
\begin{aligned}
c_{15} = &- \tfrac{206954345058376961934037 6724}{32547744208650608 4375}\zeta(3) + \tfrac{651950262982450079 36}{22370298575625}\gamma\zeta(3) - \tfrac{5049442304}{25725}\gamma^2\zeta(3) + \tfrac{1262360576}{15435}\pi^2\zeta(3) \\
&+ \tfrac{171722752}{441}\zeta(3)^2 + \tfrac{1613866959570176}{496621125}\zeta(5) - \tfrac{343445504}{441}\gamma\zeta(5) - \tfrac{146997248}{105}\zeta(7) + \tfrac{56314978304}{385875}\zeta(3)\log^2(2) \\
&- \tfrac{106445664}{343}\zeta(3)\log^2(3) + \tfrac{151670998244849797696}{22370298575625}\zeta(3)\log(2) - \tfrac{190336581632}{1157625}\gamma\zeta(3)\log(2) \\
&+ \tfrac{28863591064624341}{4909804900}\zeta(3)\log(3) - \tfrac{212891328}{343}\gamma\zeta(3)\log(3) - \tfrac{212891328}{343}\zeta(3)\log(2)\log(3) - \tfrac{77186767578125}{19876428}\zeta(3)\log \\
&- \tfrac{2039263232}{3675}\zeta(5)\log(2) - \tfrac{49128768}{49}\zeta(5)\log(3) + \tfrac{29826742751501839701973659217528941950139153944429084 9}{658761222254465322614246840503191731953125 0} \\
&- \tfrac{680766176845363776831328694806032908750141 9}{704310948124803722562607729544062500}\gamma + \tfrac{1598346944412603247831006289829388}{526171715038677033591890625}\gamma^2 - \tfrac{1007647146215971027644}{335890033113009375} \\
&+ \tfrac{461219496448}{72930375}\gamma^4 - \tfrac{28338275082077591587855063450276303790065762907243197}{99970315584514341811574404579275571200000 0}\pi^2 + \tfrac{25191178655399275691104}{67178006622601875}\gamma\pi^2 \\
&- \tfrac{230609748224}{14586075}\gamma^2\pi^2 + \tfrac{105480323357757226894713787760391180776248036241}{304245354831316028025099055320268800000}\pi^4 + \tfrac{1262360576}{385875}\gamma\pi^4 \\
&- \tfrac{62084728396129669726914571311 43}{26693015135410024611840 0}\pi^6 + \tfrac{357317878192092911828132 9}{151996487423754240}\pi^8 - \tfrac{10136323685888}{72930375}\log^4(2) + \tfrac{38438712}{2401}\log^4(3) \\
&- \tfrac{177896086126482679647872}{54963823600310625}\log^3(2) - \tfrac{89686013106176}{364651875}\gamma\log^3(2) + \tfrac{153754848}{2401}\log^3(2)\log(3) \\
&- \tfrac{131463845322790269123}{245735735245000}\log^3(3) + \tfrac{153754848}{2401}\gamma\log^3(3) + \tfrac{153754848}{2401}\log(2)\log^3(3) + \tfrac{11933074267578125}{51161925672}\log^3(5) \\
&+ \tfrac{387825867416662897459542063520020 4}{189421817413923732093080625}\log^2(2) - \tfrac{344085637991460169215116 8}{1007670099339028125}\gamma\log^2(2) - \tfrac{16582891400192}{121550625}\gamma^2\log^2(2) \\
&+ \tfrac{4145722850048}{72930375}\pi^2\log^2(2) - \tfrac{523697163373483905609}{245735735245000}\log^2(2)\log(3) + \tfrac{461264544}{2401}\gamma\log^2(2)\log(3) \\
&+ \tfrac{45454535766189065888302299261759}{65697282267898830348800 00}\log^2(3) - \tfrac{394391535968370807369}{245735735245000}\gamma\log^2(3) + \tfrac{230632272}{2401}\gamma^2\log^2(3) \\
&- \tfrac{96096780}{2401}\pi^2\log^2(3) - \tfrac{437493411770075173449}{245735735245000}\log(2)\log^2(3) + \tfrac{461264544}{2401}\gamma\log(2)\log^2(3) \\
&+ \tfrac{230632272}{2401}\log^2(2)\log^2(3) + \tfrac{11933074267578125}{17053975224}\log^2(2)\log(5) - \tfrac{250584269699314594370549804687 5}{402136320895332222431232}\log^2(5) \\
&+ \tfrac{11933074267578125}{17053975224}\gamma\log^2(5) + \tfrac{11933074267578125}{17053975224}\log(2)\log^2(5) + \tfrac{479295083164704151420102 51}{564646351702118400 00}\log^2(7) \\
&- \tfrac{1816360672168952204215377476852536997344946 59}{633879853312323350306346956589656250 0}\log(2) + \tfrac{742036621552191085437995316530101 36}{473554543534809330232701562 5}\gamma\log(2) \\
&- \tfrac{148216932652249251549939 2}{1007670099339028125}\gamma^2\log(2) - \tfrac{4905667647488}{364651875}\gamma^3\log(2) + \tfrac{371228115490667668451168}{604602059603416875}\pi^2\log(2) \\
&+ \tfrac{1226416911872}{72930375}\gamma\pi^2\log(2) + \tfrac{23792072704}{17364375}\pi^4\log(2) - \tfrac{414115837539718030238709512493585574772 7}{108266631596274488880198656000000}\log(3) \\
&+ \tfrac{945935800113157545433205527623 9}{691550339662092951040000}\gamma\log(3) - \tfrac{394391535968370807369}{245735735245000}\gamma^2\log(3) + \tfrac{153754848}{2401}\gamma^3\log(3) \\
&+ \tfrac{131463845322790269123}{196588588196000}\pi^2\log(3) - \tfrac{192193560}{2401}\gamma\pi^2\log(3) + \tfrac{8870472}{1715}\pi^4\log(3) \\
&+ \tfrac{21441150106021138984596292714838 1}{131394564535797660697600 00}\log(2)\log(3) - \tfrac{437493411770075173449}{122867867622500}\gamma\log(2)\log(3) \\
&+ \tfrac{461264544}{2401}\gamma^2\log(2)\log(3) - \tfrac{192193560}{2401}\pi^2\log(2)\log(3) + \tfrac{978612948501709853277095576118865234375}{179427491919561270211323849031 68}\log(5) \\
&- \tfrac{250584269699314594370549804687 5}{201068160447666111215616}\gamma\log(5) + \tfrac{11933074267578125}{17053975224}\gamma^2\log(5) - \tfrac{59665371337890625}{204647702688}\pi^2\log(5) \\
&- \tfrac{250584269699314594370549804687 5}{201068160447666111215616}\log(2)\log(5) + \tfrac{11933074267578125}{8526987612}\gamma\log(2)\log(5) \\
&- \tfrac{585800617379230891566511301391464808 1}{323919193207512802977792000000}\log(7) + \tfrac{479295083164704151420102 51}{282323175851059200 00}\gamma\log(7) \\
&+ \tfrac{479295083164704151420102 51}{282323175851059200 00}\log(2)\log(7) + \tfrac{740024994425816010121 1}{656763448320000 00}\log(11),
\end{aligned}
$$

