Recent progress in the search (and theories) for quantum spin liquids ground-states

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Due to air transport problems…☹ this talk has *not* been given at the conference
« Perspective in highly frustrated magnetism », MPI-PKS Dresden, Germany, April 2010.

Everything you always wanted to know about spin liquids
(but were afraid to ask)

- What is a Quantum Spin Liquid (QSL) ?
  - Is there a definition ?
  - How to decide if a given wave-function is a QSL or not ?
  - How to decide if a given Hamiltonian has a QSL ground-state ?

- What are the different types of QSL ?
  - Is a “classification” available ?

- What are the simplest *models* with a QSL ground- state ?
  - exactly solvable examples?

- Do QSL exist in *nature* ?

Mostly focus on:

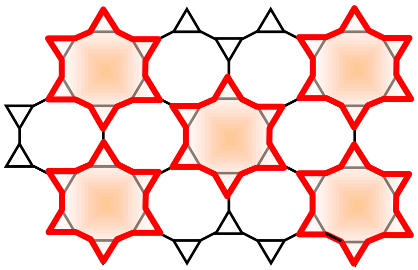
- 2d regular lattices
- Spin S=½
- SU(2) symmetry
- T=0
- Period 2005-2010

What is a (quantum) spin liquid ?

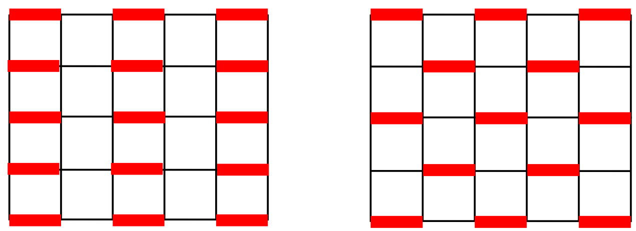
What is a quantum spin liquid ? Definition n°1

A spin liquid is a state without any spontaneously broken symmetry

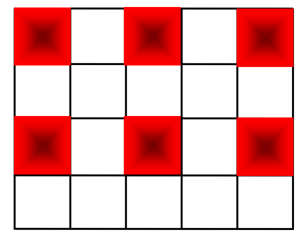
- excludes Néél phases (LRO) & spin nematics,
- excludes valence-bond crystals (also called V.-B. solids).



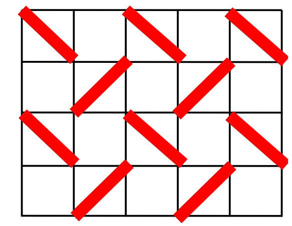
GM & Sindzingre 2007



Heisenberg model & 4-spin “ring” exchange
Läuchli *et al*. 2005

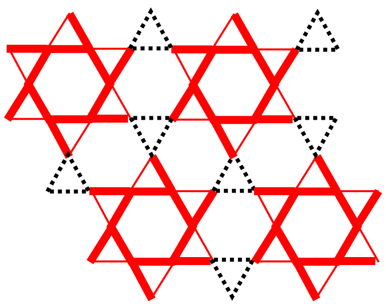


J_1-J_2-J_3 model
Mambrini *et al*., 2006



Gellé *et al.* 2007

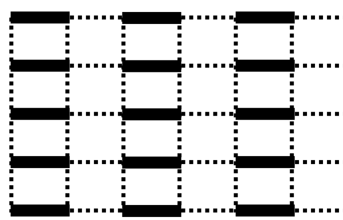
Kagome a VBC ? Singh & Huse 2007; Evenbly & Vidal 2009



example with12 sites/cell:
$Rb_2Cu_3SnF_{12}$
Morita *et al*., 2008; Ono et al., 2009;
Yang & Kim, 2009

- allows “quantum paramagnets”

integer spin/cell, and (possibly) no phase transition from T=0 to T=∞ ☹
Example: coupled dimer systems, like $TlCuCl_3$, $SrCu_2(BO_3)_2$, …



Strong explicit dimerization
J(▬▬) >> J’(··········)
→ simple unique & gapped ground-state

What is a quantum spin liquid ? Definition n°2

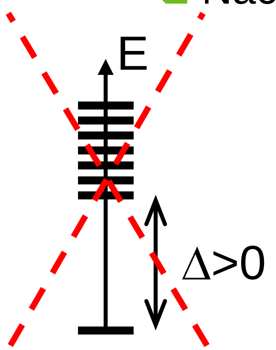
A spin liquid is a state without any spontaneously broken symmetry and a *half-odd integer spin per unit cell* *(=Mott insulator).*

→ excludes "quantum paramagnets"

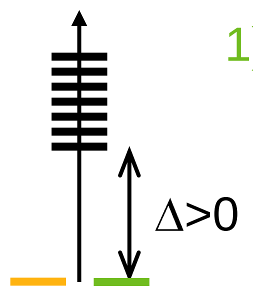
Lieb-Schultz-Mattis Theorem for spin chains (d=1), 1961

Recent proof valid in any dimension (d>1) :

- Hastings 2004 — See also Affleck 1988; Bonesteel 1989; Oshikawa 2000
- Nachtergaele & Sims, Com. Math. Phys. 276, 437 (2007).



