Shining Light on Modifications of Gravity

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Unique Lorentz invariant spin 2 effective theory = General Relativity (Weinberg 1965)

GR + ordinary matter does not lead to acceleration

Dark energy and modified gravity require extra degrees of freedom: scalars

Scalars acting on cosmological scales have a low mass and mediate a long range force

Massive gravity involves massive gravitons= 2 helicity 2, 2 helicity 1 and 1 scalar

$$\mathcal{L} = \frac{1}{M_P} h_{\mu\nu} T^{\mu\nu}$$

The coupling involves the metric:

$$h_{\mu\nu} = \hat{h}_{\mu\nu} + \beta\pi\eta_{\mu\nu} + \frac{(6c_3 - 1)}{\Lambda_3^3}\partial_\mu\pi\partial_\nu\pi$$

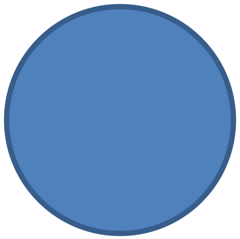
Normalised graviton

Conformal coupling of scalar, β =1 for massive gravitons

Disformal coupling

The conformal coupling is strongly constrained by the coupling to baryonic matter

$$\phi = -\frac{\beta M_c}{4\pi M_p r} e^{-mr} \ .$$



Dense body mass M radius R

The scalar force is:

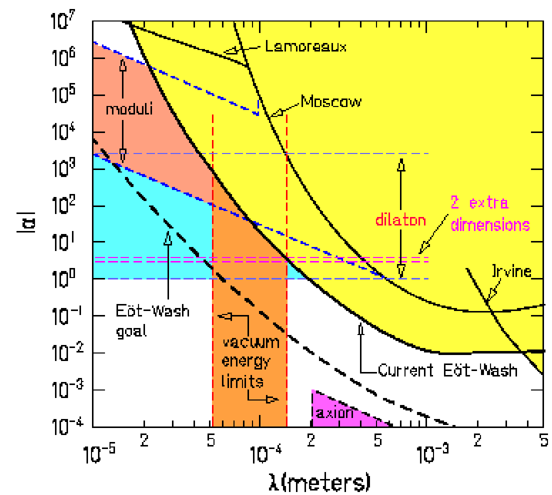
$$\left|\frac{F_\phi}{F_N}\right| = 2\beta^2 (1 + mr) e^{-mr} \ .$$

Deviations from Newton's law are parametrised by:

$$\phi_N = -\frac{G_N}{r}(1 + 2\beta^2 e^{-r/\lambda})$$

For fields of zero mass or of the order of the Hubble rate now, the tightest constraint on β comes from the Cassini probe measuring the Shapiro effect (time delay):

$$\beta^2 \leq 1.210^{-5}$$

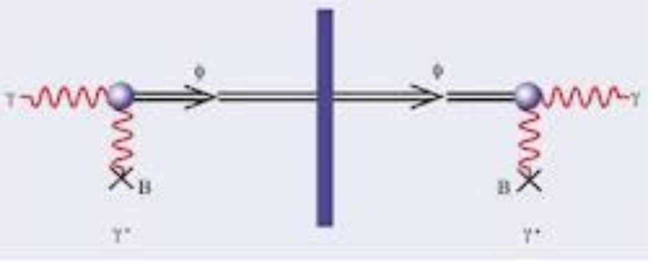


The effect of a long range scalar field must be screened to comply with this bound: Vainshtein mechanism.

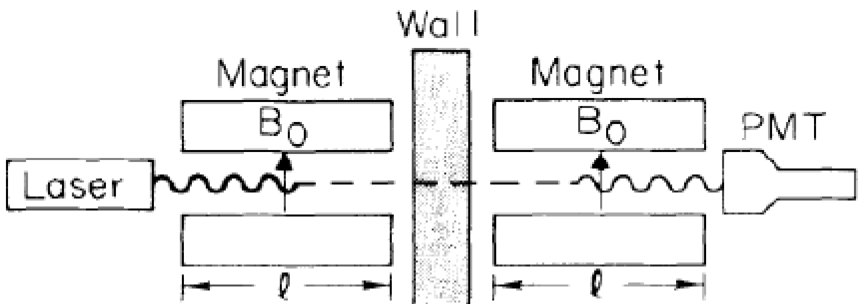
Disformal couplings not tested by static tests of gravity:

$$\frac{\partial_\mu\phi\partial_\nu\phi}{M^4}T^{\mu\nu} = \frac{\dot\phi^2}{M^4}\rho \to 0 \qquad M^4 = M_P^4 m_{\rm grav}^2$$

Disformal couplings can be tested thanks to the coupling to photons.