Numerical SF computation of a(u) function

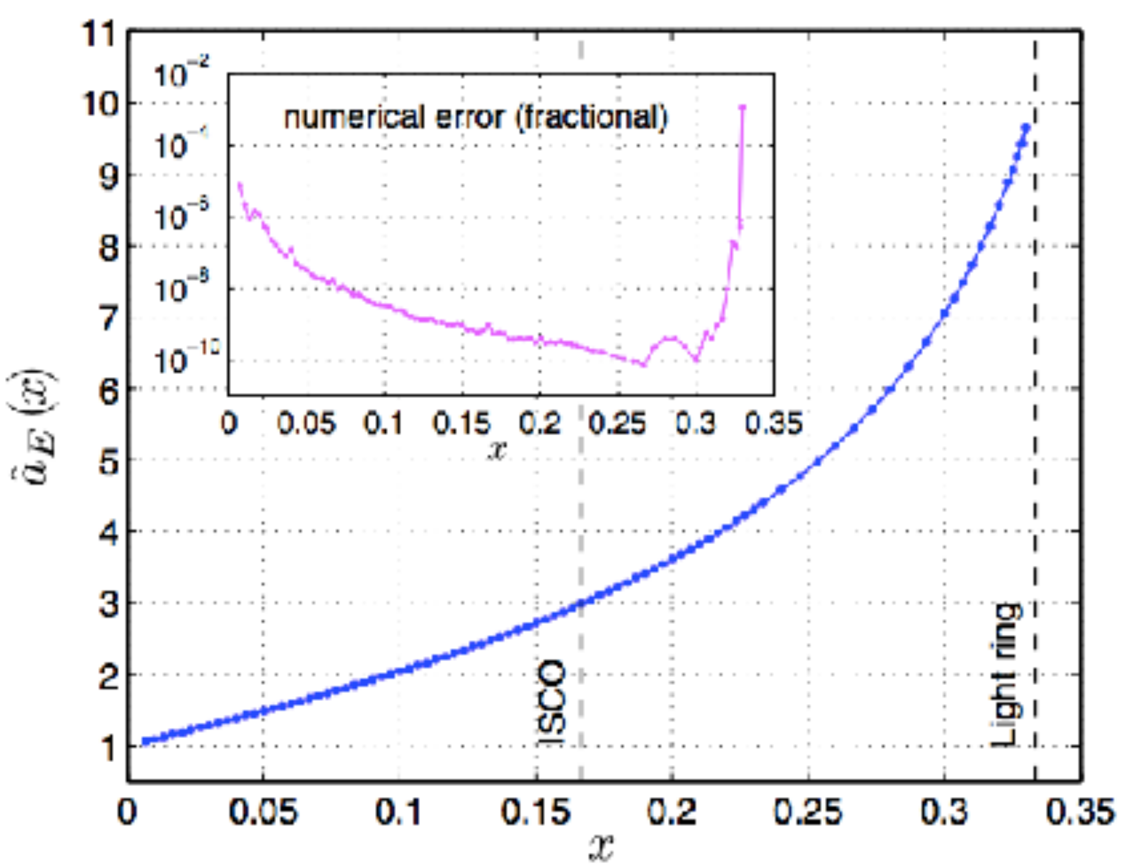
(using LeTiec-Blanchet-Whiting'12, Barausse-Buonanno-LeTiec'12), Akcay-Barack-Damour-Sago '12

