Frustrated magnets and spin liquids

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Outline

- Frustration in **classical** spin models
  Frustrated Ising models
  Pyrochlore lattice & spin ice
  3-component spins and non-planar structures

- Frustration in **quantum** spin models
  Antiferromagnetic long ranged order
  Examples of spin gap materials

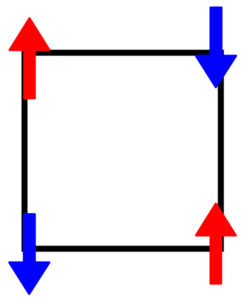
- Spin liquids
  Zero modes on the kagome lattice and spin wave theory
  Short-range RVB spin liquids, spinons and fractionalization

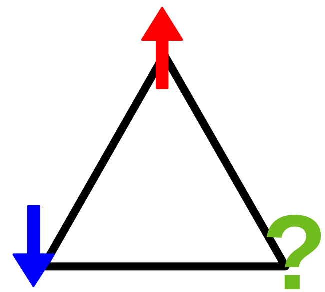
Classical spins

The simplest magnetic frustration – classical Ising models

$$E = +\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z \qquad \sigma_i^z = \pm 1$$

- Antiferromagnetism on a square and triangular lattice.
  Satisfaction (M. Jagger 1965) and *frustration*.

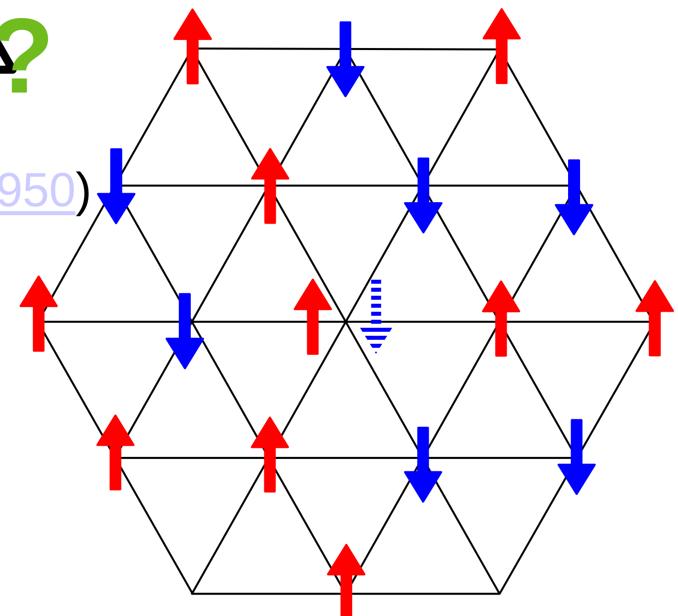




- Extensive entropy at T=0 for the Ising model (Wannier 1950)

$$S(0) = R\frac{2}{\pi}\int_0^{\pi/3} \ln(2\cos\omega)d\omega = 0.3383R.$$

- Power law spin-spin correlations at T=0.
- Effect of perturbations ?
  Example: (quantum) transverse field
  Moessner & Sondhi 2001.

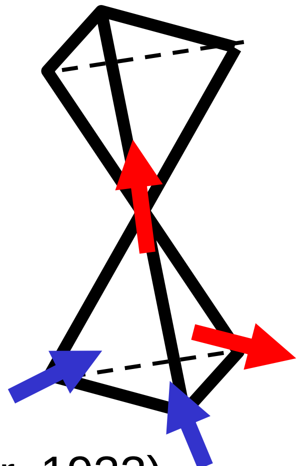


Ising(-like) spins on a frustrated 3d lattice & spin ice

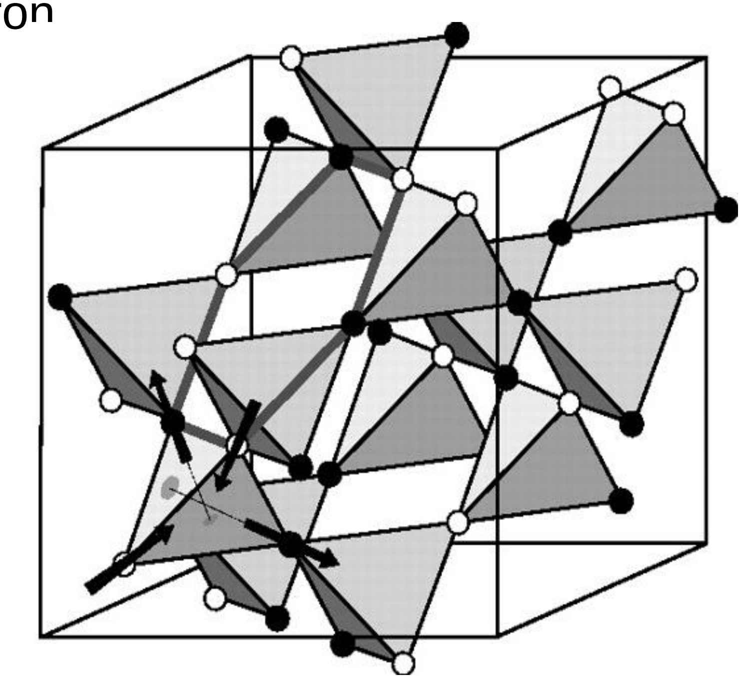
Bramwell and Gingras, Science 2001 + refs. therein