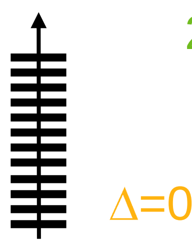
Conditions:
- half-odd-integer spin in the unit cell
- short-range interactions
- Global U(1) symmetry: $[S^z_{tot}, H]=0$
- dimensions L_1 L_2 … L_D with L_2 … L_d=odd
- periodic bound. conditions in direction 1
- thermodynamic limit



1) Ground-state degeneracy

a- "Conventional" broken symmetry

b- Topological degeneracy



2) Gapless spectrum

a- Continuous broken sym. (Néel order)

b- Critical phase (or crit. point)

spin liquids with **fractional** excitations (s=½ spinons)

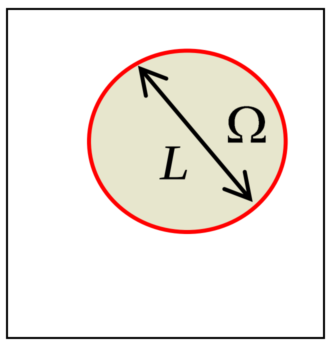
How to decide if a given wave-function is a QSL ?

- Scrutinize the wave-functions using reduced density matrices, to look for possible orderings (or absence thereof).

Correlation density matrix method : Cheong & Henley 2009
Application to kagome: Sudan & Lauchli (Orsay kagome workshop 2010, unpublished oral communication)
Another approach: Furukawa, GM & Oshikawa 2006, 2007

- Gapped QSL are characterized by some form of long-ranged entanglement.

→*Topological entanglement entropy* Levin & Wen, 2006; Kitaev & Preskill, 2006



$$S^{\Omega}_{\text{Von Neuman}}(L) \approx \alpha L + \gamma$$

γ is related to the nature of the elementary excitations in the liquid (*total quantum dimension*).
Example for a Z_2 liquid (short-ranged RVB): $\gamma = -\ln(2)$
Numerical check in a toy model [quantum dimer/RVB wave function]: Furukawa & GM 2007

How to decide if a given Hamilt. has a QSL ground-state ?

- a very hard problem…!

- Recent progress in numerical methods for frustrated quantum spin models :
  - Variationnal Monte-Carlo (→ F. Becca's talk)
  - Variationnal study in the RVB subspace (→ D. Poilblanc's talk)

  - Density Matrix Renormalization Group (DMRG) for 2D frustrated spin systems (108 sites kagome: Jiang, Weng & Sheng, 2008)
  - Tensor product representations of the many-body wave-functions. MERA & kagome : Evenbly & Vidal 2009

What are the different types of QSL ?

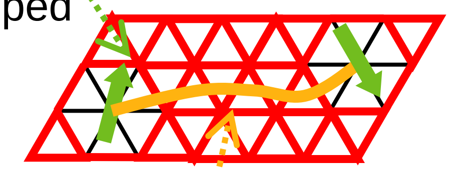
Classification of QSL phases

Hundreds of phases have already been indentified !

Lee, Nagaosa & Wen, Rev. Mod. Phys. 2006

- Elementary excitations

Different QSL can have different elementary excitations: spinons (gapped or gapless), visons, non-Abelian quasiparticles, no quasiparticle at all…



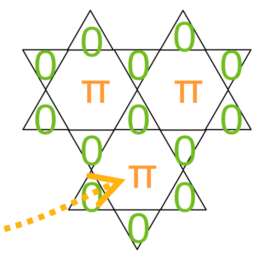
- Emergent gauge fields

The (fractionalized) quasiparticles (necessarily) interact through a gauge field, and the corresponding gauge group (Z_2, U(1) or even SU(2)) is an intrinsic property of the QSL phase.

- Classical versus topological/quantum order (X.-G. Wen)

Conventional phases are classified according to their spontaneously broken symmetries

… and gapped QSL are classified according to their “topological order”. It is often related to some “background flux” experienced by the quasiparticles.