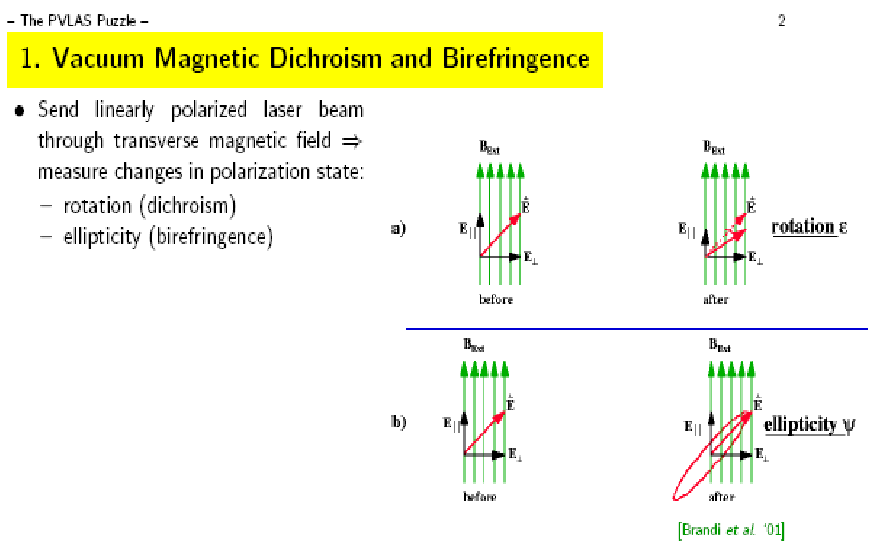


Light shining through a wall:



K. Van Bibber, et. al., PRL 59, 759 (1987)

Laser Polarisation:



The interaction Lagrangian is:

$$\mathcal{L}_{\phi,\gamma} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F^2 - \frac{\phi}{\Lambda}F^2 - \frac{1}{M^4}\partial_\mu\phi\partial_\nu\phi\left[\frac{1}{4}g^{\mu\nu}F^2 - F^\mu{}_\alpha F^{\nu\alpha}\right]$$

where the coupling involves

$$\tilde{g}_{\mu\nu} = \left(1 + \frac{\phi}{\Lambda}\right)g_{\mu\nu} + \frac{2}{M^4}\partial_\mu\phi\partial_\nu\phi$$

The Klein-Gordon equation:

$$\begin{aligned}\partial^2\phi\left(1 + \frac{1}{2M^4}F^2\right) - \frac{2}{M^4}\partial_\mu\partial_\nu\phi F^\mu{}_\alpha F^{\nu\alpha} \\ + \frac{2}{M^4}\partial_\nu\phi(2\partial^\nu\partial_\alpha A_\beta F^{\alpha\beta} + \partial^2 A_\alpha F^{\nu\alpha}) \;&=\; V' + \frac{1}{\Lambda}F^2\end{aligned}$$

Maxwell's equation:

$$\begin{aligned}0 \;=\; & \partial^2 A_\rho + \frac{4}{\Lambda}(\phi\partial^2 A_\rho + F_{\sigma\rho}\partial^\sigma\phi) \\ & + \frac{1}{M^4}\left[\begin{array}{l}\partial^2 A_\rho(\partial\phi)^2 + 4F_{\sigma\rho}\partial_\alpha\phi\partial^\sigma\partial^\alpha\phi + 2(\partial_\sigma F_{\nu\rho})\partial^\sigma\phi\partial^\nu\phi \\ + 2F_{\nu\rho}\partial^2\phi\partial^\nu\phi + 2\partial^2 A_\nu\partial_\rho\phi\partial^\nu\phi - 2F_{\nu\sigma}(\partial^\sigma\partial_\rho\phi\partial^\nu\phi - \partial_\rho\phi\partial^\sigma\partial^\nu\phi)\end{array}\right]\end{aligned}$$

in the Lorentz gauge.