- Pyrochlolre lattice: corner-sharing tetrahedron
- Strong easy axis anisotropy

→ spins must point toward the center of a tetrahedron



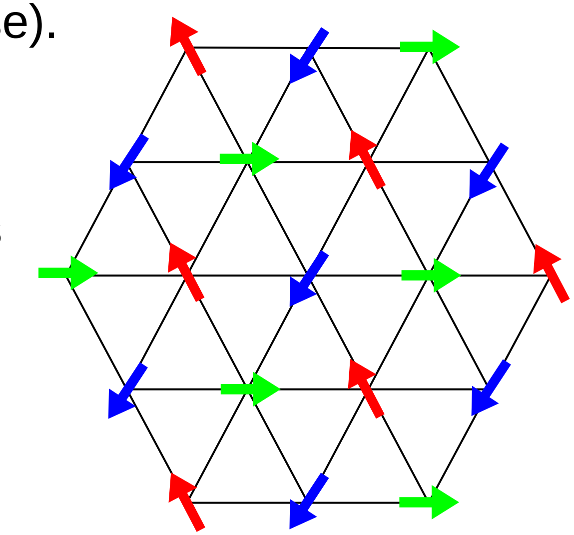
"2 **in** & 2 **out**"
= ice rules (Bernal & Fowler, 1933)



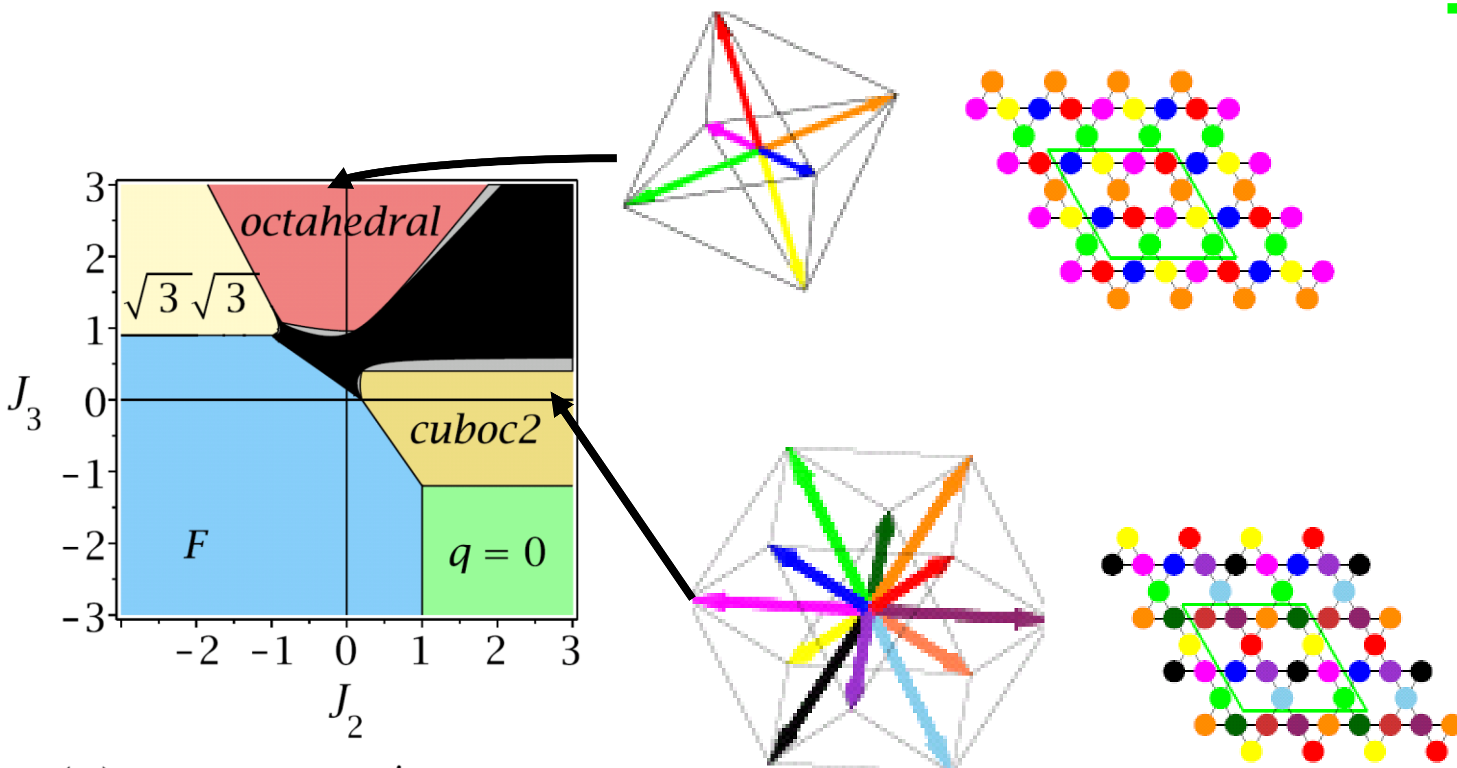
- Examples: Ho_2Ti_2O, $Dy_2Ti_2O_7$, $Ho_2Sn_2O_7$
- Extensive entropy (measured by Ramirez, Nature 1999), S_0~1/2 R ln(3/2)
- Power law spin-spin correlations ($1/r^3$), seen in neutron scattering
- Exotic excitations: "monopoles" (=half spin flips!). Castelnovo *et al.* Nature 2008