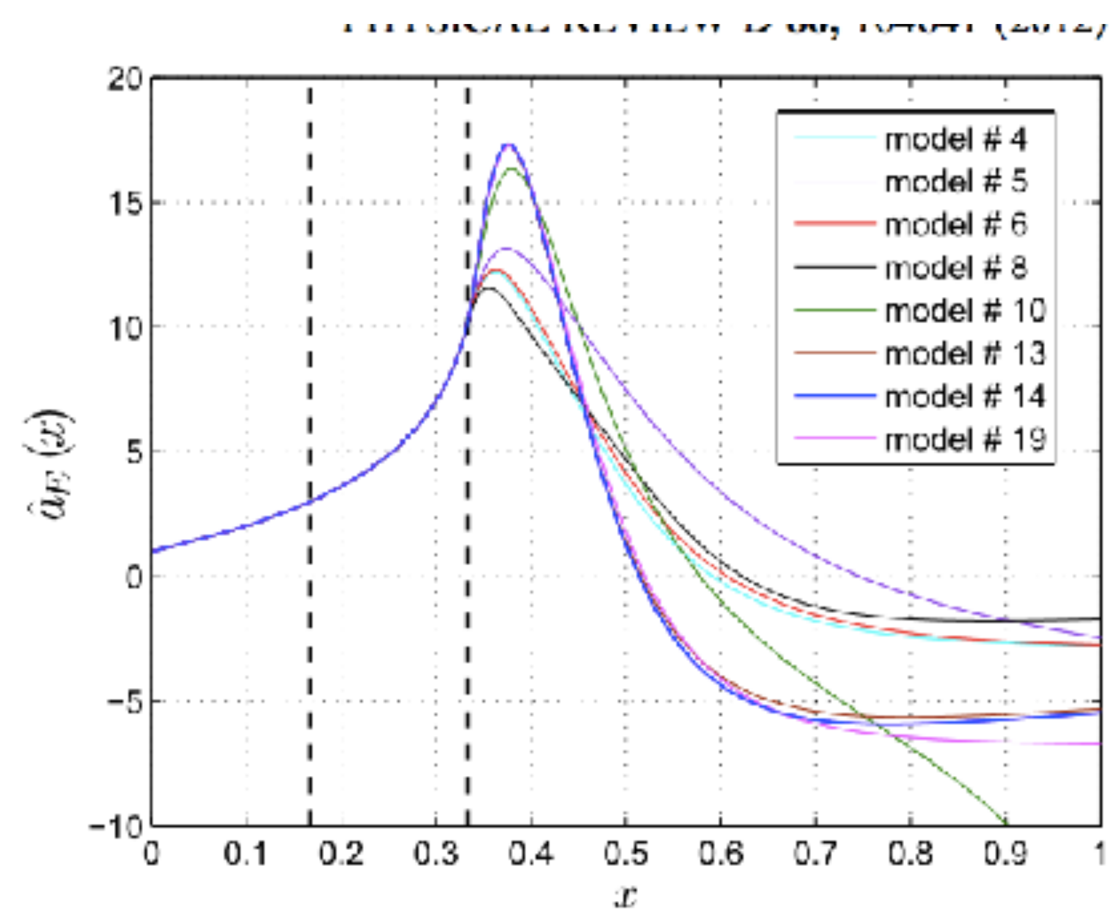
$$A(u,\nu) = 1 - 2u + \nu a(u) + O(\nu^2) \; ; \quad u \equiv \frac{GM}{c^2 R}$$