We have included a scalar potential V , the field feels the effective potential:

$$V_{\mathrm{eff}}(\phi) = V(\phi) + \frac{\phi}{\Lambda}F^2$$

In a static magnetic, we assume that this potential has a minimum (e.g massive field). We consider perturbations around this configuration

$$\begin{aligned} \phi &\rightarrow \phi_0 + \phi \, , \\ A_\mu &\rightarrow \frac{1}{2}\delta_{\mu i}\epsilon_{ijk}B_j x_k + A_\mu \end{aligned}$$

The Klein-Gordon equation:

$$\partial^2\phi\left(1 + \frac{B^2}{M^4}\right) - \frac{2}{M^4}(\nabla\phi B^2 - \partial_i\partial_j\phi B^i B^j) = m^2\phi - \frac{2}{\Lambda}\epsilon_{ijk}B_j(\partial_k A_i - \partial_i A_k)$$

Maxwell:

$$\left(1 + \frac{4\phi_0}{\Lambda}\right)\partial^2 A_\mu + \frac{4}{\Lambda}\delta_{\mu i}B_j\epsilon_{ijk}\partial_k\phi = 0$$

For the canonically normalised field, when interested in photons propagating along x in a magnetic field along z, only the y polarisation of photons is affected and mixes with scalar:

$$a = 2\sqrt{\frac{-\phi_0}{\Lambda}},\ b = \frac{B}{M^2}$$

$$\left[\omega - i\partial_x + \omega \begin{pmatrix} \frac{2\omega^2 b^2 - m^2}{2\omega(1+b^2)} & \frac{am}{\sqrt{2}\sqrt{1-a^2}\sqrt{1+b^2}} \\ \frac{am}{\sqrt{2}\sqrt{1-a^2}\sqrt{1+b^2}} & 0 \end{pmatrix}\right] \begin{pmatrix} \phi \\ A_y \end{pmatrix} = 0$$

This leads to oscillations (like neutrino flavours):

$$\begin{pmatrix} \phi(x) \\ A_y(x) \end{pmatrix} = P \begin{pmatrix} e^{-i\omega(1+\lambda_+)x} & 0 \\ 0 & e^{-i\omega(1+\lambda_-)x} \end{pmatrix} P^{-1} \begin{pmatrix} \phi(0) \\ A_y(0) \end{pmatrix},$$

The mixing matrix is :

$$P = \begin{pmatrix} \sin\vartheta & -\cos\vartheta \\ \cos\vartheta & \sin\vartheta \end{pmatrix},\ \tan 2\vartheta = \frac{4B}{\Lambda\omega}\sqrt{\frac{1+b^2}{1-a^2}}\left(\frac{m^2}{2\omega^2} - b^2\right)^{-1},$$

The propagating modes have eigenfrequencies

$$\lambda_\pm = -\lambda(\cos 2\vartheta \mp 1)\ ,\ \lambda = \frac{1}{2(1+b^2)}\left|\frac{m^2}{2\omega^2} - b^2\right|(1+\tan^2 2\vartheta)^{1/2}.$$

Most importantly, the transition probability after a length x is:

$$P_{\gamma\to\phi} = \sin^2 2\vartheta \sin^2 \lambda\omega x \ .$$

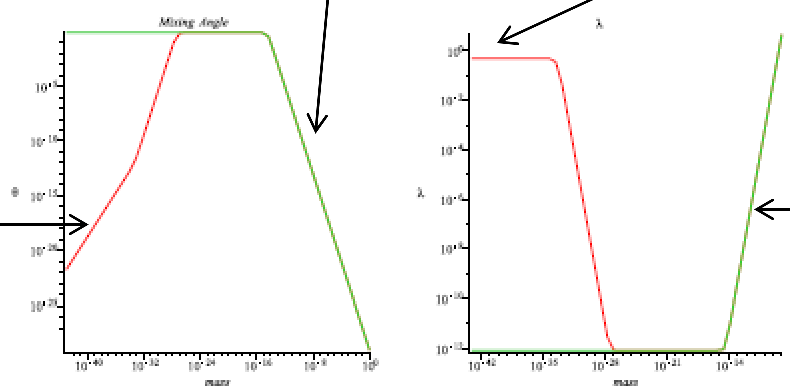
In the weak mixing angle limit:

$$\vartheta \approx \frac{2B}{\Lambda\omega}\sqrt{1+b^2}\left(\frac{m^2}{2\omega^2} - b^2\right)^{-1} \ll 1$$

Three different regimes:

ALP

Modified gravity



Modified gravity

ALP

(a) Mixing Angle, ϑ

(b) Oscillation Wavelength, λ

Figure 1. The mixing parameters ϑ and λ plotted as a function of mass m. We have taken $\Lambda = 10^5$ GeV, $\phi_0 = 10^{-2}\Lambda$ and $M^2 = mM_P$. We take $B = 5$ Tesla and $\omega = 2.33$ eV, the experimental parameters for the ALPS experiment. The green line shows the standard result for axion-like particles with $b = 0$, the red line shows how the effects of including a disformal coupling dominate at very low masses, which correspond to large b.