Classical O(3) models: non-collinear (and non-planar) structures

- **Triangular** lattice: three sublattice (120° degree) Néel for Heisenberg (or XY) spins. Degeneracy: *global* rotations (different from the Ising case).



- **Kagome** lattice with competing J_1-J_2-J_3-$J_{3'}$ interactions



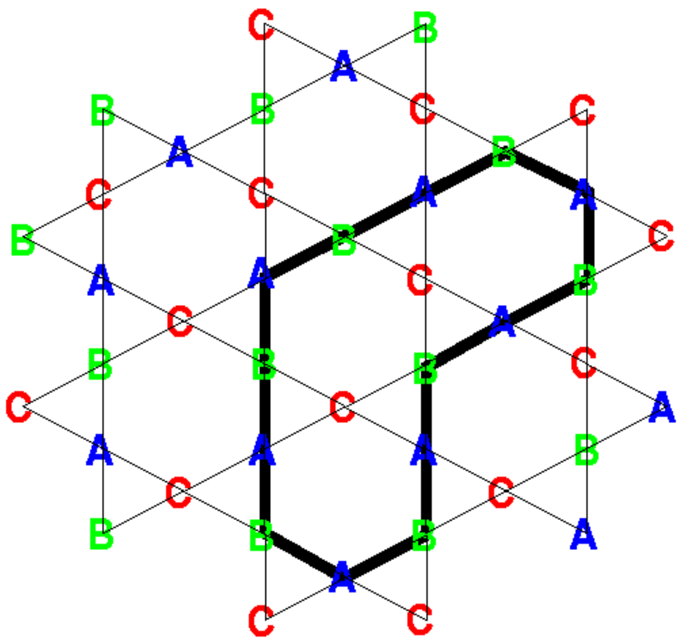
(d) $J_1 = -1,\ J_3' = 0.2$

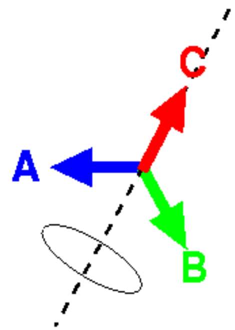
Messio, Lhuillier & GM
arXiv:1101.1212

Classical O(3) model on the kagome lattice & zero modes

- Planar ground-state ↔ three-coloring of the lattice. **“A B C”**

  → exponential number of planar ground-states





A and B can rotate freely about the C axis.