Singularity at the light-ring: u=1/3

$$a\left(x \to \frac{1}{3}\right) \sim \frac{\zeta}{4}(1-3x)^{-1/2},$$

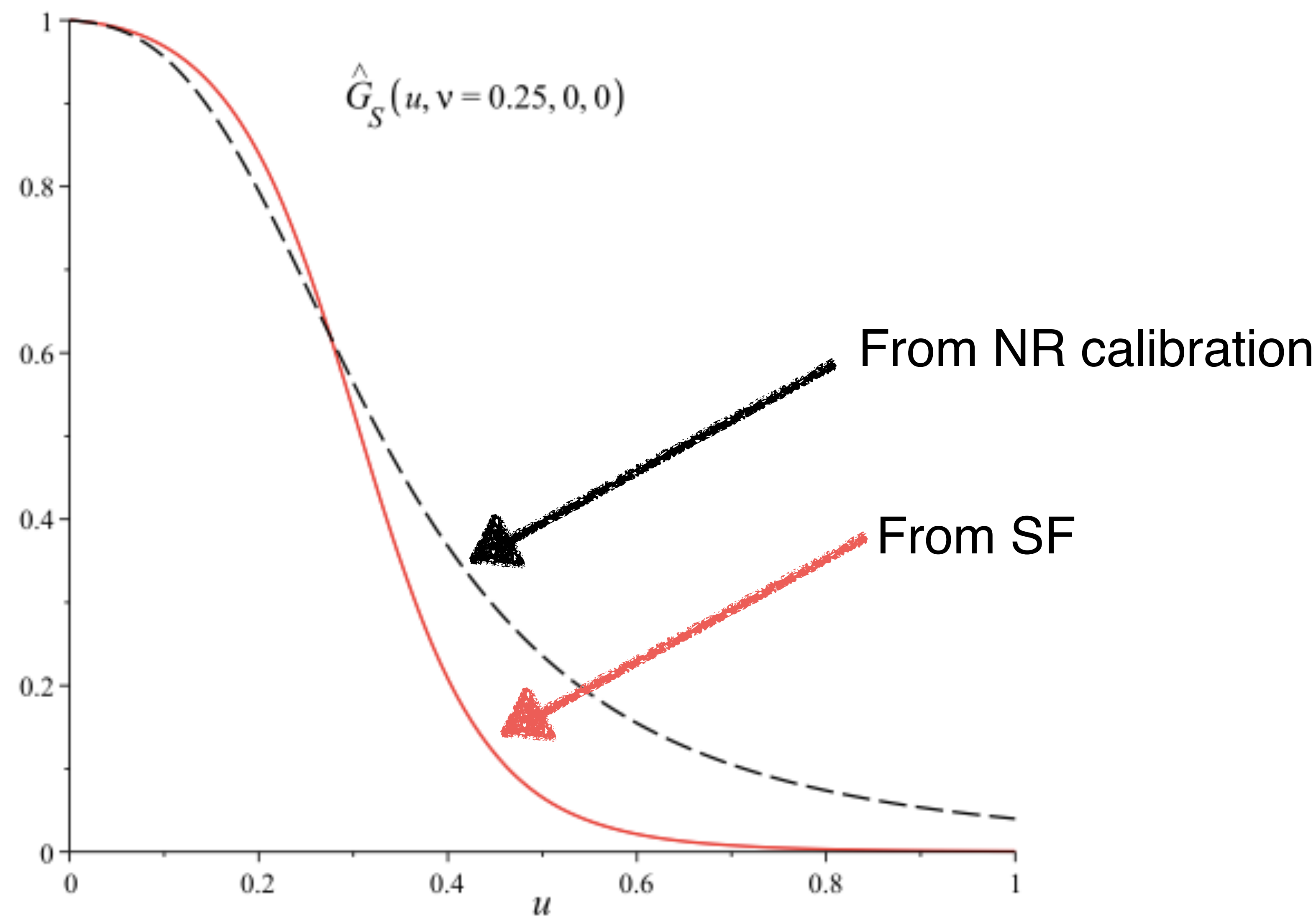
Factorize the energy $E(x) = \frac{1-2x}{\sqrt{1-3x}},$ and the LO PN: 2u^3