The light shining through a wall at DESY gives the best bound for photons of energy 2.33 eV, a magnetic field of 5T and a pipe of length 4.3m

$$\mathcal{P}_{\gamma\to\phi} < 2.08 \times 10^{-25}$$

Long range graviton:

$$M = \sqrt{M_p H_0} = 3 \times 10^{-11}\ \text{GeV}$$

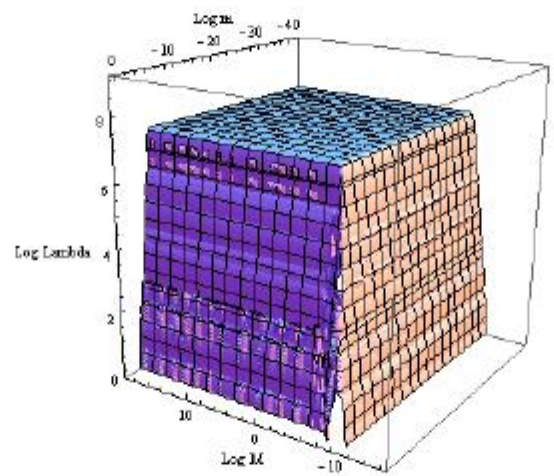


Figure 3. The constraint of the ALPS experiment on the m, M, Λ parameter space. All regions below the surface are excluded. The parameters are measured in units of GeV.

For a graviton with a range at the Hubble scale, only small values of Λ are excluded. For larger values $\Lambda \geq 10^7$ GeV no constraints.

Polarisation experiments such as PVLAS, BMV (Toulouse) etc… give complementary constraints:

$$A_\gamma = \cos^2\vartheta e^{-i\omega(1+\lambda_+)x} + \sin^2\vartheta e^{-i\omega(1+\lambda_-)x} \approx A\cos(\omega x + \delta x)$$

where the phase shift and the amplitude are:

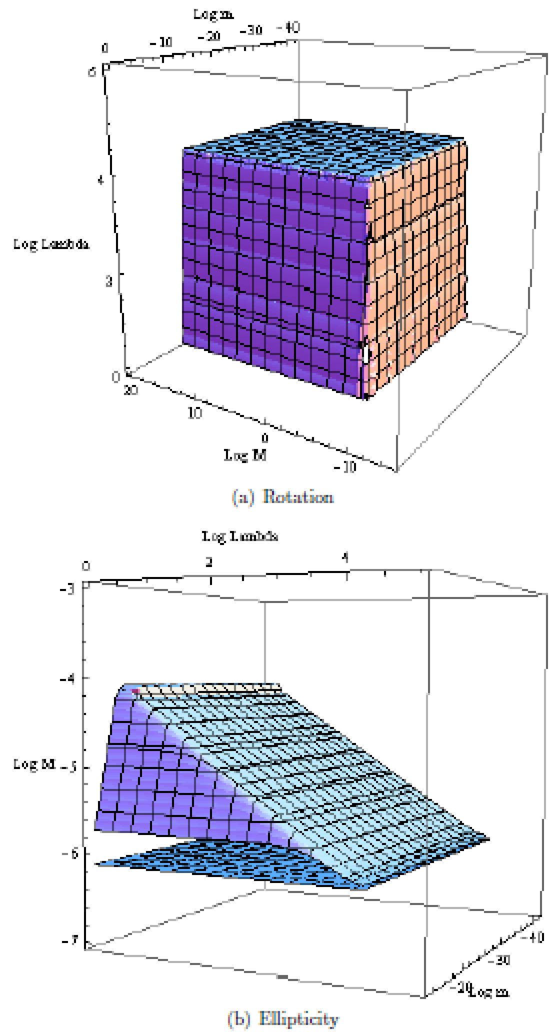
$$\delta x \approx 2\vartheta^2(\lambda\omega x - \tan\lambda\omega x), \;\; A \approx 1 - \vartheta^2\sin^2\lambda\omega x$$

The best constraints are still given by the (correct) PVLAS results:

$$\frac{|1-A|}{2} < 1.0\times 10^{-8}\ \text{rad}\ , \ \Psi = \frac{\delta x}{2} < 1.4\times 10^{-8}$$

for photons of energy 1.17 eV, a magnetic field of 2.3T and a cavity of size 1m.

Rotation better than ellipticity. Not as good as light shining through a wall.



(a) Rotation

(b) Ellipticity

Figure 5. The constraint of the PVLAS rotation and ellipticity measurements on the m, M, Λ parameter space. All regions inside the surfaces are excluded. All quantities are measured in units of GeV.

Conclusion and outlook

Matter coupled conformally and disformally is modified gravity

Disformal coupling evades static gravity tests

Optics, good testing ground! So far, weak experimental constraints.

Prospects: effects on the CMB polarisation? Effects on the opacity of the Universe?