→ non-planar ground-states

→ **Local zero mode**

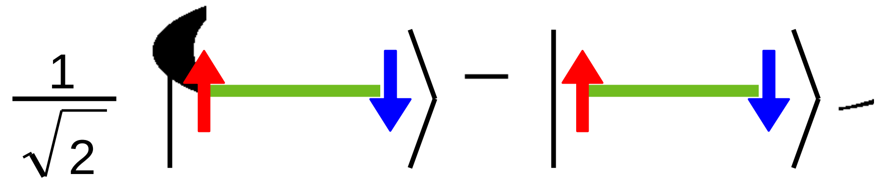
- Physics when T→0 ? Not completely settled !
  Spontaneous selection of a plane. Long range magnetic order in the plane ?

Shender, Holdsworth & Chalker, 1992. Huse & Rutenberg 1992. M. Zhitomirsky 2008. C. Henley 2008.

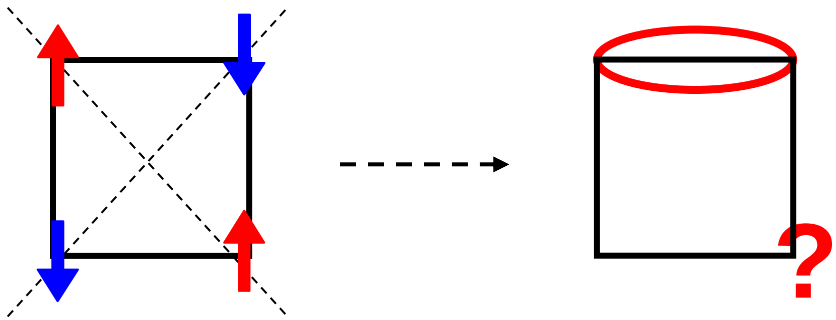
Quantum spin systems

Heisenberg antiferromagnets & quantum frustration

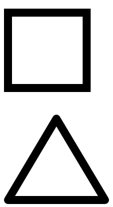
$$H = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Singlet $S_{tot}=0$



Quantum AF
are (almost) always
"frustrated" according to
the (classical) definition !



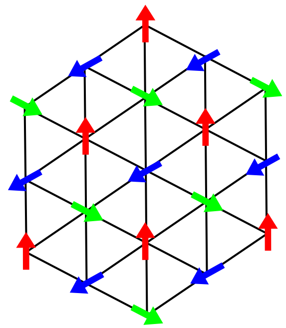
- **Sign problem** in quantum Monte Carlo
  Square (bipartite) → easy to simulate numerically
  Triangular (non-bipartite) → very hard to simulate (almost impossible in QMC)

- Other methods: exact diagonalizations (small number of spins ≤ 42)
  DMRG, and tensor networks methods ?

Spin wave theory & magnetic (Néel) long range order

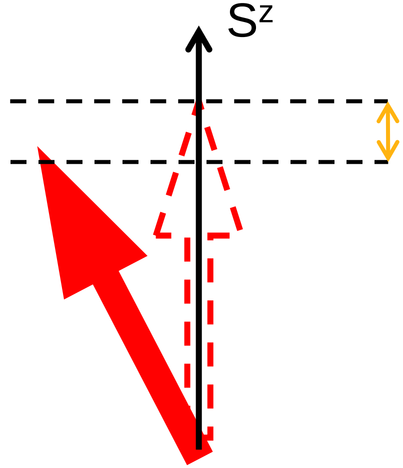
$$H = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

- Start from the **classical ground-state**



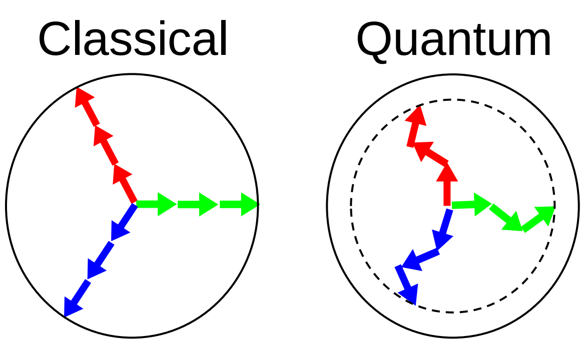
- Assume **small deviations**

  (should be ok for large enough S)



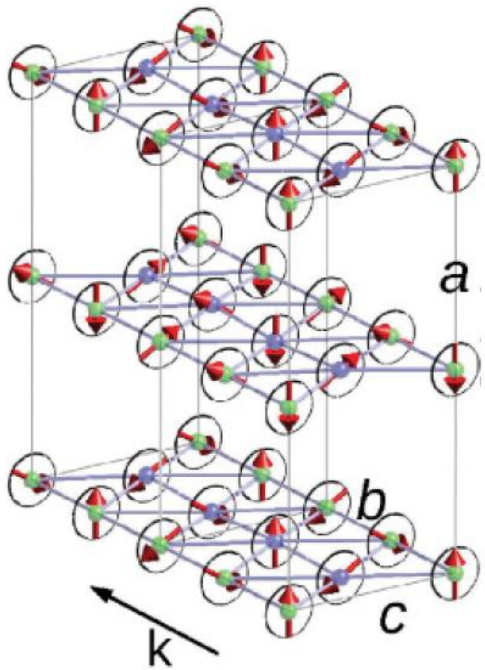
$$\langle \delta S_i^z \rangle = \langle a_i^+ a_i \rangle \ll S$$