The concept of “projective symmetry group” (*PSG*, Wen 2002) has become a practical tool to investigate QSL phases of spin models.

Examples: Ran *et al.* 2007; Wang & Vishwanath 2006; Messio, Cepas & Lhuillier 2010; Choy & Kim 2009 ; …

- Gapless QSL: the “classification” is far from complete…

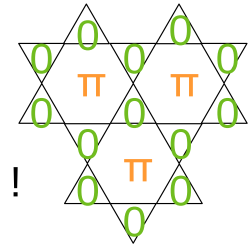
Two "prototypes" of 2d quantum spin liquids

Anderson 1973;
Read & Chakraborty 1989; Read & Sachdev 1991

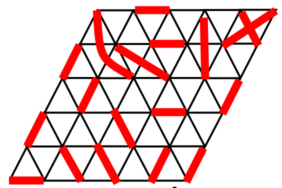
- Z_2 QSL
- Gapped magnetic excitations with a spin ½ (spinon). Gapped non-magnetic excitations (vison).
  $Cv \sim e^{-\Delta/T}$; $\chi \sim e^{-\Delta'/T}$

Rantner & Wen 2002; Hermele *et al.* 2004;
Hastings 2000; Ran *et al.* 2007; Hermele *et al.* 2008

- Algebraic/Critical/Dirac/U(1) QSL
- Gapless excitations.
  Magnetic ones with a spin ½.
  No "sharp" (=long lived) quasiparticle !
  $C_v \sim T^2$; $\chi \sim T$ (at least in ean field)

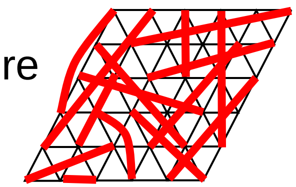


- Short-range RVB picture



- Simple effective theory (Z_2 gauge theory in its deconfined phase, coupled to gapped spinons)

- Long-range RVB picture



- Complex effective theory: multi-flavour Dirac fermions coupled to a U(1) gauge field.

- now many models !

- No "proved" microscopic realization so far
  (except in mean field / Large-N)
  The spin-½ Heisenberg model on the kagome lattice is a candidate".

- No experimental "candidate" so far
  (to my knowledge)

- Herbertsmithite ?

Many other families of QSL exists (at least theoretically!) : chiral, non-Abelian, critical vortex liquids, spinon Fermi surface, Coulomb liquids in 3d, …

GM, "Les Houches" lectures notes, arXiv '0809.2257

Do spin liquids exist ?

(as ground-states of simple *models*)

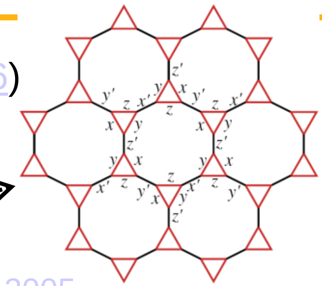
Some examples of *gapped* QSL (2d)

Exaclty solvable

- Kitaev's models ("toric code" 1997; honeycomb-lattice model 2006)

  See also Wen 2003; ...

  Non-Abelian chiral spin liquid Yao & Kivelson 2007

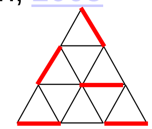


- Zoo of gapped topological liquids: *string-nets* Levin & Wen, 2005

Z_2 liquids

- Quantum dimer models

  =toy models to describe the short-range RVB physics



Triangular & kagome lattices: Moessner & Sondhi 2001; GM, Serban & Pasquier 2002; Ralko *et al.* 2005, 2006, 2007; Vernay *et al.* 2006; Ribeiro *et al.* 2007; Furukawa & GM 2007; Lauchli *et al.* 2008; GM & Mila 2008: Poilblanc *et al.* 2009; Schwandt *et al.* 2010.

Spin models with rotation symmetry

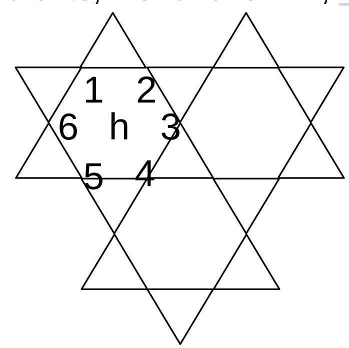
- Easy-axis kagome model

Balents, Fisher & Girvin, 2002; Sheng & Balents, 2005

$$H = J_z \sum_{h \text{ hexagon}} \left(S^z_{h1} + \cdots + S^z_{h6}\right)^2$$

$$+ J_\perp \sum_{h \text{ hexagon}} \left(S^x_{h1} + \cdots + S^x_{h6}\right)^2 + \left(S^y_{h1} + \cdots + S^y_{h6}\right)^2$$

$$J_\perp << J_z$$



- SU(2) symmetric *spin* models (inspired from quantum dimer models)