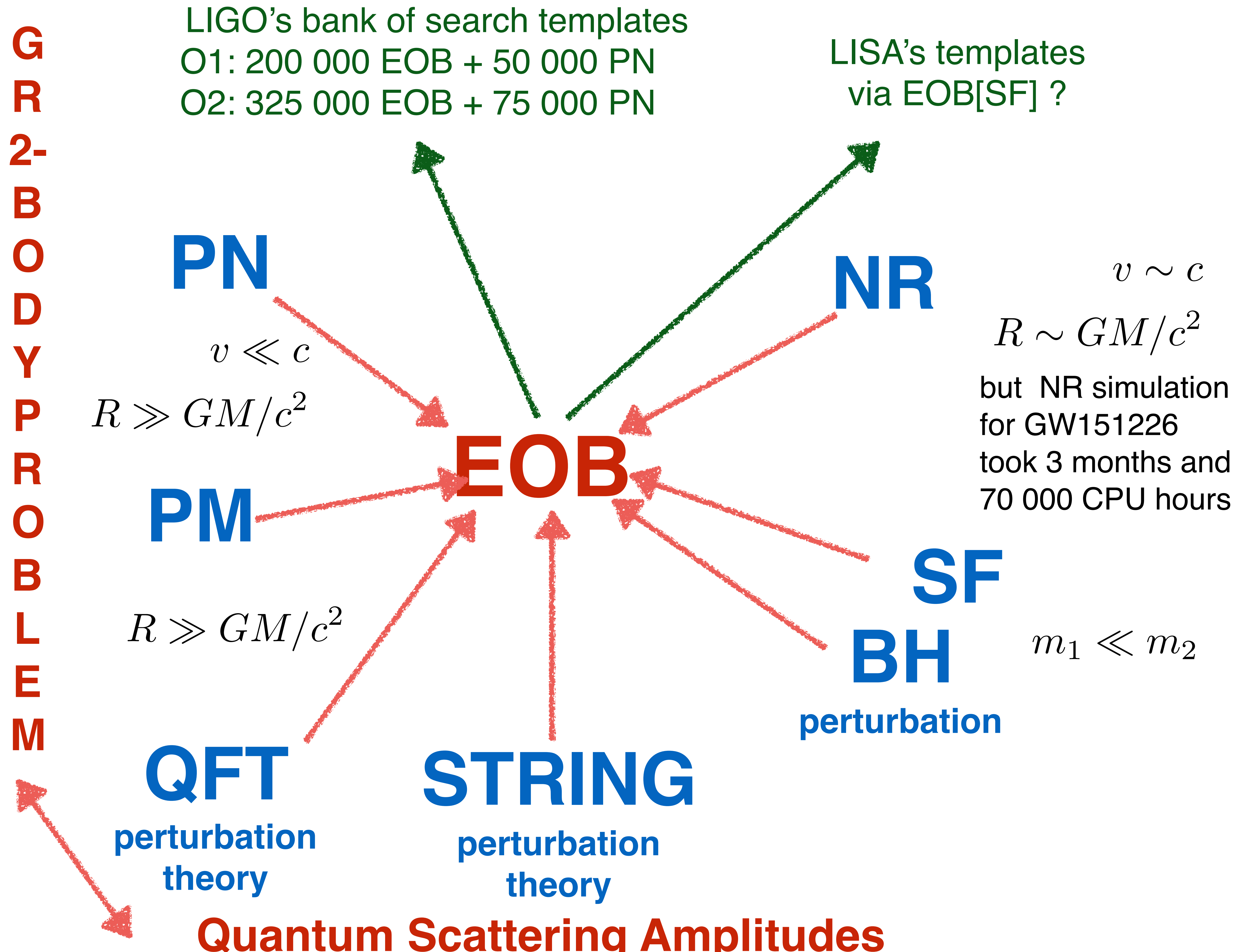




FIRST EOB GYROGRAVITOMAGNETIC RATIO FROM SF

Bini-Damour-Geralico'15




G R 2- B O D Y P R O B L E M
LIGO's bank of search templates
O1: 200 000 EOB + 50 000 PN
O2: 325 000 EOB + 75 000 PN
LISA's templates
via EOB[SF] ?
PN
$v \ll c$
$R \gg GM/c^2$
NR
$v \sim c$
$R \sim GM/c^2$
but NR simulation
for GW151226
took 3 months and
70 000 CPU hours
EOB
PM
$R \gg GM/c^2$
SF
BH
$m_1 \ll m_2$
perturbation
QFT
perturbation
theory
STRING
perturbation
theory
Quantum Scattering Amplitudes