- Treat these deviations as (coupled) **quantum harmonic oscillators**
- **Ground-state ≈ classical ground-state + zero-point fluctuations**

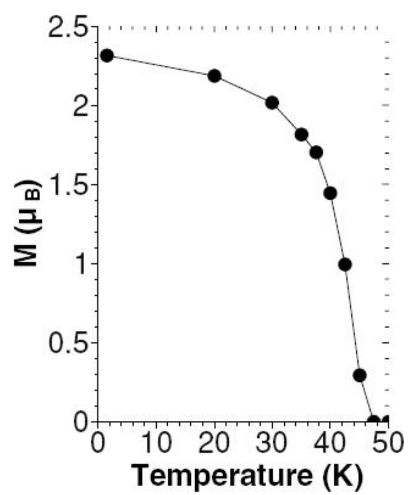


Example of the triangular AF Heisenberg model
Linear spin-wave calculation: Jolicoeur & Le Guillou, 1989
Exact diagonalization numerics for S=1/2:
Huse & Elser 1988, Bernu, Lhuillier & Pierre, 1992.

Triangular quantum AF: an example



α-$CaCr_2O_4$



Chapon *et al.*, Phys. Rev. B 2011

Do quantum antiferromagnets always order at low temperature ?

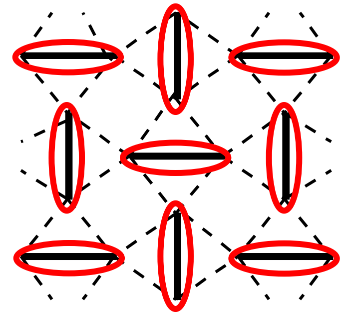
No !

"Spin gap" materials

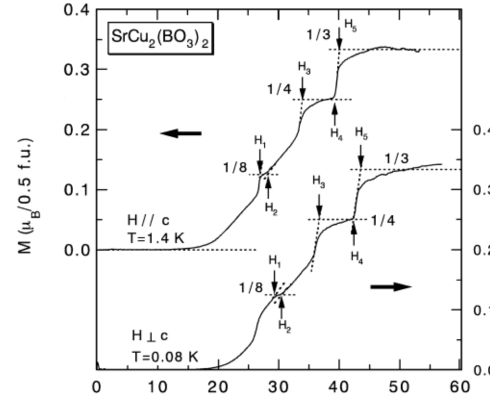
- $SrCu_2(BO_3)_2$

J ≈ 100 K

J' ≈ 68 K



Kageyama *et al. (1999)*



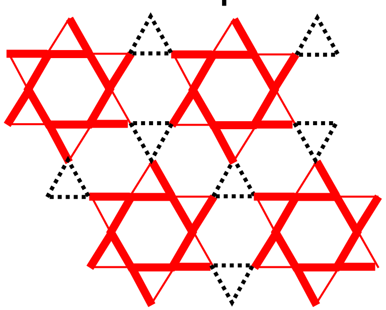
Magnetization plateaux
Onizuka *et al.* 2000

- Ground-state is (almost) a product of uncorrelated singlets

Shastry-Sutherland (1981)

 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$ S=0 singlet

- Example with a bigger unit cell



example with12 sites/cell:
$Rb_2Cu_3SnF_{12}$
Morita *et al.*, 2008; Ono et al., 2009;
Yang & Kim, 2009

- **Theoretical picture: weakly coupled units. Possible description of the wave-function in perturbation theory.**

Related phase: "valence bond crystals" with spontaneous lattice symmetry breaking.