  Raman-Moessner-Sondhi, 2005 (decorated lattices)

  Seidel 2009 (kagome lattice model with (up to) 12-spin interactions)

Some examples of *gapless* QSL models

Some candidates in 2d, but no definite conclusion so far …

- J_1-J_2 model on the square lattice ? Becca *et al.,* Trieste Lecture notes arxiv.905.4854
- Kagome ?

  Ran *et al.* 2007; Hermele *et al.;* 2008;
  Cepas *et al.* 2008; Sindzingre & Lhuillier 2009
- Ring exchange on the triangular lattice ? LiMing *et al.* 2000; Motrunich 2005

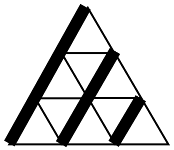
- Some realizations of a 3d-Coulomb QSL :

  Hermele, Fisher & Balents 2004, Raman, Moessner & Sondhi 2005

  (+ several other examples with bosons, dimers, …)

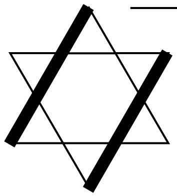
Do spin liquids exist in nature ?

List of suspects (no long range order detected down to T<<J)



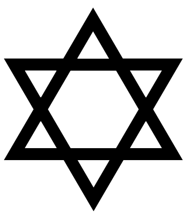
- Triangular organics
  κ-$(BEDT\text{-}TTF)_2Cu_2(CN)_3$ — Shimizu *et al.* 2003
  $EtMe_3Sb[Pd(dmit)_2]_2$ — Itou *et al.* 2008

Proximity to the Mott transition → charge fluctuations → ring exchange interactions ?
Spinon Fermi surface or other gapless QSL ? Motrunich 2005; Lee & Lee 2005; Kyung & Tremblay 2006; Tocchio *et al.* 2009
Quantum critical point in $EtMe_3Sb[Pd(dmit)_2]_2$?



- Volborthite $Cu_3V_2O_7(OH)_2+2H_2O$ — Hiroi *et al*. 2001; Bert *et al.*, 2005; Yoshida *et al.*; 2009.

low impurity concentration
but possible magnetic order (freezing?) at very low-T (T/J~1/100).
distorted kagome lattice (and maybe not kagome at all: Jason *et al.* 2010)

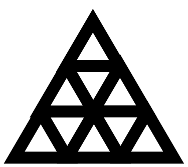


- Herbertsmithite $ZnCu_3(OH)_6Cl_2$ — Helton *et al.* 2007, Mendels *et al.* 2007, Bert *et al.* 2007, Ofer *et al.* 2007, Imai *et al.* 2007, Olariu *et al.* 2008

undistorted kagome lattice, looks critical ?
but many missing spins in the kagome planes (Cu→Zn) ☹.

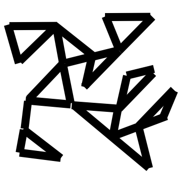
- Vesignieite BaCu3V2O8(OH)2. — Okamoto *et al.* 2009

almost undistorted kagome lattice, no antisite disorder (but some mag. Impurities ~7%).



- He^3 films (Nuclear magnetism) — Ikegami *et al.* 2000, Masutomi *et al.* 2004

Ring exchange interactions (but not precisely known).
Experiment are very difficult (J~mK).



- "Hyper kagome" $Na_4Ir_3O_8$ (3d) — Okamoto *et al.* 2007

Gapless QSL ? Lawler *et al.* 2008; Zhou *et al.* 2008 (spinon Fermi surface ?)
some anisotropies ? Chen & Balents 2008

ALL SEEM GAPLESS !

Conclusions

- Gapped QSL (Z_2/short-ranged RVB in particular) are now rather well understood theoretically, but … not experimental candidate so far (why ?).

- Gapless QSL are more difficult to treat theoretically… and there are several experimental candidates !

- New experimental probes to detect QSL ?
  "smoking gun experiment" ?

  - 2-spinon continuum in inelastic neutron scattering (Cs_2CuCl_4, Coldea *et al.* 2003)
  - $2k_F$-like oscillations in presence of spinon Fermi surface Norman & Miklitz 2009
  - Thermal Hall effect (sensitive to deconfined spinons) Katsura Nagaosa & Lee 2010