So far:

1) magnetic long-range order or

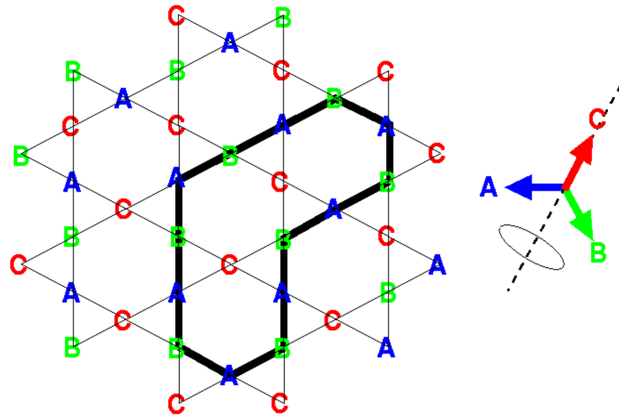
2) “weakly-coupled-singlets” ground-states

… is that all ?

No !

Spin-1/2 kagome Heisenberg model

- Many soft modes

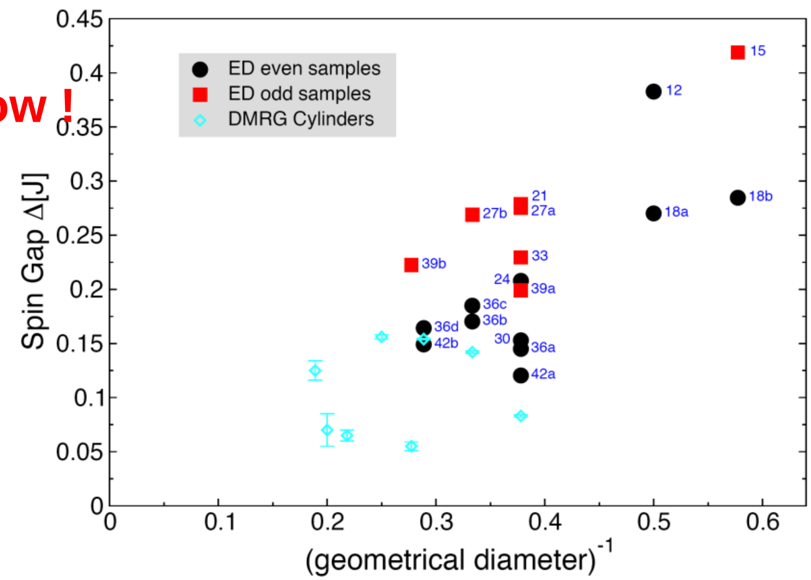


→ Break-down of the spin wave theory (at least the most naïve one) because of the divergent zero-point motion of the harmonic oscillators.

- Nature of the ground-state ?

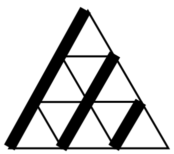
1989 (Elser) → 2011: **we still do not know !**

Numerics: no magnetic LRO at T=0.
and probably no order at all.
Diagonalization 42 spins
Lauchli *et al.*, arXiv:1103.1159
DMRG: Yan *at al.*, arXiv:1011.6114

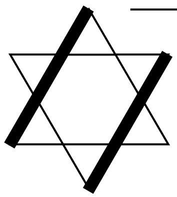


- What about experiments ?

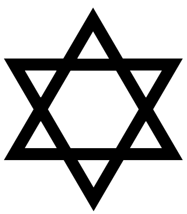
Some spin liquid candidates (no long range order detected down to T<<J)



❑ Triangular organics
κ-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$ — Shimizu *et al.* 2003
EtMe$_3$Sb[Pd(dmit))$_2$]$_2$ — Itou *et al.* 2008
Proximity to the Mott transition → charge fluctuations → ring exchange interactions ?
Spinon Fermi surface or other gapless QSL ? — Motrunich 2005; Lee & Lee 2005; Kyung & Tremblay 2006; Tocchio *et al.* 2009
Quantum critical point in EtMe$_3$Sb[Pd(dmit))$_2$]$_2$?

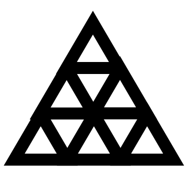


❑ Volborthite Cu$_3$V$_2$O$_7$(OH)$_2$+2H$_2$O — Hiroi *et al*. 2001; Bert *et al.*, 2005; Yoshida *et al.*; 2009.
low impurity concentration
but possible magnetic order (freezing?) at very low-T (T/J~1/100).
distorted kagome lattice (and maybe not kagome at all: Jason *et al.* 2010)

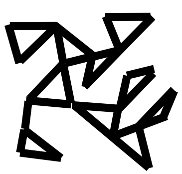


❑ Herbertsmithite ZnCu$_3$(OH)$_6$Cl$_2$ — Helton *et al.* 2007, Mendels *et al.* 2007, Bert *et al.* 2007, Ofer *et al.* 2007, Imai *et al.* 2007, Olariu *et al.* 2008
undistorted kagome lattice, looks critical ?
but many missing spins in the kagome planes (Cu→Zn) ☹.

❑ Vesignieite BaCu$_3$V$_2$O$_8$(OH)$_2$. — Okamoto *et al.* 2009
almost undistorted kagome lattice, no antisite disorder (but some mag. Impurities ~7%).



❑ He^3 films (Nuclear magnetism) — Ikegami *et al.* 2000, Masutomi *et al.* 2004
Ring exchange interactions (but not precisely known).
Experiment are very difficult (J~mK).



❑ "Hyper kagome" Na$_4$Ir$_3$O$_8$ (3d) — Okamoto *et al.* 2007
Gapless QSL ? Lawler *et al.* 2008; Zhou *et al.* 2008 (spinon Fermi surface ?)
some anisotropies ? Chen & Balents 2008

ALL SEEM GAPLESS !

Why are theoreticians so excited about systems which are just “disordered” ?

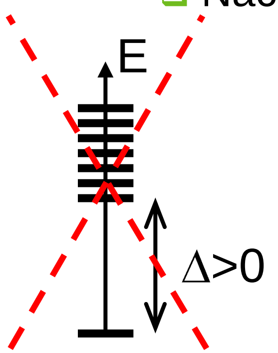
Lieb-Schultz-Mattis-Hastings theorem

The absence of order (=spontaneously broken symmetry) is a very interesting situation if the spin is half-odd integer spin per unit cell (=Mott insulator) !

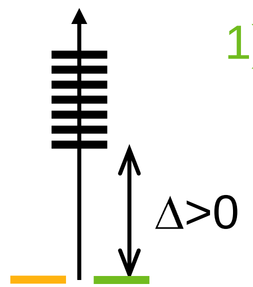
Lieb-Schultz-Mattis Theorem for spin chains (d=1), 1961

Recent proof valid in any dimension (d>1) :

- Hastings 2004 — See also Affleck 1988; Bonesteel 1989; Oshikawa 2000
- Nachtergaele & Sims, Com. Math. Phys. 276, 437 (2007).



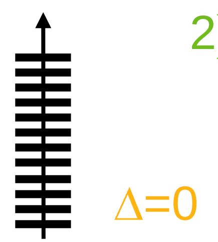
Conditions:
- half-odd-integer spin in the unit cell
- short-range interactions
- Global U(1) symmetry: $[S^z_{tot},H]=0$
- dimensions L_1 L_2 … L_D with L_2 … L_d=odd
- periodic bound. conditions in direction 1
- thermodynamic limit



1) Ground-state degeneracy

a- “Conventional” broken symmetry

b- Topological degeneracy



2) Gapless spectrum

a- Continuous broken sym. (Néel order)

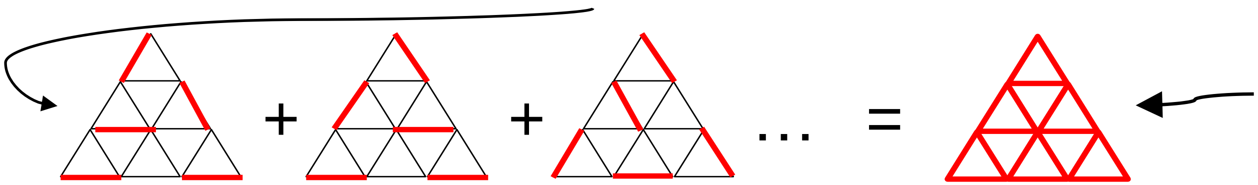
b- Critical phase (or crit. point)

spin liquids with **fractional** excitations (s=½ spinons)

Gapped quantum spin liquids & short-range RVB picture

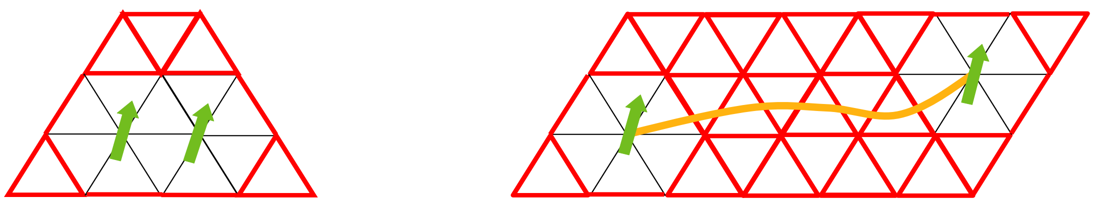
- P. W. Anderson's idea (1973) : (short-ranged) resonating valence-bond (RVB)

Linear superposition of many (exponential) low-energy short-range valence-bond configurations



Spatially *uniform* state

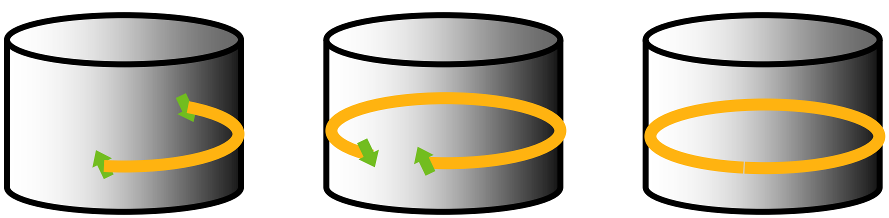
- Magnetic excitations ? Spinon with spin S=½



Solvable models realizing this idea no the kagome lattice:
GM, Pasquier & Serban 2002

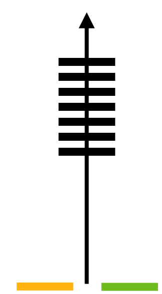
Seidel 2009

- Topological degeneracy & spinon fractionalization



adiabatic process → New ground-state

Topological degeneracy
(X. G. Wen 1991) ↔
fractionalization
See also Oshikawa & Senthil 2006



2-fold degeneracy
⇒ Satisfies LSMH

A starting point to describe QSL: large-N (mean-field) limit

Review: Lee, Nagaosa & Wen, Rev. Mod. Phys. 78, 17 (2006)

- Express the spin operators using *fermions*

$$S^z = \tfrac{1}{2}\left(c_\uparrow^+ c_\uparrow - c_\downarrow^+ c_\downarrow\right)$$

$$S^+ = c_\uparrow^+ c_\downarrow \qquad S^- = c_\downarrow^+ c_\uparrow$$

$$c_\uparrow^+ c_\uparrow + c_\downarrow^+ c_\downarrow = 1$$

$c_\uparrow^+$ (or $c_\downarrow^+$) changes S^z by $+\tfrac{1}{2}$ $(-\tfrac{1}{2})$

creates a "spinon"

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j = \sum_{\langle ij \rangle} J_{ij} \left(\ldots\right) c_{i\mu}^+ c_{i\nu} c_{j\rho}^+ c_{j\sigma}$$

- Mean-field decoupling → quadratic Hamiltonian (solvable)
  + self consistency conditions

$$H_{MF} = \sum_{\langle ij \rangle} \chi_{ij} \left(c_{i\downarrow}^+ c_{j\downarrow} + c_{i\uparrow}^+ c_{j\uparrow}\right) + \eta_{ij} \left(c_{i\uparrow} c_{j\downarrow} - c_{i\downarrow} c_{j\uparrow}\right) + H.c$$

Spinon "hopping" (χ_{ij}) Spinon "pairing" (η_{ij})

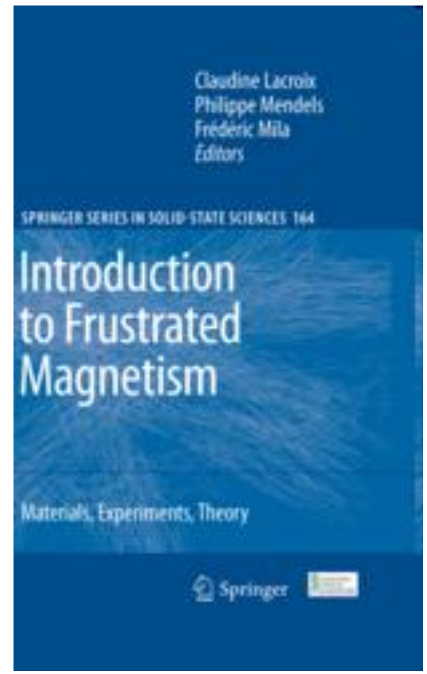
- Beyond mean-field ?
  Fluctuations ? Spinon confinement ? Stability of gapless phases ?

Summary

- Rich collective phenomena can emerge in frustrated models
- Gapless spin liquids have recently been observed
- A lot remains to be understood on the theoretical side

Some references :

Introduction to frustrated magnetism
C. Lacroix, Ph. Mendels & F. Mila (Eds), Springer 2011



"Quantum spin liquids and fractionalization" GM (in the book above)
"Quantum Spin Liquids", GM, arXiv:0809.2257 (Les Houches lecture notes)